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Comparative analysis of Bayesian PAPR Reduction Methods for OFDM-Based Massive MIMO Systems

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ABSTRACT

In this project the first module, problem of peak-to-average power ratio (PAPR) reduction in orthogonal frequency-division multiplexing (OFDM) based massive multiple-input multiple-output (MIMO) downlink systems. Specifically, given a set of symbol vectors to be transmitted to K users, the problem is to find an OFDM-modulated signal that has a low PAPR and meanwhile enables multiuser interference (MUI) cancelation. Unlike previous works that tackled the problem using convex optimization, we take a Bayesian approach and develop an efficient PAPR reduction method by exploiting the redundant degrees-of-freedom of the transmit array. The sought-after signal is treated as a random vector with a hierarchical truncated Gaussian mixture prior, which has the potential to encourage a low PAPR signal with most of its samples concentrated on the boundaries. A variational expectation-maximization (EM) strategy is developed to obtain estimates of the hyper parameters associated with the prior model, along with the signal. In addition, the generalized approximate message passing (GAMP) is embedded into the variational EM framework, which results in a significant reduction in computational complexity of the proposed algorithm. The draw backs of Module one will be corrected by considering the Improved ICTF method Simulation results show our proposed algorithm achieves a substantial performance improvement over existing methods.

KEYWORDS: ICTF, PAPR, BER, OUT-OF-BAND interference

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I. Introduction

Massive multiple-input multiple-output also known as large- scale or very-large MIMO, is a promising technology to meet the ever growing demands for higher throughput and better quality-of-service of next-generation wireless communication systems. MIMO is a method for multiplying the capacity of a radio link using multiple transmit and receive antennas to exploit multipath propagation. MIMO is fundamentally different from smart antenna techniques developed to enhance the performance of a single data signal,

as beam forming and diversity. Massive such MIMO systems are those that are equipped with a large number of antennas at the base station simultaneously serving am much smaller number of single-antenna users sharing the same timefrequency bandwidth. In addition to higher throughput, massive MIMO systems also have the potential to improve the energy efficiency and enable the use of inexpensive, low-power components. Hence, it is expected that massive MIMO will bring radical changes to future wireless communication systems. In practice, broadband wireless communications suffer may

frequency-selective fading. Orthogonal frequency division multiplexing (OFDM), a scheme of encoding digital data on multiple frequencies, has been widely used to deal with frequency-selective fading. Orthogonal Frequency Division Multiplexing or OFDM is a modulation format that is being used for many of the latest wireless and telecommunications standards. OFDM is a form of multicarrier modulation. An OFDM signal consists of a number of closely spaced modulated carriers. When modulation of any form - voice, data, etc. is applied to a carrier, then sidebands spread out either side. It is necessary for a receiver to be able to receive the whole signal to be able to successfully demodulate the data. As a result when signals are transmitted close to one another they must be spaced so that the receiver can separate them using a filter and there must be a guard band between them. This is not the case with OFDM. Although the sidebands from each carrier overlap, they can still be received without the interference that might be expected because they are orthogonal to each another. This is achieved by having the carrier spacing equal to the reciprocal of the symbol period. However, a major problem associated with the OFDM is that it is subject to a high peak-to-average power ratio owing to the independent phases of the sub-carriers. To avoid out-of-band radiation and signal distortion, handling this high PAPR requires a high-resolution digital-to-analog converter and a linear power amplifier (PA) at the transmitter, which is not only expensive power-inefficient. The situation deteriorates when the number of antennas is large leaving such systems impractical. Therefore, it is of crucial importance to reduce the PAPR of massive MIMO-OFDM systems to facilitate low-cost and power-efficient hardware implementations. Many techniques have been developed for PAPR reduction in single-input single-output (SISO) OFDM wireless systems. The most prominent are clipping, tone reservation (TR), active constellation extension (ACE), partial transmission sequence (PTS) and others Although these PAPR-reduction schemes can be extended to point-to point MIMO systems easily, extension to the multi-user (MU) MIMO downlink is not straightforward, mainly because joint receiver-side signal processing is almost impossible in practice as the users are distributed. Recently, a new PAPR reduction method was developed for massive MIMO-OFDM systems. The proposed scheme utilizes the redundant degrees-of-freedom (DoFs) resulting

