



Vibrations of Static Stresses in Thermoelastic Solid Cylinder

Manjula Ramagiri

Department of Mathematics, University Arts and Science College Autonomous, Kakatiya University, Warangal-506009, Telangana, INDIA. Email: manjularamagiri@gmail.com

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ABSTRACT

Wave propagation in thermoelastic solid cylinder with static stresses has been studied. Governing equations are derived for thermoelastic solid cylinder in the presence of static stresses. The frequency equation is obtained in the case of uniaxial static stress. Phase velocity against uniaxial stress and wave number is computed for four types of thermoelastic solids and the results are presented graphically.

KEY WORDS: Thermoelastic cylinder, Phase velocity, Static stresses, Wavenumber, Frequency equation.

1. INTRODUCTION:

The Thermoelastic stress analysis is mostly used in motorcar- and airplane Engineering. The word elasticity comprises the properties of solid materials. The variation in temperature is responsible for producing the stresses along any of dimensions in solids. The solid rod, circular plates, and rectangular plates have been considered and analyzed for variation of temperature. Hollow cylinders are attentive to transfer of heat from one place to other. Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross section is studied by (Ponnuswamy, 2007). (Dilip and Navneet Kumar, 2016) authors discussed the elastic analysis of functionally graded hollow cylinder subjected to uniform temperature field. (MottG, 1971) investigated elastic motion of an isotropic medium in the presence of body forces and static stresses. Stresses of transient thermoelastic problem in an elliptical cylinder are investigated by (Sunil and Badge, 2019). Flexural vibrations of poroelastic solid cylinder in the presence of static stresses studied by (Manjula et.al, 2015). In this paper non-dimensional phase velocity is computed as a function of static uniaxial stress and wavenumber. Effect of static stress on plane strain vibrations in poroelastic solid cylinder is discussed by (Rajitha, MallaReddy, 2015), Vibrations of poroelastic solid cylinder in the presence of static stresses in the pervious surface are studied by (Srisailiam Alety, 2014). From this paper frequency equations for pervious surface are obtained in the frame work of Biot's theory. Thermoelastic interactions in a hollow cylinder due to a continuous heat source without energy dissipation are discussed by (Ashraf and Marwan, 2020). Thermoelastic analysis of a non homogeneous hollow cylinder with internal heat generation is presented in [9]. Thermoelastic

behavior in thin hollow cylinder using internal moving heat source is studied by (Yogit et.al, 2019). In this paper authors studied thermoelastic analysis for an infinite solid cylinder due to harmonically varying heat with thermal conductivity variable. (Singh and Renu, 2017) studied displacement field due to a cylindrical inclusion thermoelastic half space. An exact solution to the elastic deformation of a finite length hollow cylinder is discussed by (Sierakoroski and Sun, 1968). Analytical solution of the coupled dynamic thermoelasticity problem in a hollow cylinder is studied by (Sharifi et.al, 2020). Thermal stresses in a finite solid cylinder are investigated by (Sunil and Bagde, 2019). On determination of dynamic and static stress components from experimental thermoelastic data is investigated by (Ryall et.al, 1992). In all the above cited literature, the authors have not taken static stress for thermoelastic solid cylinder. In this manuscript, we intend to study the vibrations of static stresses in thermoelastic solid cylinder as a novel approach.

2. GOVERNING EQUATIONS AND SOLUTION OF THE PROBLEM

Consider an isotropic thermoelastic solid cylinder in cylindrical polar coordinate system (r, θ, z) . Let $\vec{u}(u, v, w)$ are displacements. The effective stress components in the case of elastic isotropic solid is given by (Mott G, 1971) are

$$\begin{aligned}\sigma'_{rr} &= \sigma_{rr} - \sigma_{r\theta} \frac{\partial u}{\partial \theta} - \sigma_{rz} \frac{\partial u}{\partial z}, \\ \sigma'_{r\theta} &= \sigma_{r\theta} - \sigma_{rr} \frac{\partial v}{\partial r} - \sigma_{rz} \frac{\partial v}{\partial z}, \\ \sigma'_{rz} &= \sigma_{rz} - \sigma_{rr} \frac{\partial w}{\partial r} - \sigma_{r\theta} \frac{\partial w}{\partial \theta}, \\ \sigma'_{\theta\theta} &= \sigma_{\theta\theta} - \sigma_{\theta r} \frac{\partial v}{\partial r} - \sigma_{\theta z} \frac{\partial v}{\partial z}, \\ \sigma'_{\theta r} &= \sigma_{\theta r} - \sigma_{\theta\theta} \frac{\partial u}{\partial \theta} - \sigma_{\theta z} \frac{\partial u}{\partial z}, \\ \sigma'_{\theta z} &= \sigma_{\theta z} - \sigma_{\theta r} \frac{\partial w}{\partial r} - \sigma_{\theta\theta} \frac{\partial w}{\partial \theta}, \\ \sigma'_{zr} &= \sigma_{zr} - \sigma_{z\theta} \frac{\partial u}{\partial \theta} - \sigma_{zz} \frac{\partial u}{\partial z}, \\ \sigma'_{z\theta} &= \sigma_{z\theta} - \sigma_{zr} \frac{\partial v}{\partial r} - \sigma_{zz} \frac{\partial v}{\partial z}, \\ \sigma'_{zz} &= \sigma_{zz} - \sigma_{zr} \frac{\partial w}{\partial r} - \sigma_{z\theta} \frac{\partial w}{\partial \theta}.\end{aligned}$$

(1)

In eq. (1), usual stresses σ'_{ij} 's are given (Ponnuswamy, 2014)

$$\begin{aligned}\sigma_{rr} &= \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{rr} - \beta T, \\ \sigma_{\theta\theta} &= \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{\theta\theta} - \beta T, \\ \sigma_{zz} &= \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{zz} - \beta T, \\ \sigma_{r\theta} &= \mu\gamma_{r\theta}, \quad \sigma_{\theta z} = \mu\gamma_{\theta z}, \quad \sigma_{rz} = \mu\gamma_{rz},\end{aligned}$$

$$K\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}\right)T = \rho c_v \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}\right) + \beta T_0 \left(\frac{\partial^2 u}{\partial r \partial t} + \frac{1}{r} \left(\frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial \theta \partial t}\right) + \frac{\partial^2 w}{\partial t \partial z}\right). \quad (2)$$

