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An Affine Arithmetic Applied for Model Order Reduction on Discrete Uncertain Systems

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ABSTRACT

To reduce the order of discrete-uncertain systems of higher order, a modified polynomial derivative approach is put forward in this study. In order to derive denominator and numerator of lower order model by means of affine arithmetic, a modified polynomial derivative stability array is built. To assess the respectivenumerator of the reduced order model and denominator of the reduced order model, Modified Polynomial Derivatives are used. The Calculation is based on the theories of Dolgin&Zehab [14]. In order to effectivelyassess the of the suggested strategy, it has been applied to two typical numerical examples. Additionally, it has been shown that the newer approach has a lower integral square error and offers a better approximation to the initial higher order system.

Keywords:Model Order Reduction;Discrete Uncertain Systems;Affine arithmetic; Modified Partial Differentiation;

1. INTRODUCTION

Low order models are often used to analyse and create complex systems, which are well acknowledged. The literature provides a number of approaches for scaling down large continuous time fixed systems. However, it is only possible to reduce discrete time fixed systems using a few numbers of approaches [1, 2, 3–11].

In the literature, there haven't been many approaches to reducing large-discrete time interval systems, which is true. Recent approaches involving model order reduction for large-discrete type interval systems were presented by Bandyopadhyay et al. [12, 13, 14-16]. New concepts are proposed to improve the computation and output of the methods described below that may be found in the literature once they have been studied.

2. PROBLEM FORMULATION

These significant flaws and restrictions affect the approaches suggested by Bandyopadhyay and Ismail that are addressed [13]. Because this approach calls for the differentiation of the system's nth order polynomials, it is computationally laborious. The approach sometimes generates unstable lower-order interval based models for stable original interval based discrete system of high order [13, 15], which is a serious drawback. To get denominator and numerator of the models with reduced order, a comparisonofthe initial 't' interval time instants of the reduced model and'm' Markov parameters from reduced model pared to those from the original uncertain model is done. This is done so that t+m = 2r. For this process, it is necessary to determine the time-instants of the original high order system G(z)and Markov parameters from the original high order system G(z), as well as those of the previously reduced rth order. This method has the ability to provide innovative, stable high order systems unstable models [13, 16]. With this method, the system's initial 't' interval time instants and'm' Markov parameters is matched with model that is resulting in t + m = r, and the denominator is obtained by maintaining the dominating poles of G. (z). The time basedinstants and Markov based parameters of the original discrete time system of higher order must be determined before using this approach. To determine the dominating poles in advance, the original system's denominator must be factorized [12]. The r-th order reduced model is created in a way that Rr(z) approximates G(z) to within 2r points using the Pade method. Transmutation that is reciprocal is necessary. Interval Routh type arrays must be created. It is necessary to identify the high order interval system's roots. A set involving 2r number of linear interval based equations must be resolved in order to produce a rth order reduced model [14].

The first "r" number of interval time instants from the model are to be matched with the first "r" interval time instants of the original system with high order to produce the numerator polynomial, and the dominant poles from the given discrete interval based system G(z) are used to generate the r-th order denominator Dr(z) of the reduced model. This approach is computationally expensive since the dominant poles must be determined from the denominator polynomial of the original system's denominator. The determination of the dominating poles is required for this procedure in advance. The essential goal and meaning of modelling are lost after a system's poles have been determined since its stability may then be evaluated directly. Similar to this, additional computational work is needed since the interval time based instants of the original system with higher order must be determined beforehand.

Here we offer Modified Polynomial Derivative Technique, a novel and simple approach for order reduction of discretetime based interval systems of higher order, so as to overcome the drawbacks and limits discussed before. It is not dependent on long Routh type arrays, reciprocal transformations, Pade type arrays, or time-ahead calculations. The recommended method consistently yields "r-1" initial interval time moments in its reduced order model, improving the time response approximation, and low order interval models that are stable and unique.

In this study, the previously created modified polynomial derivative method used for continuous and time based interval systems is expanded to the model reduction fordiscrete-time interval systemshigher order. The suggested technique is to be used for generating the reduced order models for the initial discrete time interval systems of higher order using the following procedural procedures.