from the large number of antennas at the BS to achieve ioint multiuser interference (MUI) cancelation and PAPR reduction. Specifically, the problem was formulated as a linear constrained l∞ optimization problem and a fast truncation algorithm (FITRA) was developed. However, the FITRA algorithm shows to have a fairly low convergence rate. Also, the algorithm employs a regularization parameter to achieve balance between the PAPR reduction and the MUI cancelation (i.e. data fitting error). The choice of the regularization parameter may be tricky in practice. On the other hand, the regularization parameter may be seen instead as an additional degree of freedom that allows to regulate the operation of the algorithm., a peak signal clipping scheme was employed to reduce the PAPR and some of the antennas at the BS are reserved to compensate for peak-clipping signals. This method has a lower computational complexity. But it achieves only a mild PAPR reduction and those antennas reserved for compensation may incur large PAPRs. In this paper, we develop a novel Bayesian approach to address the joint PAPR reduction and MUI cancelation problem for downlink multi-user massive MIMO-OFDM systems. Specifically, MUI cancelation can be formulated as an underdetermined linear inverse problem which admits numerous solutions. To search for a low PAPR solution, a hierarchical truncated Gaussian mixture prior model is proposed and assigned to the unknown signal (i.e. solution). This hierarchical prior has the potential to encourage a quasi- constant magnitude solution with as many entries as possible lying on the truncated boundaries, thus resulting in a low PAPR. A variational expectation-maximization (EM) algorithm is developed to obtain estimates of the hyper parameters associated with the prior model, along with the signal. In addition, the generalized approximate message passing (GAMP) technique is employed to facilitate the algorithm development in the expectation step. This GAMP technique also helps significantly reduce the computational complexity of the proposed algorithm.

II. LITERATURE SURVEY

A. PAPR Reduction Techniques for OFDM Signals

As an attractive technology for wireless communications, Orthogonal Frequency Division Multiplexing which is one of multi-carrier modulation techniques offers a considerable high efficiency, spectral multipath delay

tolerance, immunity to the frequency selective fading channels and power efficiency. One of the challenging issues for Orthogonal Frequency system Division Multiplexing is its high Peak-to-Average Power Ratio. In this paper, it reviews and analysis different OFDM reduction techniques, based on computational complexity, bandwidth expansion, spectral spillage and performance. They are clipping, filtering, coding schemes and partial transmit sequences. The simplest and most widely used technique of PAPR reduction is to basically clip the parts of the signals that are outside the allowed region. Generally, clipping is performed at the transmitter. However the receiver need to estimate the clipping that has occurred and to compensate the received OFDM symbol accordingly. Therefore, clipping method introduces both in band distortion and out of band radiation into OFDM signals, which degrades the system performance including BER and spectral efficiency. Filtering can reduce out of band radiation after clipping although it cannot reduce in-band distortion. However, clipping may cause some peak regrowth so that the signal after clipping and filtering will exceed the clipping level at some points. . A simple block coding scheme was introduced and its basic idea is that mapping 3 bits data into 4 bits codeword by adding a Simple Odd Parity Code at the last bit across the channels. The main disadvantage of SOBC method is that it can reduce PAPR for a 4-bit codeword. Later, Wulich applied the Cyclic Coding to reduce the PAPR Fragiacomo proposed an efficient Simple Block Code to reduce the PAPR of OFDM signals. However, it is concluded that SBC is not effective when the frame size is large. In a typical OFDM system with PTS approach to reduce the PAPR, the input data block in is partitioned into disjoint sub blocks. In general, for PTS scheme, the known sub block partitioning methods can be classified into three categories: adjacent partition, interleaved partition and pseudo random partition. Then, the sub blocks are transformed into time-domain partial transmit sequences. The objective is to optimally combine the sub blocks to obtain the time domain OFDM signals with the lowest PAPR.