In eq. (2), σ'_{ij} 's stress components, e_{ij} are strain components, T is the change in temperature about the equilibrium

T_0 , ρ is the mass density, c_p is the specific heat, τ_0 is the relaxation time, β is a coupling factor that couples the heat conduction and elastic field equation $\beta = \frac{3\lambda + 2\mu}{3}$, K is the thermal conductivity t is the time, λ and μ are

Lame's constants. The strain e_{ij} related to the displacements are given by

$$\begin{aligned} e_{rr} &= \frac{\partial u}{\partial r}, \quad e_{\theta\theta} = \frac{u}{r} + \frac{1}{r} \frac{\partial v}{\partial \theta}, \quad e_{zz} = \frac{\partial w}{\partial z}, \\ \gamma_{r\theta} &= \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r} \frac{\partial u}{\partial \theta}, \quad \gamma_{r\theta} = \frac{\partial v}{\partial z} + \frac{1}{r} \frac{\partial w}{\partial \theta}, \quad \gamma_{r\theta} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \end{aligned} \quad (3)$$

Substitution of eq. (1) for stresses, the equations takes the following form

$$\begin{aligned} &\frac{\partial}{\partial r} (\sigma'_{rr} (1 + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z})) + \frac{1}{r} \frac{\partial}{\partial \theta} (\sigma'_{r\theta} (1 + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial z} (\sigma'_{rz} (1 + \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta})) \\ &+ \frac{1}{r} (\sigma'_{rr} (1 + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z}) - \sigma'_{\theta\theta} (1 + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z})) + F_r = \rho \frac{\partial^2 u}{\partial t^2}, \\ &\frac{\partial}{\partial r} (\sigma'_{\theta r} (1 + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z})) + \frac{1}{r} \frac{\partial}{\partial \theta} (\sigma'_{\theta\theta} (1 + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial z} (\sigma'_{\theta z} (1 + \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta})) \\ &+ \frac{2}{r} (\sigma'_{\theta r} (1 + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z})) + F_\theta = \rho \frac{\partial^2 v}{\partial t^2}, \\ &\frac{\partial}{\partial r} (\sigma'_{rz} (1 + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z})) + \frac{1}{r} (\sigma'_{z\theta} (1 + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z})) + \frac{\partial}{\partial z} (\sigma'_{zz} (1 + \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta})) \\ &+ \frac{1}{r} (\sigma'_{rz} (1 + \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z})) + F_z = \rho \frac{\partial^2 w}{\partial t^2}, \\ &K (\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}) T = \rho c_v (\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}) + \beta T_0 (\frac{\partial^2 u}{\partial r \partial t} + \frac{1}{r} (\frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial \theta \partial t}) + \frac{\partial^2 w}{\partial t \partial z}). \end{aligned} \quad (4)$$

In eq. (4) F_r, F_θ, F_z are the components of body force vector \vec{F} . Eqs. (4) with the eq. (1) must satisfy the properties at every point inside and on the body. The effective stresses must satisfy the conditions of the thermoelastic solid cylinder. The relation between the time variant and the static thermoelastic quantities, the displacements and the stresses are taken as follows (Mott G, 1971)

$$\begin{aligned} u &= \sum u_n(r, \theta, z, \omega_n t) = u_0 + u_1 + u_2 + \dots, \\ v &= \sum v_n(r, \theta, z, \omega_n t) = v_0 + v_1 + v_2 + \dots, \\ w &= \sum w_n(r, \theta, z, \omega_n t) = w_0 + w_1 + w_2 + \dots, \\ \sigma_{ij} &= \sum \sigma_{ij_n}(r, \theta, z, \omega_n t) = \sigma_{ij_0} + \sigma_{ij_1} + \sigma_{ij_2} + \dots. \end{aligned} \quad (5)$$

In eq. (5), ω_n is the n^{th} angular frequency. Using the equations (1), (2), (5) in (4) we get the static equations ($n = 0$) given below.

$$\begin{aligned} &(\frac{\partial}{\partial r} + \frac{1}{r}) ((\sigma_{r\theta_0} - \sigma_{r\theta_0} \frac{\partial u_0}{\partial \theta} - \sigma_{rz_0} \frac{\partial u_0}{\partial z}) (1 + \frac{\partial v_0}{\partial \theta} + \frac{\partial w_0}{\partial z})) + \frac{1}{r} \frac{\partial}{\partial \theta} ((\sigma_{r\theta_0} - \sigma_{r\theta_0} \frac{\partial v_0}{\partial r} - \sigma_{rz_0} \frac{\partial v_0}{\partial z}) \\ &\times (1 + \frac{\partial u_0}{\partial r} + \frac{\partial w_0}{\partial z})) + \frac{\partial}{\partial z} ((\sigma_{rz_0} - \sigma_{r\theta_0} \frac{\partial w_0}{\partial r} - \sigma_{r\theta_0} \frac{\partial w_0}{\partial \theta}) (1 + \frac{\partial u_0}{\partial r} + \frac{\partial v_0}{\partial \theta})) - \frac{1}{r} ((\sigma_{\theta\theta_0} - \sigma_{\theta\theta_0} \frac{\partial v_0}{\partial r} \\ &- \sigma_{\theta z_0} \frac{\partial v_0}{\partial z}) (1 + \frac{\partial u_0}{\partial r} + \frac{\partial w_0}{\partial z})) + F_{r_0} = 0, \end{aligned}$$

