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International Journal for Modern Trends in Science and Technology, 7(10): 118-122, 2021

ISSN: 2455-3778 online
DOI: https://doi.org/10.46501/IJMTST0710019
Available online at: http://www.ijmtst.com/vol7issue10.html

Upper Total Signed Unidomination Number of a Rooted Product Graph of a Path with a Star

P.V. Durgavathi[1] | Kondragunta Rama Krishnaiah[2]

[1]Research Scholar, Department of Mathematics, Krishna University, Machilipatnam,Krishna District – 521001, Andhra Pradesh, India.
[2] Department of Computer Science& Engineering, R. K. College of Engineering, Ibrahimpatnam, Vijayawada - 521456, Andhra Pradesh, India.

To Cite this Article
P.V. Durgavathi and Kondragunta Rama Krishnaiah. Upper Total Signed Unidomination Number of a Rooted Product Graph of a Path with a Star. *International Journal for Modern Trends in Science and Technology* 2021, *7*, pp. 118-122. https://doi.org/10.46501/IJMTST0710019

Article Info
Received: 06 September 2021; Accepted: 30 September 2021; Published: 09 October 2021

ABSTRACT

Graph theory is an important subject in mathematics. Applications in many fields like Logical Algebra, Coding theory, Computer networking and Engineering communications. Theory of domination is one of the most active areas of research in graph theory in last few decades.

Zelinka [9] was the person who first coined the phrase "total signed dominating function." Total signed dominating function of corona product graphs $C_n \odot K_m$, $P_n \odot K_{1,m}$ are studied by Siva Parvathi [8]. Aruna [1] introduced the total signed unidominating function and investigated these concepts for corona product graphs. Godsil and Mckay [3] introduced the rooted product, a new two-graph product denoted by $G \ o \ H$, in 1978. Shobha Rani [7] examined signed edge total domination on rooted product graphs. For a rooted product graph of a path with a star, we explore the notions of total signed unidominating function and upper total signed unidomination number in this paper.

Keywords: Graph Theory, Rooted product graph, total signed unidomination number properties

1.INTRODUCTION

Ore [6] and Berge [2] introduced the theory of domination, which has been extensively studied in graphs. Graph dominance and related topics are explored in depth in Haynes et al., two books [4, 5].The rooted product graphs are used for connecting internet to one system to other systems in internet systems. Generally Product of graphs is used in discrete mathematics. They found the upper total number of signed unidominations in a rooted product graph of a path with a star, as well as the total number of signed unidominations. Along with this, the graph's minimum and maximum number of minimally total-weighted signed unidominating functions are discovered.

The authors have determined total signed unidomination number and upper total signed unidomination number of a rooted product graph of a path with a star in this paper. Along with this, the number of total and minimal total signed unidominating functions with minimum and maximum weight of the above said graph are found.

2. ROOTED PRODUCT OF P_n AND $K_{1,m}$

The rooted product of a path P_n and with a star $K_{1,m}$ is a graph formed by taking one copy of a n – vertex graph

P_n and n copies of $K_{1,m}$ and then joining the $i^{th}-$ vertex of P_n with any one vertex in i^{th} copy of of $K_{1,m}$. Every i^{th} vertex of P_n is merging with any one vertex in every i^{th} copy of $K_{1,m}$. The rooted product of two graphs P_n and $K_{1,m}$ is denoted by $P_n oK_{1,m}$, P_n contains n vertices and $K_{1,m}$ contains m vertices in each copy of $K_{1,m}$.

The vertices in path P_n are denoted by v_i, the centre and the vertices of the star $K_{1,m}$ are denoted by u_i and u_{ij} respectively. In $P_n oK_{1,m}$, every ith vertex v_i of P_n is merging with centre u_i of $K_{1,m}$ is identifying the root in every ith copy of $K_{1,m}$.

3. TOTAL SIGNED UNIDOMINATION NUMBER OF $P_n oK_{1,m}$

Total signed unidomination number and total signed unidominating function are defined in this section. One gets total signed unidomination number and the number of total signed unidominating functions of $P_n o K_{1,m}$ are determined.

Definition 1: Assume $G(V,E)$ is a connected graph. $g: V \to \{-1,1\}$ is a total signed unidominating function of G if and only if it has the property

$$\sum_{u\in N(v)} g(u) \geq 1 \quad \forall v \in V and g(v) = 1$$

$$and \sum_{u\in N(v)} g(u) = 1 \quad \forall v \in V and g(v) = -1$$

where $N(v)$ is the open neighbourhood of the vertex v.

Definition 2: A graph's total signed unidomination number $G(V,E)$ is defined as $\min\{g(V)/g$ is a total signed unidominating function$\}$.