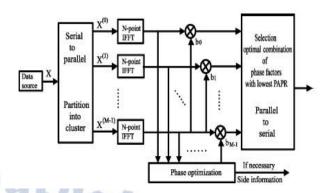


Fig. 1 Block diagram of PTS technique

III. PREVIOUS SYSTEM

Here, a novel Bayesian approach to address the joint PAPR reduction and MUI cancelation problem for downlink multi- user massive MIMO-OFDM systems is developed. Specifically, MUI cancelation can be formulated as an underdetermined linear inverse problem which admits numerous solutions. The block diagram is shown in fig.2.

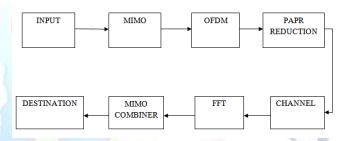


Fig. 2 System block diagram

search for a low PAPR solution, hierarchical truncated Gaussian mixture prior model is proposed and assigned to the unknown signal (i.e. solution). This hierarchical prior has the potential to encourage a quasi-constant magnitude solution with as many entries as possible lying on the truncated boundaries, thus resulting in a low PAPR. A variational expectation-maximization (EM) algorithm is developed to obtain estimates of the hyper parameters associated with the prior model, along with the signal. In addition, the generalized approximate message passing technique is employed to facilitate the algorithm development in the expectation step. This GAMP technique also helps significantly reduce the computational complexity of the algorithm. In this paper, instead of designing the precoding matrix, directly search for the signal w to achieve a joint PAPR reduction MUI cancelation. Approximate message passing (GAMP) is a very-low-complexity Bayesian iterative technique recently developed for obtaining approximate marginal posteriors and likelihoods. It therefore can be naturally embedded within the EM framework to provide an approximate posterior distribution of x and reduce the computational complexity. Specifically, the EM-GAMP framework proceeds in a double loop manner: the outer loop computes the Q-function using approximate posterior distribution of x, and maximizes the Q-function to update the model parameters the inner loop (GAMP) utilizes the newly estimated parameters to obtain a new approximation of the posterior distribution of x. However, this procedure is not suitable for our variational EM framework, because from the GAMP's point of view, the hyper parameters need to be known and fixed in order to compute an approximate posterior distribution of x, while the variational EM treats the model parameters) as latent variables. Therefore, instead of computing the approximate posterior distribution of x, in the variational EM framework, the GAMP is simply used to obtain an amiable approximation of the likelihood function and this approximation involves no latent variables. Besides, unlike the EM-GAMP framework where the inner loop (GAMP) is implemented in an iterative way, in the variational EM- GAMP framework, the GAMP only needs to go through one iteration to obtain an approximation of the likelihood function. In fact, the GAMP algorithm described here is a simplified version of the original GAMP algorithm by retaining only its first three steps and skipping its iterative procedure.

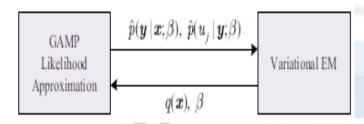


Fig. 3 Variational EM-GAMP framework

To facilitate the algorithm development, we introduce a noise term to model the mismatch between y and Ax. i.e.

$$y = Ax + \epsilon$$

Where ϵ denotes the noise vector and its entries are assumed to be i.i.d. Gaussian random variables with zero-mean and unknown variance β^{-1} . Here β is treated as an unknown parameter because the Bayesian framework allows an automatic determination of its model parameters and usually

provides a reasonable balance between the data fitting error and the desired characteristics of the solution. To reduce the PAPR associated with each transmit antenna, aim is to find a quasi-constant magnitude solution to the above underdetermined linear system. To encourage a quasi- constant magnitude solution, a hierarchical truncated Gaussian mixture prior for the signal x is proposed. In the first layer, coefficients of x are assumed independent of each other and each entry xi is assigned a truncated Gaussian mixture distribution. The second layer specifies Gamma distributions as hyper priors over the precision parameters. In general, Bayesian inference requires computing the logarithm of the prior. To address this issue, here turn the prior into an exponential form by introducing a binary latent variable. A variational expectation maximization (EM) strategy is employed for the Bayesian inference. Consider a probabilistic model with observed data y, hidden variables z and unknown deterministic parameters θ . It is straightforward to show that the marginal probability of the observed data can be decomposed into two terms

$$lnp(y; \theta) = F(q, \theta) + KL(q | | p)$$

KL(q | p) is the Kullback-Leibler divergence between $p(z|y; \theta)$ and q(z). Since $KL(q|p) \ge 0$, it follows that $F(q, \theta)$ is a lower bound of $Inp(y; \theta)$, with the equality holds only when KL(q | p) = 0, which implies $p(z|y; \theta) = q(z)$. The EM algorithm can be viewed as an iterative algorithm which iteratively maximizes the lower bound $F(q, \theta)$ with respect to the distribution q(z) and the parameters