$$\begin{aligned}
& \left(\frac{\partial}{\partial r} + \frac{2}{r}\right) \left((\sigma_{\theta r_0} - \sigma_{\theta\theta_0}) \frac{\partial u_0}{\partial \theta} - \sigma_{\theta z_0} \frac{\partial u_0}{\partial z} \right) \left(1 + \frac{\partial v_0}{\partial \theta} + \frac{\partial w_0}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left((\sigma_{\theta\theta_0} - \sigma_{\theta r_0}) \frac{\partial v_0}{\partial r} - \sigma_{\theta z_0} \frac{\partial v_0}{\partial z} \right) \left(1 + \frac{\partial u_0}{\partial r} + \frac{\partial w_0}{\partial z} \right) \\
& + \frac{\partial}{\partial z} \left((\sigma_{\theta z_0} - \sigma_{\theta\theta_0}) \frac{\partial w_0}{\partial \theta} - \sigma_{\theta r_0} \frac{\partial w_0}{\partial r} \right) \left(1 + \frac{\partial u_0}{\partial r} + \frac{\partial v_0}{\partial \theta} \right) + F_{\theta_0} = 0, \\
& \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \left((\sigma_{z r_0} - \sigma_{z z_0}) \frac{\partial u_0}{\partial z} - \sigma_{z\theta_0} \frac{\partial u_0}{\partial \theta} \right) \left(1 + \frac{\partial v_0}{\partial \theta} + \frac{\partial w_0}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left((\sigma_{z\theta_0} - \sigma_{z z_0}) \frac{\partial v_0}{\partial z} - \sigma_{z r_0} \frac{\partial v_0}{\partial r} \right) \left(1 + \frac{\partial u_0}{\partial r} + \frac{\partial w_0}{\partial z} \right) \\
& + \frac{\partial}{\partial z} \left((\sigma_{z z_0} - \sigma_{z\theta_0}) \frac{\partial w_0}{\partial \theta} - \sigma_{z r_0} \frac{\partial w_0}{\partial r} \right) \left(1 + \frac{\partial v_0}{\partial \theta} + \frac{\partial u_0}{\partial r} \right) + F_{z_0} = 0, \\
& K \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right) T = \rho c_v \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \beta T_0 \left(\frac{\partial^2 u}{\partial r \partial t} + \frac{1}{r} \left(\frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial \theta \partial t} \right) + \frac{\partial^2 w}{\partial t \partial z} \right).
\end{aligned} \tag{6}$$

Similarly, the harmonic equations ($n = 1$) are as follows

$$\begin{aligned}
& \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \left((\sigma_{r r_1} - \sigma_{r\theta_1}) \frac{\partial u_1}{\partial \theta} - \sigma_{r\theta_0} \frac{\partial u_1}{\partial \theta} - \sigma_{r z_1} \frac{\partial u_1}{\partial z} - \sigma_{r z_0} \frac{\partial u_1}{\partial z} \right) \left(1 + \frac{\partial v_1}{\partial \theta} + \frac{\partial w_1}{\partial z} \right) + \left((\sigma_{r r_0} - \sigma_{r\theta_0}) \frac{\partial u_0}{\partial \theta} - \sigma_{r z_0} \frac{\partial u_0}{\partial z} \right) \\
& \times \left(\frac{\partial v_1}{\partial \theta} + \frac{\partial w_1}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left((\sigma_{r\theta_1} - \sigma_{r r_1}) \frac{\partial v_1}{\partial r} - \sigma_{r\theta_0} \frac{\partial v_1}{\partial r} - \sigma_{r z_1} \frac{\partial v_1}{\partial z} - \sigma_{r z_0} \frac{\partial v_1}{\partial z} \right) \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + (\sigma_{r\theta_0} - \sigma_{r r_0}) \\
& \times \left(\frac{\partial v_0}{\partial \theta} - \sigma_{r z_0} \frac{\partial v_0}{\partial z} \right) \left(\frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + \frac{\partial}{\partial z} \left((\sigma_{r z_1} - \sigma_{r r_1}) \frac{\partial w_1}{\partial r} - \sigma_{r\theta_0} \frac{\partial w_1}{\partial r} - \sigma_{r\theta_1} \frac{\partial w_1}{\partial \theta} - \sigma_{r\theta_0} \frac{\partial w_1}{\partial \theta} \right) \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} \right) \\
& + (\sigma_{r z_0} - \sigma_{r r_0}) \frac{\partial w_0}{\partial r} - \sigma_{r\theta_0} \frac{\partial w_0}{\partial \theta} \left(\frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} \right) - \frac{1}{r} \left((\sigma_{\theta\theta_1} - \sigma_{\theta r_1}) \frac{\partial v_1}{\partial r} - \sigma_{\theta r_0} \frac{\partial v_1}{\partial r} - \sigma_{\theta z_1} \frac{\partial v_1}{\partial z} - \sigma_{\theta z_0} \frac{\partial v_1}{\partial z} \right) \\
& \times \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + \left((\sigma_{\theta\theta_0} - \sigma_{\theta r_1}) \frac{\partial v_0}{\partial r} - \sigma_{\theta z_0} \frac{\partial v_0}{\partial z} \right) \left(\frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + F_{r_1} = \frac{\partial^2}{\partial t^2} (\rho_{11} u_1 + \rho_{12} U_1), \\
& \left(\frac{\partial}{\partial r} + \frac{2}{r}\right) \left((\sigma_{\theta r_1} - \sigma_{\theta\theta_1}) \frac{\partial u_1}{\partial \theta} - \sigma_{\theta\theta_0} \frac{\partial u_1}{\partial \theta} - \sigma_{\theta z_1} \frac{\partial u_1}{\partial z} - \sigma_{\theta z_0} \frac{\partial u_1}{\partial z} \right) \left(1 + \frac{\partial v_1}{\partial \theta} + \frac{\partial w_1}{\partial z} \right) + (\sigma_{\theta r_0} - \sigma_{\theta\theta_0}) \frac{\partial u_0}{\partial \theta} - \sigma_{\theta z_0} \frac{\partial u_0}{\partial z} \\
& \times \left(\frac{\partial v_1}{\partial \theta} + \frac{\partial w_1}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left((\sigma_{\theta\theta_1} - \sigma_{\theta r_1}) \frac{\partial v_1}{\partial r} - \sigma_{\theta\theta_0} \frac{\partial v_1}{\partial r} - \sigma_{\theta z_1} \frac{\partial v_1}{\partial z} - \sigma_{\theta z_0} \frac{\partial v_1}{\partial z} \right) \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + \left((\sigma_{\theta\theta_0} - \sigma_{\theta r_0}) \frac{\partial v_0}{\partial r} \right. \\
& \left. - \sigma_{\theta z_0} \frac{\partial v_0}{\partial z} \right) \left(\frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + \frac{\partial}{\partial z} \left((\sigma_{\theta z_1} - \sigma_{\theta\theta_1}) \frac{\partial w_1}{\partial r} - \sigma_{\theta\theta_0} \frac{\partial w_1}{\partial r} - \sigma_{\theta r_1} \frac{\partial w_1}{\partial \theta} - \sigma_{\theta r_0} \frac{\partial w_1}{\partial \theta} \right) \times \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} \right) + (\sigma_{\theta z_0} \\
& - \sigma_{\theta\theta_0}) \frac{\partial w_0}{\partial \theta} - \sigma_{\theta r_0} \frac{\partial w_0}{\partial r} \left(\frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} \right) + F_{\theta_1} = \rho \frac{\partial^2 v_1}{\partial t^2}, \\
& \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \left((\sigma_{z r_1} - \sigma_{z z_1}) \frac{\partial u_1}{\partial z} - \sigma_{z z_0} \frac{\partial u_1}{\partial z} - \sigma_{z\theta_1} \frac{\partial u_1}{\partial \theta} - \sigma_{z\theta_0} \frac{\partial u_1}{\partial \theta} \right) \left(1 + \frac{\partial v_1}{\partial \theta} + \frac{\partial w_1}{\partial z} \right) + (\sigma_{z r_0} - \sigma_{z z_0}) \frac{\partial u_0}{\partial z} \\
& - \sigma_{z\theta_0} \frac{\partial u_0}{\partial \theta} \left(\frac{\partial v_1}{\partial \theta} + \frac{\partial w_1}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left((\sigma_{z\theta_1} - \sigma_{z z_1}) \frac{\partial v_1}{\partial z} - \sigma_{z z_0} \frac{\partial v_1}{\partial z} - \sigma_{z r_1} \frac{\partial v_1}{\partial r} - \sigma_{z r_0} \frac{\partial v_1}{\partial r} \right) \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) \\
& + (\sigma_{z\theta_0} - \sigma_{z z_0}) \frac{\partial v_0}{\partial z} - \sigma_{z r_0} \frac{\partial v_0}{\partial r} \left(\frac{\partial u_1}{\partial r} + \frac{\partial w_1}{\partial z} \right) + \frac{\partial}{\partial z} \left((\sigma_{z z_1} - \sigma_{z\theta_1}) \frac{\partial w_1}{\partial \theta} - \sigma_{z\theta_0} \frac{\partial w_1}{\partial \theta} - \sigma_{z r_1} \frac{\partial w_1}{\partial r} - \sigma_{z r_0} \frac{\partial w_1}{\partial r} \right) \\
& \times \left(1 + \frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} \right) + (\sigma_{z z_0} - \sigma_{z\theta_0}) \frac{\partial w_0}{\partial \theta} - \sigma_{z r_0} \frac{\partial w_0}{\partial r} \left(\frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} \right) + F_{z_1} = \rho \frac{\partial^2 w_1}{\partial t^2}, \\
& K \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right) T = \rho c_v \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \beta T_0 \left(\frac{\partial^2 u}{\partial r \partial t} + \frac{1}{r} \left(\frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial \theta \partial t} \right) + \frac{\partial^2 w}{\partial t \partial z} \right).
\end{aligned} \tag{7}$$