3. PROPOSED METHODOLOGY

Give the initial high order discrete time interval based system the following form:

$$G_n(z) = \frac{N_m(z)}{D_n(z)} = \frac{[E_m^-, E_m^+]\, z^m + \ldots\ldots + [E_1^-, E_1^+]\, z + [E_0^-, E_0^+]}{[D_n^-, D_n^+]\, z^n + \ldots\ldots\ldots .+ [D_1^-, D_1^+]\, z + [D_0^-, D_0^+]} \quad \ldots\ldots(1)$$

A low order model is advised to be obtained for the original discrete time interval system of higher order discussed above.

This is the definition for the suggested reduced order model with order "r" (r n):

$$R_r(z) = \frac{n(z)}{d_r(z)} = \frac{[e_{r-1}^-, e_{r-1}^+]\, z^{r-1} + \ldots.. + [e_1^-, e_1^+]\, z + [e_0^-, e_0^+]}{[d_r^-, d_r^+]\, z^r + [d_{r-1}^-, d_{r-1}^+]\, z^{r-1} + \ldots.. + [d_1^-, d_1^+]\, z + [d_0^-, d_0^+]}$$

……(2)

Reduction procedure:

Step1: The original high order of transfer function from the p-domain will be derived as follows by applying a linear transformation, z = (p+1):

$$G_n(z)\Big|_{z=(p+1)} = G_n(p) = \frac{N_m(p)}{D_n(p)} \quad \ldots\ldots(4.37)$$

$$G_n(p) = \frac{[B_m^-, B_m^+]\, p^m + [B_{m-1}^-, B_{m-1}^+]\, p^{m-1} + \ldots.. + [B_1^-, B_1^+]\, p + [B_0^-, B_0^+]}{[A_n^-, A_n^+]\, p^n + [A_{n-1}^-, A_{n-1}^+]\, p^{n-1} + \ldots\ldots\ldots + [A_1^-, A_1^+]\, p + [A_0^-, A_0^+]}$$

Step2:

Using the suggested procedure, generate the model $R_r(p)$ of reduced order for aforementioned higher order system, $G_n(p)$, as shown below.

$$R_r(p)=\frac{n(p)}{d_r(p)}=\frac{[b_{r-1}^-,b_{r-1}^+]\,p^{r-1}+\ldots\ldots+[b_1^-,b_1^+]\,p+[b_0^-,b_0^+]}{[a_r^-,a_r^+]\,p^r+[a_{r-1}^-,a_{r-1}^+]\,p^{r-1}+\ldots\ldots\ldots+[a_1^-,a_1^+]\,p+[a_0^-,a_0^+]}$$

…(3)

Step3: *Reduced order denominator, $d_r(p)$:*

The following novel procedures are suggested to produce the reduced order model's low order denominator polynomials, $d_r(p)$ ($r <= n$):

For r = 1, $d_1(p)=\left(\frac{{}^{1}K_1}{{}^{2}K_1}\right)[A_1^-,A_1^+]\,p+\left(\frac{{}^{2}K_1}{{}^{2}K_1}\right)[A_0^-,A_0^+]$

For r = 2,

$$d_2(p)=\left(\frac{{}^{1}K_1}{{}^{3}K_1}\right)[A_2^-,A_2^+]\,p^2+\left(\frac{{}^{2}K_1}{{}^{3}K_1}\right)[A_1^-,A_1^+]\,p+\left(\frac{{}^{3}K_1}{{}^{3}K_1}\right)[A_0^-,A_0^+]$$

… … ….

… … ….

and in general,

$$d_r(p)=\left(\frac{{}^{n-r}K_{n-r}}{{}^{n}K_{n-r}}\right)[A_r^-,A_r^+]\,p^r+\sum_{j=1}^{r}\left(\frac{{}^{n-j+1}K_{n-r}}{{}^{n}K_{n-r}}\right)[A_{j-1}^-,A_{j-1}^+]\,p^{j-1}$$

…(4)

where, ${}^{P}K_Q=\frac{P\,!}{P\,!\,(P-Q)!}$ and ${}^{P}K_Q=1$.