PAPR presence has been area of concern in OFDM and vast amount of research has been carried out using different techniques like Clipping and Filtering (CF), Tone Reservation (TR), Companding Transform (CT), etc. But none of the above techniques succeed in achieving the desired Clipping and filtering technique architecture remains easy to tackle the issue of PAPR but presence of significant OBI, in-band distortion and nonlinear processing makes this technique unused in real time. Compare to in-band distortion, OBI is more critical because it severely interferes with the radio communications in adjacent channels.

In the paper, enlightened by the iterative filtering approach in ICF method, an Iterative CT and Filtering (ICTF) technique is proposed for reducing the PAPR of OFDM signal. By using an iterative procedure, ICTF can obtain a significant PAPR reduction as well as an improved BER performance simultaneously Moreover, to tackle with the OBI issue, a frequency-domain filtering is adopted for minimizing the out-of-band spectral re growth. In addition, when compared to classic ICF method, ICTF dramatically decreases the number of required iterations to obtain a desired PAPR with lower computation complexity. Specifically, it is shown that the ICTF without de-companding operation at the receiver offers a good BER performance.

IV. PROPOSED METHOD

A. Description of Proposed Method

International telecom union for frequencies (ITU- RF) has approved the bit error rate (BER) as performance evaluation parameter to get desired statistics of the signal. In OFDM, BER is gradually declined due to companding distortion and the sudden declination in BER performance is due to signal attenuation factor which compresses the original symbols. Minimization of OBI is another prominent factor to be considered and to successfully handle the problem of OBI a frequency-domain filtering is utilized in the latter stage. Finally the compression of peak signals came into control by performing the companding and filtering approaches for multiple times respectively till to achieve the desired result.

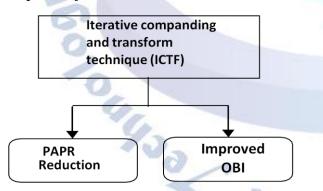


Figure 4: Iterative companding and transform technique and its mechanism

OFDM offers high data rate at one end and on other end its performance is badly impacted by PAPR. The above block diagram design intended to mitigate the peak values in accurate way. Here OFDM system model is comprises of transmitter and receiver, while ICTF technique is deployed at transmitter end for PAPR reduction. Over-sampled Inverse IFFT operation is used to convert the

complex vector $X \in \mathbb{C}^N$ in accurate manner. Here two constants K_1 and K_2 are used to switch the single and multiple operations in respective iteration level.

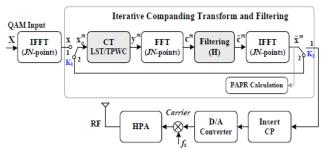


Fig.5. Block diagram of OFDM transmitter using ICTF technique.

If K₁ value is set to 1, then OFDM symbol $X \in C^{JN}$ is given as input to ICTF at the iteration, M=1 and these iterations are processed based on symbol-by-symbol process. In case, if both K1 and K_2 are set to 2, then in that stage both companding and ICTF are used for the same OFDM symbol. In last iteration both constants values are set to 1 again to get the output as $\tilde{X}^m \in C^{JN}$ reactively. Assume $c^m \in C^{JN}$ and $\tilde{c}^m \in C^{JN}$ represented the frequency-domain OFDM symbol at mth iterative level (before and after filtering process). The proposed figure area as follows The proposed method intends to decrease the PAPR impact and achieves improved OBI performance. The presence companding noise and channel noise results in companding distortion. Finally BER analysis is carried out using companding distortion. Two companding transform techniques is initialized by linear (LST) companding transform technique and non linear companding scheme (TPCW).

