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Propagation of dust acoustic waves in super thermal magnetized plasma

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ABSTRACT

Structure of dust acoustic traveling waves is studied in homogeneous, collisionless, magnetized dusty plasma with Boltzmann distributed ions and super thermal distributed electrons. Using reductive perturbation technique we have derived the Zakharov-Kuznesstov-Burgers equation. In absence of burger term analytical solution is obtained.

KEYWORDS: *Keyword 1, Keyword 2, Keyword 3, Keyword 4, Keyword 5*

INTRODUCTION

Dust in plasma is ubiquitous in solar system, namely, comet tails, asteroid zones, planetary ring, interstellar medium, and the lower part of the Earth's ionosphere and magnetosphere radio frequency plasma discharge [1]-[5] etc. To study this fascinating behaviour which is hidden within the nonlinear phenomena, many researchers [6]-[14] have engaged themselves to investigate the characteristics of solitary wave structures
viz. amplitude, width, collision [8]-[10] in collision-less plasma in planar geometry. Kortewg de Vries (KdV) [11] equation in one dimension and Kadomtsev-Petviashvili (KP)[12] equation in two dimension describe the soliton structure in unmagnetized dissipation-less plasma where as the Korteweg-de Vries Burgers (KdVB)[13] equation and Kadomtsev-Petviashivili-Burgers (KPB)[14] equation are deduced from the plasma system when the dissipation becomes effective along with the system. El Bedwehy and Moslem [15] have derived Zakharov-Kuznetsov-Burgers (ZKB) equation in superthermal electron-positron-ion plasma from uid continuity, momentum, and Poisson equations.

Recently, Sahu and Roychoudhury [16] have investigated Zakharov-Kuznetsov (ZK) equation for ion acoustic waves with superthermal electrons in cylindrical geometry.They have shown that in cylindrical geometry a new type of soliton like structuresappears as time goes by.

Some important features had been overlooked in those studies. Most of these investigations [1]-[16] are confined to study either the structure of the localized solitarywaves or higher order correction, locally through the reductive perturbative techniquewhich leads to them to some standard equations like Kortewg-de Vries (KdV), mKdVequations etc.

Firstly, it is important to clarify that solitons exist because of the balance between theeffects of nonlinearity and dispersion, while the dissipative effect is very weak or negligible in comparison with that of nonlinearity and dispersion. This dissipation-lessequation i.e KdV equation can be solved analytically and it gives a soliton structure. Similarly, when the dissipative effect is comparable to or more dominant than thedispersion effect, one encounters shock waves. But it may be more interesting tostudy the behavior of the solution of the equation incorporating dissipation, dispersion and nonlinearity all together. Whether the shock persists after the dispersioneffect comes to act and what would be the nature of the solution, are main aims ofthis presentation.

Basic equations and derivation of Basic ZKB equation

One can consider homogeneous, collisionless, magnetoplasma consisting dust, Boltzmann distributed ions and superthermal distributed electrons. The plasma isconfinedin an external magnetic field $\boldsymbol{B_0} = \hat{x}B_0$ where $\hat{x}$is the unit vector along the x-directionand B0 is the external static magnetic field. The basic system of normalized equations in planar geometry in such a plasma model is governed by the following equations

$$\frac{\partial n_d}{\partial t} + \nabla.(n_d.u_d) = 0$$

$$\frac{\partial u_d}{\partial t} + (u_d.\nabla)u_d = -\nabla\phi + \Omega(u_d \times \hat{x}) + \mu\nabla^2 u_d$$