Step4: *Reduced Order Numerator, $n_r(p)$:*

New techniques have been introduced for the derivation of the numerator polynomials with the first interval Markov parameter and (r-1) first interval time based moments of the initial interval system.

For $r=1$, $n_1(p)=[b_0^-,b_0^+]$

For $r=2$, $n_2(p)=[b_1^-,b_1^+]\,p+[b_0^-,b_0^+]$

For $r=3$, $n_3(p)=[b_2^-,b_2^+]\,p^2+[b_1^-,b_1^+]\,p+[b_0^-,b_0^+]$

…. … …..

…. … …..

and in general,

$$n_r(p)=[b_{r-1}^-,b_{r-1}^+]\,p^{r-1}+\ldots\ldots+[b_1^-,b_1^+]\,p+[b_0^-,b_0^+]$$

where, $[b_0^-,b_0^+]=\frac{[B_0^-,B_0^+]}{[A_0^-,A_0^+]}[a_0^-,a_0^+]=[B_0^-,B_0^+]$ *Let* $\left([a_0^-,a_0^+]\;/\;[A_0^-,A_0^+]\right)=1;$

$[b_{r-1}^-,b_{r-1}^+]=d\,[a_r^-,a_r^+];$ with $\mathrm{d}=\text{mean of }\frac{[B_m^-,B_m^+]}{[A_n^-,A_n^+]}$ and

for $i=1,2,3,\ldots\ldots\ldots\;\ldots.,\;(r-2)$

$$[b_i^-,b_i^+]=[B_i^-,B_i^+]+\frac{[B_0^-,B_0^+]}{[A_0^-,A_0^+]}\left\{[a_i^-,a_i^+]-[A_i^-,A_i^+]\right\}+\sum_{j=1}^{i-1}(-1)^j\left\{\frac{[A_i^-,A_i^+]}{[A_0^-,A_0^+]}[b_j^-,b_j^+]-\frac{[B_j^-,B_j]}{[B_0^-,B_0^+]}[a_j^-,a_j^+]\right\}$$

.5

Step5:

$R_r(w)$ is then transformed back into z-domain from the linear transformation's inverse, $w = (z - 1)$, i.e.

$$R(z)=R(w)\Big|_{w=(z-1)}$$ ……(6)

By using the suggested strategy as in eqn, this yields the necessary lower order model (6).

4. NUMERICAL EXAMPLES

In order to assess the recommended approach's adaptability and effectiveness, typical numerical instances are considered to apply to it, and the conclusions are successfully proven.

Example 1:

Consider the following stable discrete temporal system of fourth order [17]:

$$G_4(z)=\frac{N(z)}{D_4(z)}$$

$$=\frac{[2.0,2.21]\,z^3-[4.424,4.911]\,z^2+[3.3634,3.7334]\,z-[0.8721,0.968]}{[1.0,1.05]\,z^4-[3.037,3.189]\,z^3+[3.425,3.596]\,z^2-[1.6935,1.7782]\,z+[0.3084,0.3238]}$$

The following procedure is suggested for the above original system in order to produce a 2nd order reduced model:

Use of the suggested approach:

Step1: Convert $G_4(z)$ to $G_4(p)$ using a linear transformation as shown below.

$$G_n(p)=G_n(z)\Big|_{z=(p+1)}=\frac{N_m(p)}{D_n(p)}$$

$$i.e.,\;G_4(p)=\frac{N_3(p)}{D_4(p)}$$

$$=\frac{[2.0,2.21](p+1)^3-[4.424,4.911](p+1)^2+[3.3634,3.7334](p+1)-[0.8721,0.968]}{[1.0,1.05](p+1)^4-[3.037,3.189](p+1)^3+[3.425,3.596](p+1)^2-[1.6935,1.7782](p+1)+[0.3084,0.3238]}$$

$$=\frac{[2.0,2.21]\,p^3+[1.576,1.719]\,p^2+[0.5154,0.5414]\,p+[0.0644,0.0673]}{[1.0,1.05]\,p^4+[0.963,1.011]\,p^3+[0.314,0.329]\,p^2+[0.0455,0.0468]\,p+[0.0026,0.0029]}$$