It is represented by $\gamma_{tsu}(G)$.

$$g(V) = \sum_{u\in V} g(u)$$

denotes the weight of the total signed unidominating function g.

That is, the total signed unidomination number of a graph $G(V,E)$ is equal to the minimum of the weights of total signed unidominating functions of G.

Theorem 3.1: The total signed unidomination number of the rooted product graph $P_n oK_{1,m}$ is

$$\begin{cases} n & when\ m\ is\ even, \\ 2n & when\ m\ is\ odd. \end{cases}$$

Proof: Consider the rooted product graph $P_n oK_{1,m}$.

The following scenarios arise when trying to determine the total signed unidomination number for $P_n oK_{1,m}$

Case 1: Assume m is an even number.

Describe a function $g: V \to \{-1,1\}$ by

$$g(v_i) = 1,$$

$v_i = u_i$ for $i = 1,2, \dots\dots, n$ where u_i is merged with the vertex v_i also u_i is center of $K_{1,m}$

and

$$g(u_{ij}) = \begin{cases} -1 & for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m}, \\ 1 & for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m} \end{cases}$$

for $i = 1,2, \dots\dots, n$ and $j = 1,2, \dots\dots, m$

Let $i \neq 1$ and $i \neq n$.

If $v_i \in P_n$ then

$$\sum_{u\in N(v_i)} g(u) = g(v_{i-1}) + g(v_{i+1}) + g(u_{i1}) + g(u_{i2}) + \cdots + g(u_{im}) = 1 + 1 + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(1)\right] = 2.$$

Let $i = 1$ or $i = n$.

If $v_1 \in P_n$ then

$$\sum_{u\in N(v_1)} g(u) = g(v_2) + g(u_{11}) + g(u_{12}) + \cdots + g(u_{1m}) = 1 + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(1)\right] = 1.$$

If $v_n \in P_n$ then

$$\sum_{u\in N(v_n)} g(u) = g(v_{n-1}) + g(u_{n1}) + g(u_{n2}) + \cdots + g(u_{nm}) = 1 + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(1)\right] = 1.$$

If $u_{ij} \in K_{1,m}$ then $g(u_{ij}) = 1$ or $g(u_{ij}) = -1$.

Let $u_{ij} \in K_{1,m}$ and $g(u_{ij}) = 1$. Then

$$\sum_{u\in N(u_{ij})} g(u) = g(v_i) = 1.$$

Let $u_{ij} \in K_{1,m}$ and $g(u_{ij}) = -1$. Then

$$\sum_{u\in N(u_{ij})} g(u) = g(v_i) = 1$$

That is g is satisfying the conditions of a total signed unidominating function.

Hence g is a total signed unidominating function.

$$\text{Now } g(V) = \sum_{u\in P_n} g(u) + \sum_{u\in K_{1,m}} g(u)$$

$$= \underbrace{(1 + 1 + 1 \dots + 1)}_{(n-times)} + \underbrace{\underbrace{\{(-1) + (-1) + \cdots (-1)\}}_{(\frac{m}{2}-\text{times})} + \underbrace{\{1 + 1 + \cdots + 1\}}_{(\frac{m}{2}-\text{times})}}_{n-times}$$

$$= n + \left(\frac{m}{2}\right)(-n) + \left(\frac{m}{2}\right)(n) = n.$$

Thus $g(V) = n$.

P_n and each copy of $K_{1,m}$ has the functional values of 1 and -1, but we can find that these functions are not total signed unidominating functions for any other assignment.

There is only one total signed unidominating function, which is the one described above.

As a result, when m is even, $\gamma_{tsu}\left(P_n o K_{1,m}\right) = n$.

Case 2: Assume m is odd number.

Describe a function $g: V \to \{-1,1\}$ by

$$g(v_i) = 1,$$

$v_i = u_i$ for $i = 1,2, \dots \dots, n$ where u_i is merged with the vertex v_i also u_i is center of $K_{1,m}$

and

$$g(u_{ij}) = \begin{cases} -1 \ \ for \ \frac{m-1}{2} \ vertices \ \ in \ \ each \ \ copy \ \ of \ K_{1,m}, \\ 1 \ \ for \ \frac{m+1}{2} \ vertices \ \ in \ \ each \ \ copy \ \ of \ K_{1,m} \end{cases}$$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$

Let $i \neq 1$ and $i \neq n$.

If $v_i \in P_n$ then

$$\sum_{u \in N(v_i)} g(u) = g(v_{i-1}) + g(v_{i+1}) + g(u_{i1}) + g(u_{i2}) + \cdots + g(u_{im})$$

$$= 1 + 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2}\right)(1)\right]$$

$$= 3.$$

Let $i = 1$ or $i = n$.