$$\nabla^2\phi = n_e + \alpha n_d - (1+\alpha)n_i$$

In the above equations the subscripts i and e refer to ion and electron respectively. u_d, v_d and w_d are the dust fluid speed in the directions x, y and z respectively. n_d is the dust number normalized to its unperturbed equilibrium plasma density $n_{e0}.(u_d, v_d, w_d)$ is the dust fluid normalized to the dust acoustic velocity $v_s = (K_B T_i/m_d)^{1/2}$, and ϕ is normalized by $K_B T_e/e$ where K_B is the Boltzmann constant. The time and space variables of the dust plasma period $w_{pd}^{-1} = \sqrt{\frac{m_d}{4\pi e^2 n_{e0}}}$ and the Debye radius $\lambda_D = \sqrt{\frac{K_B T_e}{4\pi e^2 n_{e0}}}$, respectively. $\Omega = (\frac{eb_0}{m_d c})/w_{pd}$ is the dust cyclotron frequency normalized to w_{pd} and $\alpha = n_{d0}/n_{e0}$. μ is the kinetic viscosity and where $n_e = \left(1 - \frac{\phi}{\kappa - \frac{1}{2}}\right)^{-\kappa+1/2}$, $n_i = e^{\sigma\phi}$ and e is the velocity of light $\sigma = \frac{T_i}{T_e}$. It is often seen that most of the systems are composed with Boltzmann distributed electrons and it is well known that the Maxwell distributuion is taken to be valid for the macroscopic ergodic equilibrium state. Since Maxwell distribution may be inadequate to describe the long range interactions in unmagnetized collision less plasma where the non-equilibrium stationary state exists. The distribution function that can better model such particle velocity distribution function is known as the generalised Lorengian or kappa distribution [17] with functional dependence of the form$f_0(v) \approx [1 + \frac{v^2}{k\theta^2}]^{-(k+1)}$. The spectral index κ, is a measure of the slope of theenergy spectrum of the superthermal electrons forming the tail of the velocity distribution function. Several observations in astrophysical plasmas, namely, solar wind, auroral zone plasma magnetosphere [18]-[26] insists one to follow the superthermal or non Maxwellian distribution. Moreover the kappa (superthermal) distribution has also been used to analyse and interpret spacecraft data on the earth's magnetospheric plasma sheet [21], Jupiter [22] and Saturn [23]. In limit $\kappa \to \infty$ the distribution approaches a Maxwellian distribution $[\approx \exp(-\frac{v^2}{\theta^2})]$

We now follow the reductive perturbation technique and construct a weakly nonlinear theory for ion acoustic waves with small but finite (1) amplitude, which leads to a stretching of the independent variables as

(2)

$$\xi = \epsilon^{\frac{1}{2}}(x - v_0 t), \eta = \epsilon^{1/2} y$$

$$\zeta = \epsilon^{\frac{1}{2}} z, and\ \tau = \epsilon^{\frac{3}{2}} t, \mu = \epsilon^{\frac{1}{2}}\mu_0 \quad (3)$$

The dependent variables are expanded as

$$n_d = 1 + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \cdots \quad (5)$$

$$u_d = \epsilon u_d^{(1)} + \epsilon u_d^{(2)} + \quad (6)$$

$$v_d = \epsilon^{3/2} v_d^{(1)} + \epsilon^2 w_d^{(2)} + \cdots \quad (7)$$

$$w_d = \epsilon^{\frac{3}{2}} w_d^{(1)} + \epsilon^2 w_d^{(2)} + \quad (8)$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \cdots \quad (9)$$

Substituting equation(4) and (5) -(9) into the equations (1) -(3) and equating the lowest power of ϵone can get

$$n_d^{(1)} = -\frac{1}{v_0^2}\phi^{(1)}, u_d^{(1)} = -\frac{1}{v_0}\phi^{(1)}, v_d^{(1)} = \frac{1}{\Omega}\frac{\partial\phi^{(1)}}{\partial\zeta}$$

$$w_d^{(1)} = -\frac{1}{v_0\Omega}\frac{\partial\phi^{(1)}}{\partial\eta}, and\ v_0 = \left(\frac{\alpha}{1 + (1+\alpha)\sigma}\right)^{1/2} \quad (10)$$

To the next order of ϵ one can get

$$\frac{\partial v_d^{(1)}}{\partial\xi} = -\frac{\Omega}{v_0} w_d^{(2)} \quad (11)$$

$$\frac{\partial w_d^{(1)}}{\partial\xi} = \frac{\Omega}{v_0} v_d^{(2)} \quad (12)$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2} + \frac{\partial^2\phi^{(1)}}{\partial\eta^2} + \frac{\partial^2\phi^{(1)}}{\partial\zeta^2} = (1 + (1+\alpha)\sigma)\phi^{(2)} + \alpha n_d^{(2)} + \beta(\phi^{(1)})^2 \quad (13)$$

Where $\beta = \frac{\kappa + 1/2}{2(\kappa * \frac{1}{2})} - \frac{(1+\alpha)\sigma^2}{2}$;