(i) Linear symmetrical transform (LST)

Linear symmetrical transform (LST) is a companding transform profile and its respective companding transform function is as follows

$$f(x) = (k \cdot |x| + b) \cdot \operatorname{sgn}(x)$$
 (1)

The above equation is composed of sign function and two parameters. These parameters are used to specify companding profile. PAPR reduction and improved BER is achieved by selecting the parameters in linear regions of companding profile. Average power alternation is concerned area in

companding transform and by $k^2 + \sqrt{\pi} \cdot \frac{kb}{\sigma} + \frac{b^2}{\sigma^2}$ is

used to maintain the average power in unchanged

form. The decompanding function is notated as follows

$$f^{-1}(x) = \frac{|x| - b}{k}.\operatorname{sgn}(x) \tag{2}$$

The PAPR and the transform gain G is defined as the ratio of the PAPR of original symbol to that of the companded symbol are as follows

$$PAPR_{LST}(db) = 10\log \frac{\max_{n \in [0,JN-1]\{|y_n|^2\}}}{\frac{1}{JN} \sum_{n=0}^{JN-1} |y_n|^2}$$

$$= 20\log \frac{k.v+b}{\sigma} \qquad (3)$$

$$G_{LST}(dB) = 10\log \frac{PAPR_{org}}{PAPR_{LST}} = 20\log \frac{k.v+b}{\sigma} \qquad (4)$$

(ii) Two-Piecewise Companding (TPWC)

There are four classified companding transform profiles in both linear and non-linear way, Linear Nonsymmetrical Transform (LNST) has achieved best results in terms of PAPR reduction and BER performance over remaining companding transform profiles. The companding function is as follows

$$f(x) = \begin{cases} u_1 |x| \cdot \operatorname{sgn}(x) & |x| \le v \\ (u_2 |x| + s) \cdot \operatorname{sgn}(x), |x| > v^1 \end{cases}$$
 (5)

For $u_1 > 1$, $0 < u_2 < 1$, $s = (u_1 - u_2)v > 0$ and $0 \le v \le V$ is the cutoff point with $V = \max_{0 \le n \le JN - 1\{|x_n|\}}$ with

maximum value $V = \max_{0 \le n \le JN-1\{|x_n|\}}$

The de-companding function of TPWC is given by

$$f^{-1}(x) = \begin{cases} \frac{1}{u_1} |x| \cdot \operatorname{sgn}(x) & |x| \le u_1 v \\ \left(\frac{1}{u_2} |x| - s\right) \cdot \operatorname{sgn}(x), |x| > u_1 v \end{cases}$$
 (6)

For a single TPWC-CT approach, Its achievable PAPR is given by

$$PAPR_{TPWC}(dB) = 20\log\frac{u_2V + s}{\sigma}$$
 (7)

The corresponding transform gain G is written by

$$G_{TPWC}(dB) = 20\log \frac{V}{u_2V + s}$$
 (8)

(B) Companding Distortion Analysis

(i) Companding noise

The BER performance in ICTF procedure is investigated. Based on Buss gang theorem for real and complex Gaussian Signal, the companded signal can be approximately decomposed into two parts the attenuated signal component and companding noise, ρ_n , i.e. $y_n = SAF \times x_n + \rho_n$ Thus, the transmitted symbol with 'm' iterations using ICTF can be approximately decomposed $x_n^m = SAF^m \times x_n + \rho_n$

(ii) Channel noise

Improved BER is achieved even in the absence of decompounding operation at receiver end. The necessary theoretical analysis as follows;

(a) ICTF-LST

At receiver end, When decompounding operation is performed and if m=1 (Iteration 1) the received signal is as follows

$$\tilde{x}_n = f^{-1} \left(f\left(x_n\right) + \omega_n \right) = x_n + \frac{\omega_n}{k} \tag{9}$$

Where ω_n = channel noise

If number of iterations increased then it results in noise,

$$e_n^{(m)} = \left| \frac{\omega_n}{k^m} \right| \tag{10}$$

As a result, it is preferable to abandon the decompanding at the receiver. It is noteworthy that this is quite advantageous for practical OFDM systems.

