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Algorithm for Finding Roman Dominating Number of Extended Duplicate Graph of Star Families

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ABSTRACT

In this paper, we have determine the roman dominating number for extended duplicate graph of star, double star and bistar by using algorithm.

KEY WORDS: Roman, Extended duplicate, Star, Bistar, Double star

1. INTRODUCTION

Graph theory is one of the most applied research area in Mathematics. Graph theory evolved when Leonhard Euler, a Swiss mathematician solved the famous Koningsberg bridge problem around the year 1735.

A Roman domination is a coloring of the vertices of a graph with the colors {0, 1, 2} such that every vertex colored 0 is adjacent to at least one vertex of colored 2. The idea is that colors 1 and 2 represent either one or two Roman legions stationed at a given location (vertex). A nearby location (an adjacent vertex u) is considered to be unsecured if no legions are stationed there. (i.e. $f(u) = 0$) .

An unsecured location (u) can be secured by sending a legion to u from an adjacent location (v). But Emperor Constantine the Great, in the fourth Centaury A.D, decreed that a legion cannot be send from a location unsecured (i.e. if $f(v) = 1$). Thus two legions must be stationed at a location $f(v) = 2$ before one of the legions can be sent to an adjacent location.

Ernie J.et al [1] introduced the Roman domination of a graph and then the study on Roman domination was carried out by many researchers especially by O.Favaran [2]. S.P.Jeyakokila and P.Sumathi [3] studied about star graph and bistar graphs. K.Thirusangu and M.Madhu [4] introduced extended duplicate graph of star and bistar graphs. S K Vaidya and N H Shah [5] studied some Star and Bistar Related Divisor Cordial graphs. This paper determines the roman dominating number for extended duplicate graph of star, double star and bistar and gives an algorithm to find the roman dominating number.

2. PRELIMINARIES

Definition-2.1 (Star graph)

A star graph K1,n is a complete bipartite graph if one vertex belongs to one set and all theremaining vertices belong to the other set.

Definition-2.2(Bistar graph)

A bistar graph Bn,n, n>1 is obtained from two disjoint copies of K1,n by joining the centrevertices by an edge .

Definition-2.3(Double star graph)

A double star graph K1,n,n is obtained from K1,n by adding a new pendant edge of theexisting n pendant vertices.

Definition-2.4 (Duplicate graph)

A duplicate graph of G is DG=(P, Q) where the vertex set P=$p \cup p'$ and $p \cap p' = \emptyset$ and $f: p \rightarrow p'$ is bijective and the edge set Q of DG is defined as the edge q_1q_2 is in G if and only ifboth q_1q' and $q'q_2$ are edges in duplicate graph.

Definition-2.5 (Extended duplicate graph of star)

The extended duplicate graph of star is obtained from duplicate graph of star by joining the edges p and p'.

Definition-2.6 (Extended duplicate graph of Bistar)

The extended duplicate graph of star is obtained from duplicate graph of bistar by joining the edges p and p'.

Definition-2.7 (Extended duplicate graph of Double star)

The extended duplicate graph of star is obtained from duplicate graph of double star by joining the edges p and p'.

Definition-2.8 (Roman dominating number)

A Roman dominating function on a graph is a function $f: V \leftarrow \{0,1,2\}$ satisfying thecondition that every vertex u for which f(u)=0 is adjacent to atleast one vertex v for whichf(v)=2. The weight of a Roman dominating function is the value $f(V) = \sum_{u \in V} f(u)$. The minimum weight of a Roman dominating function on a graph G is called Roman domination number of agraph.

3. MAIN RESULT

Algorithm 3.1

Procedure-(Roman domination of Extended duplicate graph of Star K1,n)

V← $\{p, p', p_1, p_2, \dots, p_n, p_1', p_2', \dots, p_n'\}$

E← $\{e_1, e_2, \dots, e_{2n+1}\}$

$p \leftarrow 2$

$p' \leftarrow 2$

for i=1 to n do

$p_i \leftarrow 0$

$p' \leftarrow 0$

end for

end if

end procedure.

Theorem-3.1 For n>1, R(EDG(K1,n))=4.

