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Some Introductory Properties of Fibonacci Sequence and Fibonacci like Matrices

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ABSTRACT

Some basic properties of Fibonacci Sequence and Fibonacci like Matrices are discussed.

KEY WORDS: Binet's Formula, Fibonacci Like Matrices

1. INTRODUCTION

As far as sequences of integers are concerned, the Fibonacci Sequen $\{F_n\}$,

$F_{n+2} = F_{n+1} + F_n$, $n \geq 0$ $F_0=0$, $F_1=1$ (1.1)

is important not only from historic point of view but also has applicable value from mathematics point of view. The other being Lucas Sequence defined by the recurrence relation

$L_{n+2} = L_{n+1} + L_n$, $n \geq 0$ $L_0 = 2, L_1 = 1$ (1.2)

Fibonacci Sequence is more famous than Lucas Sequence , since the first one is invented much earlier than the Lucas Sequence.

The Fibonacci Sequence have more interesting and amazing properties than the Lucas Sequence.

The two sequences gave birth to a class of

$a_{n+2} = x \cdot a_{n+1} + y\, a_n$, $n \geq 0$ $a_0 = a$, $a_1 = b$ (1.3)

The Binets Formula is

$Fn = \frac{\alpha^n - \beta^n}{\alpha - \beta}$,where α & β are the roots of the equation $x^2 - x - 1 = 0$ Note that $\alpha + \beta = 1$, $\alpha\beta = -1$, we have

$\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$. Thus $\alpha - \beta = \sqrt{5}$ (1.4)

2. PRELIMINARIES

In this section, we proved some Basic Properties of Fibonacci Sequence.

(i) $F_1 + F_2 + \ldots + F_n = F_{n+2} - 1$

(ii) $F_{n+1} \cdot F_{n-1} - F_n^2 = (-1)^n$, $n \geq 2$

(iii) $F_{m+n} = F_{m-1} \cdot F_n + F_m \cdot F_{n+1}$

(iv) $F_{2n+1} = F^2_n + F^2_{n+1}$

PROOF:

(i) is proved by using direct definition of Fibonacci Sequence i.e. $F_{n+1} = F_n + F_{n-1}$, by using the values of $F_0, F_1, F_2 \ldots F_{n-1}, F_n$. Putting value of F_n in F_{n-1}, F_{n-2} again in previous equation and so on finally we get our required result.

(ii) By using Induction Principle this result can be proved.

(iii) This result is proved by induction on n.

(iv) In equation (iii), $F_{m+n} = F_{m-1} \cdot F_n + F_m \cdot F_{n+1}$ Putting m=n+1 we get required result.

3. FIBONACCI LIKE MATRICES

Let us determine the values of a, b, c such that

$M^2 = M + I$ (3.1)

$I = \begin{bmatrix} 1, & 0 \\ 0, & 1 \end{bmatrix}$ (3.2)

Where $M = \begin{bmatrix} a, & b \\ b, & c \end{bmatrix}$ i.e. $a c - b^2 \neq 0$ (3.3)

And $b \neq \alpha$ or β ; α and β are the roots of $x^2 - x - 1 = 0$, using (3.1) now we prove the following theorem.

Theorem 3.1

To prove $M^{n+2} = M^{n+1} + M^n$, for integers $n \geq 0$ (3.4)

After establishing (3.4) we shall also show that the sequence

$A_n = \{ M^n \}_{n=1}^{\infty}$ is a Fibonacci Like Sequence for different values of a.

PROOF:

Proof of (3.4) can be find by using

$$A_n = M^n = \begin{bmatrix} F \;\; + F_{-1}, & F\sqrt{1 + \;\; -^2} \\ F\sqrt{1 + \;\; -^2}, & F_{+1} - \;\; F \end{bmatrix} \quad (3.5)$$

where $\{ F_n \}$ is the usual Fibonacci Sequence defined by $F_{n+2} = F_{n+1} + F_n$, $n \geq 0$ with $F_0 = 0$, $F_1 = 1$ and to find (3.5) we use Induction Principle .

(3.4) can be solved by using (3.3), which is solvable by taking square of (3.1) and then we find the value of M+I , therefore it satisfies $M^2 = M+I$. Multiplying this equation both sides by M^n result in our required (3.4) equation i.e. $M^{n+2} = M^{n+1} + M^n$, where $A_n = M^n$, $n \geq 0$.

Note that here we conclude that $A_n = \{ M^n \}_{n=1}^{\infty}$ is a Fibonacci Like Matrix, for different values of a , because $A_{n+2} = A_{n+1} + A_n$ follows from (3.4) .

When $a = 1$, we have

$$A_n = \begin{bmatrix} F + \;\; F_{-1}, & F \\ F, & F_{+1} - \;\; F \end{bmatrix} \text{ i.e.}$$

$$A_n = \begin{bmatrix} F_{+1}, & F \\ F, & F_{-1} \end{bmatrix} \quad (3.6)$$

4. SOME BASIC IDENTITIES

In this section we discuss some basic identities by using Fibonacci Like Matrices.

4.1 (i) $\alpha^n = F_n \alpha + F_{n-1}$ (4.1a) and

$\beta^n = F_n \beta + F_{n-1}$ (4.1b)

we find a result $L_n = \alpha^n + \beta^n$ by adding (4.1a) and (4.1b) .

(ii) If $A^n = \begin{bmatrix} , & 0, & 0 \\ F, & , & F \\ 0, & 0, & \end{bmatrix}$ (4.2a) and

$B^n = \begin{bmatrix} , & 0, & 0 \\ F, & , & F \\ 0, & 0, & \end{bmatrix}$ (4.2b)

A and B are not commutative , also for each I+A (or I+B) = A^2 (or B^2) i.e. $B^2 + B + I$, A^n and B^n are both FLMS.

(iii) $A^{m+n} = A^m \times A^n = A^n \times A^m$ (4.3)

we get

$F_{m+n} = \alpha^n . F_m + \beta^m . F_n = \alpha^m . F_n + \beta^n . F_m$ (4.4)

By using (4.4) we have a result $F_{2n} = L_n F_n$

(iv) By using $L_n = F_{n+1} + F_{n-1}$ we can prove that $L_n = \alpha^n + \beta^n$ and to prove this identity we use $F_{m+n} = \beta^m . F_n + \alpha^n . F_m$, by putting m=1 and m=-1 we can find our result.

5. CONCLUSION

This is a collection of very basic properties useful for advance research of Fibonaci Like Matrix and Fibonacci Matrix Sequences.

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Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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