If $v_1 \in P_n$ then

$$\sum_{u \in N(v_1)} g(u) = g(v_2) + g(u_{11}) + g(u_{12}) + \cdots + g(u_{1m})$$

$$= 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2}\right)(1)\right]$$

$$= 2.$$

If $v_n \in P_n$ then

$$\sum_{u \in N(v_n)} g(u) = g(v_{n-1}) + g(u_{n1}) + g(u_{n2}) + \cdots + g(u_{nm})$$

$$= 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2}\right)(1)\right]$$

$$= 2.$$

If $u_{ij} \in K_{1,m}$ then $g(u_{ij}) = 1$ or $g(u_{ij}) = -1$.

Let $u_{ij} \in K_{1,m}$ and $g(u_{ij}) = 1$. Then

$$\sum_{u \in N(u_{ij})} g(u) = g(v_i) = 1.$$

Let $u_{ij} \in K_{1,m}$ and $g(u_{ij}) = -1$. Then

$$\sum_{u \in N(u_{ij})} g(u) = g(v_i) = 1$$

That is g is satisfying the conditions of a total signed unidominating function.

So g is a total signed unidominating function.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in K_{1,m}} g(u)$$

$$= \underbrace{(1 + 1 + 1 \dots + 1)}_{(n-times)} + \underbrace{\underbrace{\{(-1) + (-1) + \cdots (-1)\}}_{(\frac{m-1}{2}-\text{times})} + \underbrace{\{1 + 1 + \cdots + 1\}}_{(\frac{m+1}{2}-\text{times})}}_{n-times}$$

$$= n + \left(\frac{m-1}{2}\right)(-n) + \left(\frac{m+1}{2}\right)(n) = 2n.$$

Thus $g(V) = 2n$.

Other than this, assigning the functional values 1 and -1 to P_n and each copy of $K_{1,m}$ results in functions which are not total signed unidominating functions.

There is only one total signed unidominating function, and that is the one described above as a result, when m is odd, $\gamma_{tsu}\left(P_n o K_{1,m}\right) = 2n$.

Theorem 3.2: In the even number m, the total number of signed unidominating functions with minimum weight n of $P_n o K_{1,m}$ is 1, while in the odd number m, the minimum weight of $P_n o K_{1,m}$ is 1.

Proof: Theorem 3.1 follows.

4. UPPER TOTAL SIGNED UNIDOMINATION NUMBER OF $P_n o K_{1,m}$

The terms "minimal total signed unidominating function" and "upper total signed unidominating number" are defined in this section. The number of minimal total signed unidominating functions of maximum weight of of $P_n o K_{1,m}$ is determined, as well as the upper total signed unidomination number.

Definition 1: Consider the connected graph $G(V, E)$. If for all $h < g$, h is not a total signed unidominating function, the total signed unidominating function $g: V \to \{-1,1\}$ is called a minimal total signed unidominating function.

Definition 2: A connected graph $G(V, E)$'s upper total signed unidomination number is defined as max $\{|g(V)|\ g$ is a minimal total signed unidominating function$\}$

It may be represented by $\Gamma_{tsu}(G)$.

Theorem 4.1: The upper total signed unidomination number of the rooted product graph $P_n \text{o} K_{1,m}$ is

$$\begin{cases} n \ \ when \ \ m \ \ is \ \ even, \\ 2n \ \ when \ \ m \ \ is \ \ odd. \end{cases}$$

Proof: Let $P_n \text{o} K_{1,m}$ be the given rooted product graph. The following cases arise when determining the upper total signed unidomination number of $P_n \text{o} K_{1,m}$.

Case 1: Assume m is even number.

Describe a function $g: V \to \{-1,1\}$ by

$$g(v_i) = 1,$$

$v_i = u_i$ for $i = 1,2, \dots \dots, n$ where u_i is merged with the vertex v_i also u_i is center of $K_{1,m}$

and

$$g(u_{ij}) = \begin{cases} -1 & for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m}, \\ 1 & for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m} \end{cases}$$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$

To show that it is a total signed unidominating function, we use the function defined in Case 1 of Theorem 3.1.

At this time, we prove for the minimality of g.

$$h(v_i) = \begin{cases} -1 & for\ v_i = v_k \in P_n\ for\ some\ k, \\ 1 & otherwise, \end{cases}$$

and

$$h(u_{ij}) = \begin{cases} -1 & for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m}, \\ 1 & for\ \frac{m}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m} \end{cases}$$

Suppose $i = k$.

For $v_k \in P_n$ we have $h(v_k) = -1$ and

$$\sum_{u \in N(v_k)} h(u) = h(v_{k-1}) + h(v_{k+1}) + h(u_{k1}) + h(u_{k2}) + \cdots + h(u_{km}) = 1 + 1 + \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2}\right)(1)\right] = 2 \neq 1.$$

It fails in the vicinity of the vertex $v_k \in P_n$ where $h(v_k) = -1$ when the total signed unidominating function fails.