(7)

In eq. (7) $F_{r_1}, F_{\theta_1}, F_{z_1}$ are the body forces which have the frequency ω . The eqs. (6) and (7) are the static equations at first harmonic equations.

3. EQUATION OF UNIAXIAL STATIC STRESS

When the large static uniaxial stress is applied on a solid medium one can neglect the effect of body forces (Mott G, 1971). If we assume that applied static uniaxial stress along the z -axis, then we have

$$\sigma_{ij_0} = e_{ij_0} = 0 \quad (i \neq j), \quad \sigma_{r_0} = \sigma_{\theta_0} = 0, \quad F_{r_0} = F_{\theta_0} = F_{z_0} = 0, \quad F_{r_1} = F_{\theta_1} = F_{z_1} = 0, \quad (8)$$

where σ_{z_0} is static uniaxial stress. Substituting the eqs. (8) in the eqs. (7), we can see that the eqs. (6) must satisfy and the static strains are obtained as follows:

$$e_{r_0} = e_{\theta_0} = -\mathcal{G} e_{z_0}, \quad T = T_0. \quad (9)$$

Substituting the above eqs.(8)and (9) in eq. (7) we the get the equations in the following form

$$\begin{aligned} & \left[1 + (1 - \mathcal{G}) \frac{\partial w_0}{\partial z} \right] \left[\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta r}}{\partial \theta} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) \right] + \left[1 - 2\mathcal{G} \frac{\partial w_0}{\partial z} \right] \frac{\partial \sigma_{rz}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2}, \\ & \left[1 + (1 - \mathcal{G}) \frac{\partial w_0}{\partial z} \right] \left[\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{2\sigma_{\theta r}}{r} \right] + \left[1 - 2\mathcal{G} \frac{\partial w_0}{\partial z} \right] \frac{\partial \sigma_{\theta z}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2}, \\ & \left[1 + (1 - \mathcal{G}) \frac{\partial w_0}{\partial z} \right] \left[\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\sigma_{zr}}{r} \right] + \left[1 - 2\mathcal{G} \frac{\partial w_0}{\partial z} \right] \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2}, \\ & K \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \right) T = \rho c_v \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \beta T_0 \left(\frac{\partial^2 u}{\partial r \partial t} + \frac{1}{r} \left(\frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial \theta \partial t} \right) + \frac{\partial^2 w}{\partial t \partial z} \right). \end{aligned} \quad (10)$$