Next higher order of ϵ gives

$$\frac{\partial n_d^{(1)}}{\partial\tau} - v_0\frac{\partial n_d^{(2)}}{\partial\xi} + \frac{\partial}{\partial\xi}(n_d^{(1)} v_d^{(1)}) + \frac{\partial u_d^{(2)}}{\partial\xi} + \frac{\partial v_d^{(2)}}{\partial\eta} + \frac{\partial w_d^{(2)}}{\partial\zeta} = 0 \quad (15)$$

$$\frac{\partial u_d^{(1)}}{\partial \tau} - v_0 \frac{\partial u_d^{(2)}}{\partial \xi} + u_d^{(1)} \frac{\partial u_d^{(1)}}{\partial \xi} = \frac{\partial \phi^{(2)}}{\partial \xi}$$

Eliminating $u_d^{(2)}$ we get

$$-\frac{2}{v_0^2}\frac{\partial \phi^{(1)}}{\partial \tau} - v_0 \frac{\partial n_d^{(2)}}{\partial \xi} + \frac{3}{v_0^3}\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\partial v_d^{(2)}}{\partial \eta} + \frac{\partial w_d^{(2)}}{\partial \zeta} - \frac{1}{v_0}\frac{\partial \phi^{(2)}}{\partial \xi} + \frac{u_0}{v_0^2}\left(\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}\right) = 0$$

After simplification and eliminating the second order of n_d and ϕ we finally obtain

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + C\frac{\partial}{\partial \xi}\left(\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}\right) + D\left(\frac{\partial^2 \phi^{(1)}}{\partial \eta^2} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi^{(1)}}{\partial \zeta^2}\right) = 0$$

Which is ZKB equation, where

$$A = -\frac{v_0^3}{2\alpha}\left(\frac{3\alpha}{v_0^4} - 2\beta\right)$$

$$B = \frac{v_0^3}{2\alpha}$$

$$C = \frac{v_0^3}{2\alpha}\left(1 - \frac{\alpha}{\Omega^2}\right)$$

$$D = \frac{\mu_0}{2}$$

Which is known as ZKB equation

DISCUSSION

To obtain the solution of the planar ZKB equation (17), one can transform this equation into ordinary differential equation by introducing a single independentvariable $x = l\xi + m\eta + n\zeta - u\tau$, where l, m, n are direction cosines of the wave propagationvector $\kappa$with respect toξ, η and ζ axes respectively. $u$is the velocity of the movingframe normalized by dust acoustic speed. Let us put$\Phi(x) = \phi^{(1)}(\xi, \eta, \zeta, \tau)$. Integhrating equation (17) with respect to $x$and using the boundary condition of Φ and theirderivatives up to second order for $x \to \infty$ i.e. $\Phi = \frac{d\Phi}{dx} = \frac{d^2\Phi}{dx^2} = 0$ as $x \to \infty$ yield

$$\frac{d^2\Phi}{dx^2} - c\frac{d\Phi}{dx} - a\Phi + b\Phi^2 = 0$$

Where the a, b and c are given by

$$a = \frac{u}{Bl^3 + lC(m^2 + n^2)}, b = \frac{Al}{2(Bl^3 + lC(m^2 + n^2))}, c = \frac{D}{Bl^3 + lC(m^2 + n^2)}$$

One can consider equation (18) as an equation of motion for a pseudo-time $x$and pseudo-position Φ in a force field with potential

$$V = \frac{1}{3}b\Phi^3 - \frac{1}{2}a\Phi^2$$

and a frictional force with the coefficient c which is associated with viscosity.In absence of viscosity (c = 0) the equation (18) can be rewritten as

$$\frac{d^2\Phi}{dx^2} - a\Phi + b\Phi^2 = 0 \qquad (16)$$

Analytic solution of equation (19) can be written as [20]

$$\Phi = \Phi_0 Sech^2\left(\frac{x}{W}\right) \qquad (20)$$

Where $\Phi_0 = 3\frac{a}{b}$ is the amplitude of the soliton, $W = \sqrt{\frac{4}{a}}$ is the width of the solution.

CONCLUSION

Here, we have studied dust acoustic traveling waves in homogeneous, collisionless,magnetized dusty plasma with Boltzmann distributed ions and superthermal distributed electrons. By employing the reductive perturbation technique, we have derived Zakharov-Kuznetsov-Burgers' equation. In absence of burger term, we have determined analytical solution of the Zakharov- Kuznetsov-Burgers' equation.

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