Step2: In order to get a model R2(p) with lower order for aforementioned system, G4(p), the following proposed SPDT is suggested:

$$R_2(p)=\frac{n(p)}{d_2(p)}=\frac{[b_1^-,b_1^+]\,p+[b_0^-,b_0^+]}{[a_2^-,a_2^+]\,p^2+[a_1^-,a_1^+]\,p+[a_0^-,a_0^+]}$$

Reduced order denominator, d2(p):

$$d_2(p)=[a_2^-,a_2^+]\,p^2+[a_1^-,a_1^+]\,p+[a_0^-,a_0^+]$$

$$=\left(\frac{{}^{2}K_2}{{}^{4}K_2}\right)[A_2^-,A_2^+]\,p^2+\left(\frac{{}^{3}K_2}{{}^{4}K_2}\right)[A_1^-,A_1^+]\,p+[A_0^-,A_0^+]$$

$$with\ ,\quad K_2=\left(\frac{{}^{2}K_2}{{}^{4}K_2}\right)=\frac{1}{6};\quad K_1=\left(\frac{{}^{3}K_2}{{}^{4}K_2}\right)=\frac{1}{2};$$

Thus

$$d_2(p)=[0.0523\ ,0.0548\ ]\,p^2+[0.0228\ ,0.0234\ ]\,p+[0.0026\ ,0.0029\ ]$$

Reduced order numerator, n(p):

$$n_r(p)=[b_1^-,b_1^+]\,p+[b_0^-,b_0^+]$$

$$where\ ,\quad [b_1^-,b_1^+]=d\ [a_2^-,a_2^+];$$

$$\text{with}\quad d=\text{mean of}\ \frac{[B_m^-,B_m^+]}{[A_n^-,A_n^+]}=\text{mean of}\ \frac{[2.0,2.21]}{[1.0,1.05]}=2.0574$$

Thus

$$[b_1^-,b_1^+]=[0.1076\ ,0.1127\ ]\quad and\quad [b_0^-,b_0^+]=[B_0^-,B_0^+]=[0.0644\ ,0.0673\ ]$$

Step3: Transform the reduced order model from the p-domain to the z-domain using the inverse linear transformation, as shown below:

$$R_2(z)=R_2(p)\Big|_{p=(z-1)}=\frac{n(z)}{d_2(z)}$$

$$R_2(z)=\frac{[0.1076\ ,0.1127\ ]\,z-[0.0403\ ,0.0483\ ]}{[0.0523\ ,0.0548\ ]\,z^2-[0.0818\ ,0.0862\ ]\,z+[0.0324\ ,0.034\ ]}$$

(Stable)(Proposed Method)

The step responses from original discrete time interval system withhigher order and 2nd order model developed using the recommended technique is compared in Figures 1a and 1b.

The recommended approach results in stable form and reduced order models for a stable original discrete time interval based system of higher order, as shown in Figures 1a and 1b.