Substituting equations (9) in the first equation of the eq. (2), we obtain the following equation

$$e_{z_0} = \frac{\sigma_{z_0} + \beta T_0}{Y}. \quad (11)$$

In the above, $\mathcal{G} = \frac{\lambda}{2(\lambda + \mu)}$ is the Poisson's ratio and $Y = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)}$ is the Young's modulus. Substituting the

eqs. (2), (3), (11) in eq. (10), we get

$$\begin{aligned} & \lambda \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) + \mu \left(\frac{2\partial^2 u}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{2}{r} \frac{\partial^2 u}{\partial r^2} - \frac{2u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) + \left(\frac{\lambda + \mu}{r} \right) \frac{\partial^2 v}{\partial \theta \partial z} + \left(\frac{2\lambda - \mu}{r^2} \right) \frac{\partial v}{\partial \theta} + \\ & (\lambda + \mu) \frac{\partial^2 w}{\partial r \partial z} + \left(\frac{\lambda}{r} - \frac{1}{r} \right) \frac{\partial w}{\partial z} + \beta \left(\frac{T}{r} - \frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial T}{\partial r} \right) + \frac{1}{2(\lambda + \mu)} \left(\frac{\sigma_{z_0} + \beta T_0}{Y} \right) \left((\lambda + 2\mu) \left(\lambda \frac{\partial^2 u}{\partial r^2} + \frac{\lambda}{r} \frac{\partial u}{\partial r} + \right. \right. \\ & \left. \left. \frac{\lambda}{r} \frac{\partial^2 v}{\partial \theta \partial r} + \lambda \frac{\partial^2 w}{\partial r \partial z} + 2\mu \frac{\partial^2 u}{\partial r^2} - \beta \frac{\partial T}{\partial r} + \frac{\mu}{r} \frac{\partial^2 v}{\partial r \partial \theta} - \frac{\mu}{r^2} \frac{\partial v}{\partial \theta} + \frac{\mu}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\lambda}{r^2} \frac{\partial u}{\partial r} + \frac{\lambda}{r} \frac{\partial v}{\partial \theta} + \frac{\lambda}{r} \frac{\partial w}{\partial z} + \frac{2\mu}{r^2} \frac{\partial^2 u}{\partial r^2} \right. \right. \\ & \left. \left. - \frac{\beta}{r} \frac{\partial T}{\partial r} - \frac{\lambda u}{r^2} + \frac{\lambda}{r^2} \frac{\partial v}{\partial \theta} - \frac{1}{r} \frac{\partial w}{\partial z} - \frac{2\mu u}{r^2} - \frac{2\mu}{r^2} \frac{\partial v}{\partial \theta} + \frac{\beta T}{r} \right) - (2\lambda \mu \frac{\partial^2 w}{\partial r \partial z} + 2\lambda \mu \frac{\partial^2 u}{\partial z^2}) = \rho \frac{\partial^2 u}{\partial t^2}, \end{aligned}$$

$$\begin{aligned}
& \left(\frac{\mu+\lambda}{r}\right)\frac{\partial^2 u}{\partial r\partial\theta} + \left(\frac{\lambda}{r^2} + \frac{2\mu}{r} + \frac{2\mu}{r^2}\right)\frac{\partial u}{\partial\theta} + \mu\left(\frac{\partial^2 v}{\partial r^2} - \frac{1}{r}\frac{\partial v}{\partial r} + \frac{2}{r}\frac{\partial v}{\partial r} + \frac{2v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right) + \left(\frac{\lambda+2\mu}{r^2}\right)\frac{\partial^2 v}{\partial\theta^2} + \\
& \left(\lambda + \frac{\mu}{r}\right)\frac{\partial^2 w}{\partial\theta\partial z} - \beta\frac{\partial T}{\partial\theta} + \frac{1}{2(\lambda+\mu)}\left(\frac{\sigma_{zz_0} + \beta T_0}{Y}\right)\left((\lambda+2\mu)\left(\mu\frac{\partial^2 v}{\partial r^2} - \frac{\mu}{r}\frac{\partial v}{\partial r} + \frac{\mu}{r}\frac{\partial^2 u}{\partial r\partial\theta} + \frac{\lambda}{r^2}\frac{\partial u}{\partial\theta} + \frac{\lambda}{r^2}\frac{\partial^2 v}{\partial\theta^2} + \right.\right. \\
& \left.\left.\lambda\frac{\partial^2 w}{\partial\theta\partial z} + \frac{2\mu}{r}\frac{\partial u}{\partial\theta} + \frac{2\mu}{r^2}\frac{\partial^2 v}{\partial\theta^2} - \beta\frac{\partial T}{\partial\theta} + \frac{2\mu}{r}\frac{\partial v}{\partial r} - \frac{2\mu v}{r^2} + \frac{2\mu}{r^2}\frac{\partial u}{\partial\theta}\right) - 2\lambda\left(\mu\frac{\partial^2 v}{\partial z^2} + \frac{\mu}{r}\frac{\partial^2 w}{\partial\theta\partial z}\right) = \rho\frac{\partial^2 v}{\partial t^2}, \\
& (\mu+\lambda+\sigma_{zz_0})\frac{\partial^2 u}{\partial r\partial z} + \left(\frac{\mu+\lambda}{r} + \sigma_{zz_0}\right)\frac{\partial^2 v}{\partial z\partial\theta} + \left(\frac{\mu+\lambda}{r}\right)\frac{\partial u}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) + (\lambda+2\mu)\frac{\partial^2 w}{\partial z^2} - \\
& \beta\frac{\partial T}{\partial z} + \frac{1}{2(\lambda+\mu)}\left(\frac{\sigma_{zz_0} + \beta T_0}{Y}\right)\left((\lambda+2\mu)\left(\mu\frac{\partial^2 u}{\partial r^2} + \mu\frac{\partial^2 v}{\partial z\partial r} + \frac{\mu}{r}\frac{\partial^2 v}{\partial z\partial\theta} + \frac{\mu}{r^2}\frac{\partial^2 w}{\partial\theta^2} + \frac{\mu}{r}\frac{\partial w}{\partial r} + \frac{\mu}{r}\frac{\partial u}{\partial z}\right) - \right. \\
& \left. 2\lambda\left(\lambda\frac{\partial^2 u}{\partial r\partial z} + \frac{\lambda}{r}\frac{\partial u}{\partial z} + \frac{\lambda}{r}\frac{\partial^2 v}{\partial z\partial\theta} + \lambda\frac{\partial^2 w}{\partial z^2} + 2\mu\frac{\partial^2 w}{\partial z^2} - \beta\frac{\partial T}{\partial z}\right)\right) = \rho\frac{\partial^2 w}{\partial t^2}, \\
& K\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}\right)T = \rho c_v\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \beta T_0\left(\frac{\partial^2 u}{\partial r\partial t} + \frac{1}{r}\left(\frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial\theta\partial t}\right) + \frac{\partial^2 w}{\partial t\partial z}\right).
\end{aligned} \tag{12}$$