Proof

Let EDG(K1,n) be the extended duplicate graph of star K1,n.From algorithm 3.1, we observe that p and p' are labeled by 2 and the remaining vertices are labeled by 0. Therefore R(EDG(K1,n))=$l(p) + l(p') + l(p_1) + l(p_2) + \cdots + l(p_n) + l(p') + l(p') + \cdots + l(p')$

=2+2+0+0+…+0+0+0+…+0=2+2+n(0)+n(0)=4.

Hence the roman domination number of extended duplicate graph star R(EDG(K1,n))=4.

bistar byjoining the edges p

Algorithm 3.2

Procedure-(Roman domination of Extended duplicate graph of double Star K1,n,n , n>1)

V← $\{p, p', p_1, p_2, \dots, p_n, q_1, q_2, \dots, q_n, p', p', \dots, p', q', q', \dots, q'\}$

E← $\{e_1, e_2, \dots, e_{4n+1}\}$

if n>1

$p \leftarrow 2$

$p' \leftarrow 2$

for i=1 to n do

$p_i \leftarrow 0$

$p' \leftarrow 0$

$q_i \leftarrow 1$

$q'_i \leftarrow 1$
end for
end if
end procedure.

Theorem-3.2 For n>1, R(EDG(Bn,n))=8.

Proof

Let EDG(Bn,n) be the extended duplicate graph of bistar Bn,n.From algorithm 3.2, weobserve that , q , p' and $q'$are labeled by 2 and the remaining vertices are labeled by 0. Therefore

R(EDG(Bn,n))=$l(p) + l(q) + l(p')+ + l(q') + l(p_1) + l(p_2) + \cdots + l(p_n) + l(q_1) + l(q_2) + \cdots + l(q_n) + l(p_1') + l(p_2') + \cdots + l(p_n') + l(q_1') + l(q_2') + \cdots + l(q_n')$

=2+2+2+2+0+0+…+0+0+0+…+0+0+0+…+0+0+0+…+0

=2+2+2+2+n(0)+n(0)+n(0)+n(0)=8.

Hence the roman domination number of extended duplicate graph bistar R(EDG(Bn,n))=8.

Algorithm 3.3
Procedure-(Roman domination of Extended duplicate graph of Bistar Bn,n , n>1)
V← $\{p, q, p_1, p_2, \ldots, p_n, q_1, q_2, \ldots, q_n\}$
E← $\{e_1, e_2, \ldots, e_{4n+1}\}$
if n>1
$p \leftarrow 2$
$p' \leftarrow 2$
$q \leftarrow 2$
$q' \leftarrow 2$
for i=1 to n do
$p_i \leftarrow 0$
$p'_i \leftarrow 0$
$q_i \leftarrow 0$
$q'_i \leftarrow 0$
end for
end if
end procedure.

Theorem-3.3 For n>1, R(EDG(K1,n,,n))=2n+4.

Proof
Let EDG(K1,n,,n) be the extended duplicate graph of double star K1,n,,n.

From algorithm 3.3, we observe that p and p' are labeled by 2 and $q_1, q_2, \ldots q_n, q', q', \ldots, q'$ are labeled by 1 and $p_1, p, \ldots p_n, p', p', \ldots, p'$ vertices are labeled by 0.

Therefore R(EDG(K1,n,,n))=$l(p) + l(q) + l(p')+ + l(q') + l(p_1) + l(p_2) + \cdots + l(p_n) + l(p_1') + l(p_2') + \cdots + l(p_n') + l(q_1) + l(q_2) + \cdots + l(q_n) + l(q_1') + l(q_2') + \cdots + l(q_n')$

=2+2+2+2+0+0+…+0+0+0+…+0+1+1+…+1+1+1+…+1

=2+2+2+2+n(0)+n(0)+n(1)+n(1)=2n+4. Hence the roman domination number of extended duplicate graph double star R(EDG(K1,n,,n))=2n+4.

4. CONCLUSION

In this paper, we have proved determined roman dominating number for Extended duplicate graph of star, bistar and double star by using algorithm.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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