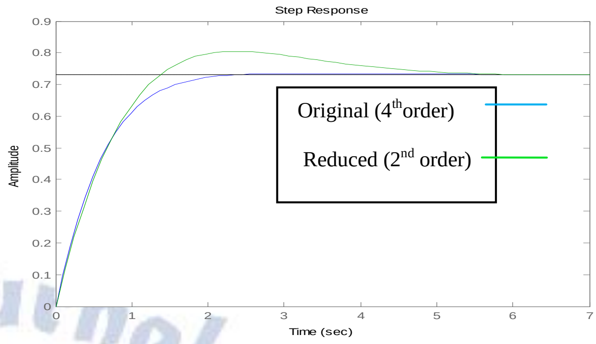

Fig. 1a: Step response of original 4thorderand 2nd order reducedsystem

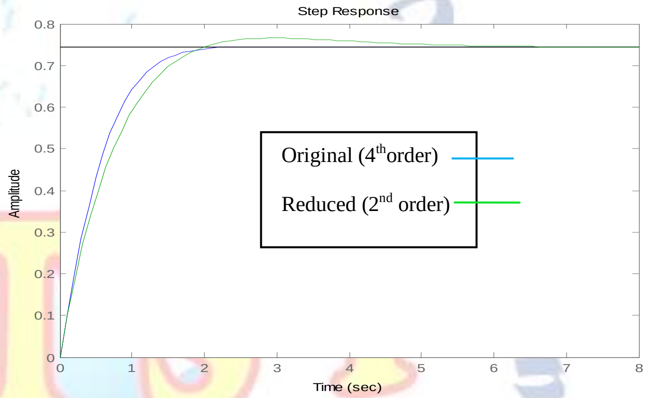

Fig. 1b: Step response of reduced 2nd order and reduced 4th order systems

Example 2:

Consider the following stable 3rd order original discrete time interval system:

$$G_3(z)=\frac{N(z)}{D_3(z)}=\frac{[3.25\ ,3.35\ ]\,z^2+[3.5,3.65\ ]\,z+[2.8,3.0\ ]}{[5.4,5.5]\,z^3+[1.0,1.1]\,z^2+[1.5,1.6]\,z+[2.1,2.15\ ]}$$

The suggested technique yields the following 2nd order reduced model for the aforementioned original 3rd order interval system:

$$R_2(z)=\frac{[3.472\ ,3.553\ ]\,z+[6.078\ ,6.447\ ]}{[5.733\ ,5.867\ ]\,z^2+[1.667\ ,1.799\ ]\,z+[2.6,2.684\ ]}$$

(Using Affine Arithmetic)

The 2nd order model obtained is:

$$R_2'(z)=\frac{[0.4717\ ,0.5998\ ]\,z+[0.9966\ ,1.0075\ ]}{[1.4148\ ,1.4589\ ]\,z^2-[0.8054\ ,0.8674\ ]\,z+[1.0,1.0]}$$

(Using Interval arithmetic)

Step responses from the original discrete time interval system, the 2nd order reduced-models created by recommended technique, and the interval arithmetic method are contrasted in Figures 2a and 2b.

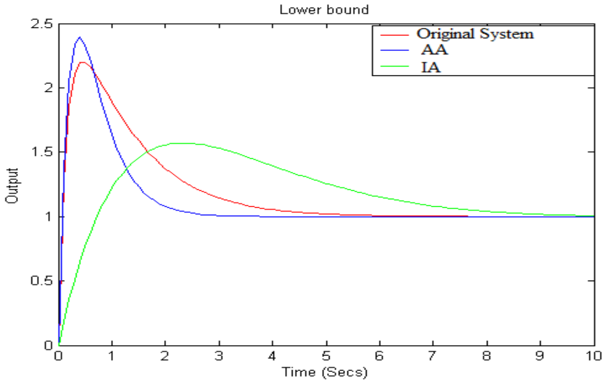

Fig. 2a: Step responses for original, AA and IA

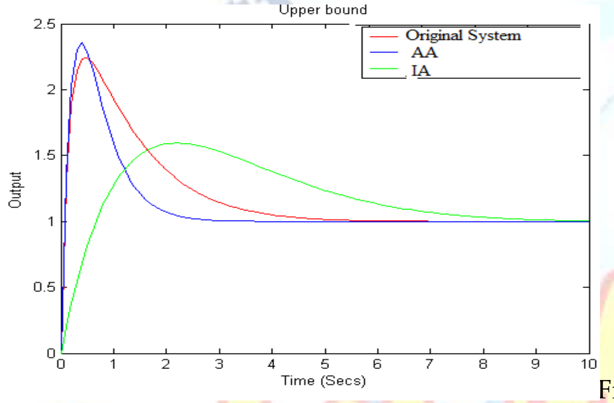

Fig. 2b: Original Step reactions, AA, and IA responses are related to one another.

5. CONCLUSION

This work presents a modified polynomial derivative method for reducing the higher order discrete time interval based systems. Denominator polynomial and numerator polynomial are created from modified partial differentiation, respectively. Due to improved outcomes when compared to the current approaches, the suggested method's findings demonstrate its viability as a novel model order reduction method. The suggested approach is readily calculable. Additionally, the suggested technique produces reduced levels of integral square mistakes as compared to other already in use methods.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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