The solution of above eq. (12) takes the following form:

$$(u, v, w, T)(r, \theta, z, t) = (C_1, C_2, C_3, C_4)e^{j\omega t - j(k_1 r + k_2 \theta + k_3 z)}. \tag{13}$$

In the above, C_1, C_2, C_3, C_4 are arbitrary constants, k_i ($i = 1, 2, 3$) is the wave number in the j^{th} direction, such that the wave number $k = \sqrt{k_1^2 + k_2^2 + k_3^2}$. substituting the eq. (13) in the eq. (12), we get equations in the following form:

$$\begin{aligned}
& \left(\lambda(j^2 k_1^2 - \frac{j}{r}k_1 + \frac{1}{r}jk_1 - \frac{u}{r^2}) + \mu(2j^2 k_1^2 + \frac{1}{r^2}j^2 k_2^2 + \frac{2}{r^2} + j^2 k_3^2) + \left(\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y}\right)((\lambda+2\mu)(\lambda j^2 k_1^2 + \right. \\
& \left. \frac{\lambda}{r}jk_1 + 2\mu j^2 k_1^2 + \frac{\mu}{r^2}j^2 k_2^2 - \frac{\lambda}{r^2}jk_1 + \frac{2\mu}{r^2}j^2 k_1^2 - \frac{\lambda}{r^2} - \frac{2\mu}{r^2} - 2\lambda\mu j^2 k_3^2 - j^2 \omega^2)\right)C_1 + \left(\frac{3\mu}{r^2}jk_2 + \left(\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y}\right)\right) \times \\
& (\lambda+2\mu)\left(\frac{\lambda}{r}j^2 k_1 k_2 + \frac{\mu}{r}j^2 k_1 k_2 + \frac{\mu+\lambda}{r^2}jk_2 + \frac{2\mu}{r^2}jk_2\right)C_2 + (\lambda+\mu)j^2 k_1 k_3 - \frac{\lambda-1}{r}jk_3 + \left(\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y}\right)(\lambda+2\mu) \times \\
& \left(\lambda j^2 k_1 k_2 - \frac{\lambda+1}{r}jk\right) - (2\lambda\mu j^2 k_1 k_3)C_3 + \left(\beta\left(\frac{1}{r} + \frac{1}{r}jk_1 - jk_1\right) + \left(\frac{\sigma_{zz_0} + \beta T_0}{2(\lambda+\mu)Y}\right)\left(\beta jk_1 + \frac{\beta}{r}jk_1 + \frac{\beta}{r}\right)\right)C_4 = 0,
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\frac{\mu + \lambda}{r} \right) j k_1 k_2 - \left(\frac{\lambda}{r^2} + \frac{2\mu}{r} + \frac{2\mu}{r^2} \right) j k_2 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) (\lambda + \mu) \left(\frac{\mu}{r} j^2 k_1 k_2 + \frac{\lambda}{2} j^2 k_1 k_2 - \frac{\lambda}{r^2} j k_2 - \frac{2\mu}{r} j k_2 + \right. \right. \\
& \left. \left. \frac{2\mu}{r^2} j k_2 \right) \right) C_1 + \left((\mu j^2 k_1^2 + \frac{2\mu}{r^2} j k_1 + \frac{2\mu}{r^2} j k_1 + \mu j^2 k_3^2 + \frac{\lambda + 2\mu}{r^2} j^2 k_2^2 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) ((\lambda + 2\mu)(\mu j^2 k_1^2 + \right. \right. \\
& \left. \left. \frac{\mu}{r} j k_1 + \frac{\lambda}{r^2} j^2 k_2^2 + \frac{2\mu}{r^2} j^2 k_2^2 - \frac{2\mu}{r} j k_1 - \frac{2\mu}{r^2} j k_1) - 2\lambda \mu j^2 k_3 - j^2 \rho \omega^2 \right) \right) C_2 + \left((\lambda + \frac{\mu}{r}) j^2 k_2 k_3 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) \times \right. \\
& \left. (\lambda + 2\mu) \lambda j^2 k_2 k_3 - \frac{2\lambda \mu}{r} j^2 k_2 k_3 \right) C_3 + \left(\beta j k_2 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) (\lambda + 2\mu) - \beta j k_1 \right) C_4 = 0, \\
& \left((\mu + \lambda + \sigma_{z_0}) j^2 k_1 k_3 - \left(\frac{\lambda + \mu}{r} \right) j k_3 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) ((\lambda + 2\mu)(\mu j^2 k_1^2 + \mu j^2 k_1 k_3 - \frac{\mu}{r} j k_3 - 2\lambda^2 j^2 k_1 k_3 - \right. \right. \\
& \left. \left. \frac{\lambda}{r} j k_3) \right) \right) C_1 + \left(\left(\frac{\mu + \lambda}{r} + \sigma_{z_0} \right) j^2 k_2 k_3 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) ((\lambda + 2\mu) \left(\frac{\mu}{r} j^2 k_2 k_3 - \frac{2\lambda^2}{r} j^2 k_2 k_3 \right) \right) C_2 + \left((\mu j^2 k_1^2 \right. \\
& \left. + \frac{\mu}{r} j^2 k_2^2 - \frac{\mu}{r} j k_1 + (\lambda + 2\mu) j^2 k_3^2 + \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) ((\lambda + 2\mu) \left(\frac{\mu}{r^2} j^2 k_2^2 - \frac{\mu}{r} j k_1 - 2\lambda^2 j^2 k_3^2 - 4\lambda \mu j^2 k_3^2 - \right. \right. \\
& \left. \left. \omega^2 j^2 \right) \right) C_3 + \left(\beta j k_3 - \left(\frac{\sigma_{z_0} + \beta T_0}{2(\lambda + \mu)Y} \right) (2\lambda \beta j k_3) \right) C_4 = 0, \\
& (\beta T_0 j^2 k_1 + \beta T_0 \frac{1}{r} j \omega) C_1 + (\beta T_0 \frac{1}{r} j^2 k_2) C_2 + (\beta T_0 \frac{1}{r} j^2 k_3 \omega) C_3 + (K j^2 k_1^2 + K \frac{1}{r} j k_1 + K \frac{1}{r^2} j k_2^2 + \\
& K j^2 k_3^2 - \rho c_v j \omega - \rho \tau_0 \omega^2 j^2) C_4 = 0.
\end{aligned} \tag{14}$$