That's why it's not possible to write total signed unidominating function for h.

Given that h has been arbitrarily defined, it follows that no $h < g$ exists that makes h a total signed unidominating function.

In other words, the function g is a minimal total signed unidominating one.

In addition, g is the only minimal total signed unidominating function because assigning the functional values $-1,1$ to the vertices of P_n and $K_{1,m}$ does not create g any longer a total signed unidominating function.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in K_{1,m}} g(u)$$

$$= \underbrace{(1 + 1 + 1 \dots + 1)}_{(n-times)} + \underbrace{\underbrace{\{(-1) + (-1) + \cdots (-1)\}}_{(\frac{m}{2}-\text{times})} + \underbrace{\{1 + 1 + \cdots + 1\}}_{(\frac{m}{2}-\text{times})}}_{n-times}$$

$$= n + \left(\frac{m}{2}\right)(-n) + \left(\frac{m}{2}\right)(n) = n.$$

Thus $g(V) = n$.

In other words, because there's only one minimal total signed unidominating function,

so $\max\{g(V)\} = n$.

As a result, when m is even, $\Gamma_{tsu}(P_n o K_{1,m}) = n$.

Case 2: Assume m is odd number.

Describe a function $f: V \to \{-1,1\}$ by

$$g(v_i) = 1,$$

$v_i = u_i$ for $i = 1,2, \dots \dots, n$ where u_i is merged with the vertex v_i also u_i is center of $K_{1,m}$

and

$$g(u_{ij}) = \begin{cases} -1 & for\ \frac{m-1}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m}, \\ 1 & for\ \frac{m+1}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m} \end{cases}$$

for $i = 1,2, \dots \dots, n$ and $j = 1,2, \dots \dots, m$

To show that g has total signed unidominating function, the function is compared to the one defined in Case 2 of Theorem 3.1.

Now we prove for the minimality of g.

$$h(v_i) = \begin{cases} -1 & for\ v_i = v_k \in P_n\ for\ some\ k, \\ 1 & otherwise, \end{cases}$$

and

$$h(u_{ij}) = \begin{cases} -1 & for\ \frac{m-1}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m}, \\ 1 & for\ \frac{m+1}{2}\ vertices\ in\ each\ copy\ of\ K_{1,m} \end{cases}$$

Suppose $i = k$.

For $v_k \in P_n$ we have $h(v_k) = -1$ and

$$\sum_{u \in N(v_k)} h(u) = h(v_{k-1}) + h(v_{k+1}) + h(u_{k1}) + h(u_{k2}) + \cdots + h(u_{km}) = 1 + 1 + \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2}\right)(1)\right] = 3 \neq 1.$$

It fails in the vicinity of the vertex $v_k \in P_n$ where $h(v_k) = -1$ when the total signed unidominating function fails.

That's why it's not possible to write total signed unidominating function for h.

Since h is defined arbitrarily, there is no $h < g$ such that h is a total signed unidominating function.

In other words, the function g is a minimal total signed unidominating one.

In addition, g is the only minimal total signed unidominating function because assigning the functional values -1,1 to the vertices of P_n and $K_{1,m}$ does not create g any longer a total signed unidominating function.

$$\text{Now } g(V) = \sum_{u \in P_n} g(u) + \sum_{u \in K_{1,m}} g(u)$$

$$= \underbrace{(1+1+1\ldots+1)}_{(n-times)} + \underbrace{\underbrace{\{(-1)+(-1)+\cdots(-1)\}}_{(\frac{m-1}{2}-\text{times})} + \underbrace{\{1+1+\cdots+1\}}_{(\frac{m+1}{2}-\text{times})}}_{n-times}$$

$$= n + \left(\frac{m-1}{2}\right)(-n) + \left(\frac{m+1}{2}\right)(n) = 2n.$$

Thus $g(V) = 2n$.

Since g is the only minimal total signed unidominating function,

$max\,\{g(V)/g \;\; is\ a\ minimal\ total\ signed\ unidominating\ function\} = 2n.$

$\Gamma_{tsu}\left(P_n \text{o} K_{1,m}\right) = 2n$, when m is odd, so this is the case.

Theorem 4.2: If m is even then the number of minimal total signed unidominating functions of $P_n \text{o} K_{1,m}$ with maximum weight n is 1 and if m is odd then the number of minimal total signed unidominating functions of $P_n \text{o}\, K_{1,m}$ with maximum weight $2n$ is 1.

Proof: Follows by Theorem 4.1.

5. CONCLUSION

The authors of this paper examine the rooted product graph's total signed unidominating function and minimum total signed unidominating function. Finding out more about signed unidominating functions and their upper signed unidominating functions in above-rooted graphs is made easier with this knowledge.

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