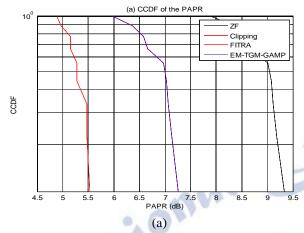
(b) ICTF-TPWC

ICTF-TPWC analysis is carried out in same way as ICTF-LST. Assuming that m iterative de-companding operations are performed at the receiver, the recovery error is approximately given by

by
$$e_{n}^{(m)} = \begin{cases} \left| \frac{\omega_{n}}{u_{1}^{m}} \right|, n \in \phi_{1}(v) \\ \left| \frac{\omega_{n}}{u_{2}^{m}} \right|, n \in \phi_{2}(v) \end{cases}$$

$$(11)$$

V. RESULTS AND DISCUSSION



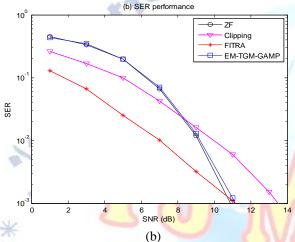


Fig. 6. PAPR and symbol error rate (SER) performance for various schemes. (a) CCDF of the PAPR, (b) SER performance.

To better evaluate the PAPR reduction performance, we plot the CCDF of the PAPR for respective schemes in Fig. 6(a). The number of trials is chosen to be 1000 in our experiments. Note that PAPRs associated with all M antennas are taken in account in calculating the empirical CCDF. it reduces the PAPR by **2dB** The SER performance of respective schemes is shown in Fig. 6(b), We observe that the proposed algorithm incurs an SNR-performance loss of 2.5 dB and 1.7 dB (at SER = 10⁻³) compared to the ZF and FITRA schemes, respectively.

Assume that, the number of subcarriers is N=128 using quaternary phase shift keying (QPSK) or 16 Quadrature amplitude modulation (16-QAM). Over sampling j=4 is used in proposed work to accurate PAPR estimation.

BER performance is analyzed in both the AWGN channel and multipath fading channels (RACIAN and SUI) are applied. A CP with length of ¹/₄ symbols is inserted to control ICF. ICTF achieves better PAPR reduction (**6.57dB PAPR** reduction is achieved) over traditional CT techniques and ICF.

The impact of PAPR reduction technique on OFDM system is observed using BER and OBI.

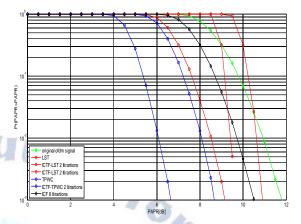


Fig. 6. CCDF statistics of OFDM symbol for different PAPR-reduction schemes (N=1024, QPSK, and the over-sampling ratio J=4).

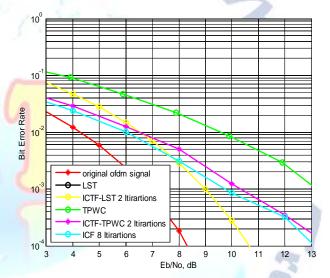


Fig. 7. BER comparison for different PAPR-reduction schemes through AWGN channel for OFDM system (N=1024, QPSK).

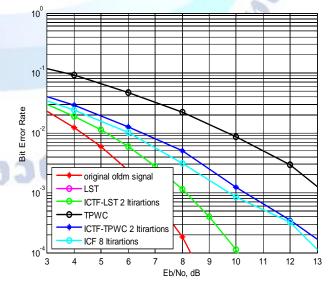


Fig. 8. BER comparison for different PAPR-reduction schemes through Rician fading channel for OFDM system (N=1024, QPSK).

VI. CONCLUSION

ICTF is an equipped approach has ability to reduce PAPR and improved BER performance. Both AWGN channel and SUI channel for reliability and efficiency. ICTF achieves good results over conventional techniques. The proposed ICTF technique not only obtains significant PAPR reduction with improved BER and OBI performance, but also dramatically decreases the iterations number. In addition, ICTF procedure can also be extended to other well known linear and nonlinear companding profiles.

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