4. NUMERICAL RESULTS AND DISCUSSION

The wave propagation is considered along z -direction and at $r = a$. In this case $k_1 = k_2 = 0$. Then the equations reduces to

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = 0.$$

Where

$$\begin{aligned}
A_{11} &= \frac{2\mu}{(k_3 a)^2} - \frac{(3\sigma_{z_0} + (3\lambda + 2\mu)\alpha_r T_0)(\lambda + 2\mu)(2\lambda\mu - c^2)}{18\mu\lambda + 12\mu^2}, \\
A_{13} &= \frac{1}{j} \left(\frac{1 - \lambda}{k_3 a} + \frac{(3\sigma_{z_0} + (3\lambda + 2\mu)\alpha_r T_0)}{12(\lambda + \mu)(3\lambda\mu + 2\mu^2)} - \frac{\lambda}{k_3 a} + \frac{1}{k_3 a} \right),
\end{aligned}$$

$$\begin{aligned}
A_{22} &= (\mu - \frac{(3\sigma_{zz_0} + (3\lambda + 2\mu)\alpha_i T_0)(\lambda + 2\mu)(\frac{2\mu}{k_3 a} + 2\lambda\mu - \rho c^2)}{18\mu\lambda + 12\mu^2}), \\
A_{31} &= \frac{1}{j} (\frac{\mu + \lambda}{k_3 a} + \frac{(3\lambda + 2\mu)\alpha_i T_0 + 3\sigma_{zz_0}}{2(\lambda + \mu)(3\lambda + 2\mu)} (\frac{\mu}{k_3 a} + \frac{\lambda}{k_3 a})), \\
A_{33} &= ((\lambda + 2\mu) + \frac{(3\lambda + 2\mu)\alpha_i T_0 + 3\sigma_{zz_0}}{2(\lambda + \mu)(3\lambda + 2\mu)}) (\lambda + 2\mu)(2\lambda^2 - 4\lambda\mu - c^2), \\
A_{34} &= \frac{\beta}{jk_3} (1 - \frac{(3\lambda^2 + 2\lambda\mu)\alpha_i T_0}{3\lambda + 3\mu}), \quad A_{43} = \frac{\beta T_0 c}{k_3 a}, \\
A_{44} &= K - \frac{1}{j} (\rho c_v c^2 - \rho \tau_0 c^2), \quad A_{12} = A_{14} = A_{21} = A_{23} = A_{24} = A_{32} = A_{41} = A_{42} = 0.
\end{aligned}
\tag{16}$$

In the above eq. (16), $c = \frac{\omega}{k}$, c is the phase velocity. For a non-trivial solution, the determinant of above coefficient matrix is zero. This leads to the frequency equation:

$$\begin{vmatrix}
A_{11} & 0 & A_{13} & 0 \\
0 & A_{22} & 0 & 0 \\
A_{31} & 0 & A_{33} & A_{34} \\
0 & 0 & A_{43} & A_{44}
\end{vmatrix} = 0
\tag{17}$$

In the above eq. (17) we have calculated real phase velocity. If we consider the imaginary part of phase velocity the determinant becomes zero. The theoretical results are computed for the Magnesium, Copper, Zinc is given by (Ponnuswamy, 2014, Erigen, 1984, Sherief and Salah, 2005, Jaswant Singh and Tomar, 2011). The physical data are given below:

Magnesium

$$\begin{aligned}
\lambda &= 9.4 \times 10^{10} \text{ Nm}^{-2}, \quad \mu = 4.0 \times 10^{10} \text{ Nm}^{-2}, \quad \rho = 1.74 \times 10^3 \text{ Kgm}^{-3}, \quad c_v = 1.04 \times 10^3 \text{ JKg}^{-1} \text{ deg}^{-3}, \\
T_0 &= 298 \text{ K}, \quad K = 1.7 \times 10^2 \text{ Wm}^{-1} \text{ K}^{-1}, \quad \alpha_i = 2.68 \times 10^6 \text{ Nm}^{-2} \text{ deg}^{-1}.
\end{aligned}$$

Copper

$$\begin{aligned}
\lambda &= 7.76 \times 10^{10} \text{ Nm}^{-2}, \quad \mu = 3.86 \times 10^{10} \text{ Nm}^{-2}, \quad \rho = 8954 \times 10^3 \text{ Kgm}^{-3}, \quad c_v = 383.1 \text{ JKg}^{-1} \text{ deg}^{-3}, \\
T_0 &= 293 \text{ K}, \quad K = 386 \text{ Wm}^{-1} \text{ K}^{-1}, \quad \alpha_i = 1.78 \times 10^{-5} \text{ K}^{-1}.
\end{aligned}$$

Zinc

$$\begin{aligned}
\lambda &= 0.385 \times 10^{11} \text{ Nm}^{-2}, \quad \mu = 0.508 \times 10^{11} \text{ Nm}^{-2}, \quad \rho = 7.14 \times 10^3 \text{ Kgm}^{-3}, \quad c_v = 383.1 \text{ JKg}^{-1} \text{ deg}^{-3}, \\
T_0 &= 296 \text{ K}, \quad K = 1.24 \times 10^2 \text{ Wm}^{-1} \text{ deg}^{-1}, \quad \alpha_i = 1.778 \times 10^5 \text{ K}^{-1}.
\end{aligned}$$

Cobalt

$$\begin{aligned}
\lambda &= 1.027 \times 10^{11} \text{ Nm}^{-2}, \quad \mu = 1.510 \times 10^{11} \text{ Nm}^{-2}, \quad \rho = 8.836 \times 10^3 \text{ Kgm}^{-3}, \quad c_v = 4.27 \times 10^2 \text{ JKg}^{-1} \text{ deg}^{-3}, \\
T_0 &= 298 \text{ K}, \quad K = 0.690 \text{ Wm}^{-1} \text{ deg}^{-1}, \quad \alpha_i = 3.688 \times 10^5 \text{ K}^{-1}.
\end{aligned}$$

Applying these parameter values in Eq. (17), the implicit relation between the phase velocity and wavenumber for fixed static uniaxial stress is obtained. Phase velocity is computed against wavenumber for static uniaxial stress. Figure.1-4 shows the plots of phase velocity against the wavenumber for (static stress) ST=100,200,300 in the case of magnesium, copper, zinc, cobalt materials. In all the cases as the uniaxial static stress and wavenumber increases phase velocity decreases. In general, cobalt material values are less when compare with the magnesium. But the

phase velocity values are higher in the case of cobalt material. In general zinc material values are greater than copper. Phase velocity is more in case of zinc material.

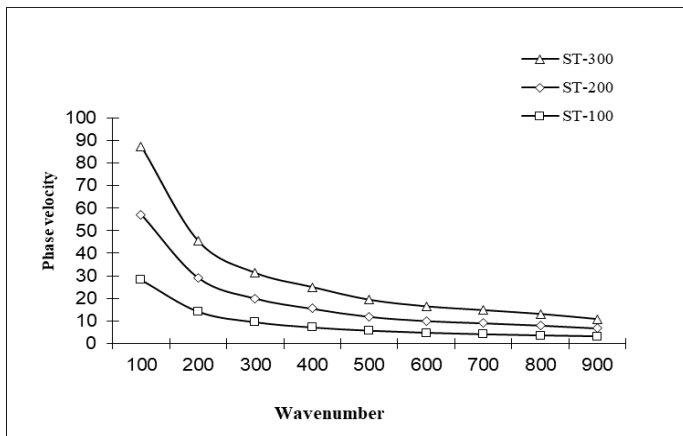


Fig.1. Phase velocity against the wavenumber for magnesium

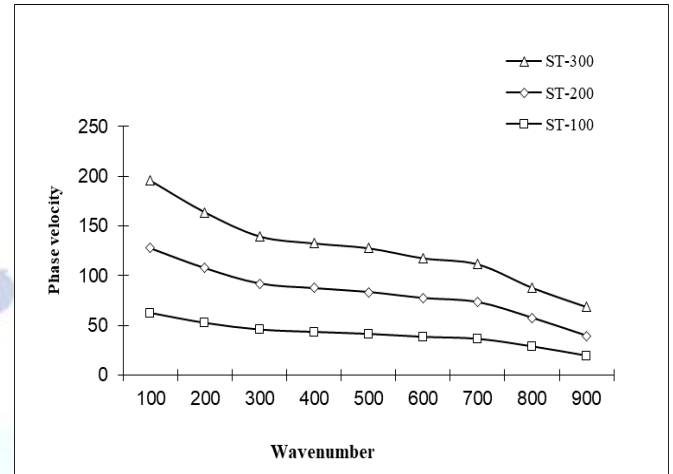


Figure.4 Phase velocity against the wavenumber for cobalt

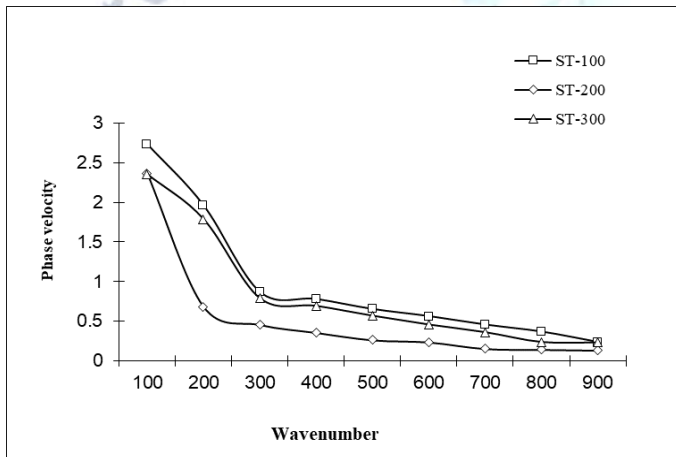


Figure.2 Phase velocity against the wavenumber for copper

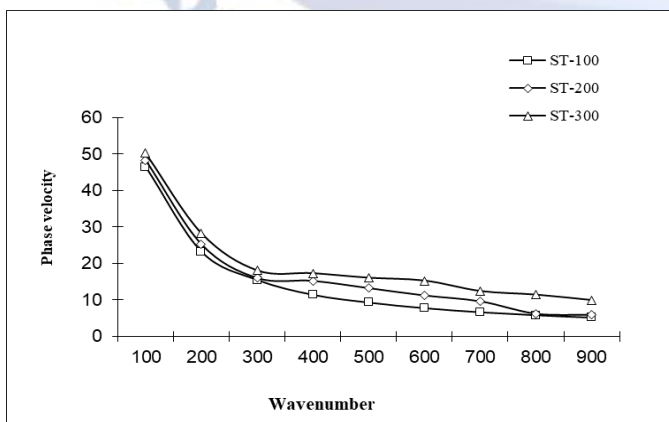


Figure.3 Phase velocity against the wavenumber for zinc

5. CONCLUSIONS

Vibrations of static stresses in thermoelastic solid cylinder are investigated for isotropic solids. For the numerical process four types of solids are considered namely magnesium, copper, cobalt, zinc employed and the results are presented graphically. Phase velocity is computed against wavenumber for different uniaxial static stress. From the numerical results, it is clear that as the wavenumber and static stress increases phase velocity decreases. This type of investigation can be done for any thermoelastic solid cylinder if the parameter values are available.

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Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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