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Image Restoration and Topological Optimization Using Tychonoff Regularization

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ABSTRACT

In this paper, The dissertation deals with the restoration techniques in Image Processing and the theoretical and numerical aspects of image restoration..Tychonoff Regularization method is one of the well known techniques followed in the last two decades. We consider a non linear boundary value problem from which will derive the asymptotic notation of Hilbert space function. More surprisingly, Image restoration and segmentation are achieved simultaneously with the help of topological gradient. In this method, the value of the diffusivity C in the smooth regions, the number of holes drilled in each iteration and the total numbers of holes are the main controlling parameters. We give a mathematical expression of the topological gradient as well as a numerical integration of the results in the restoration of images. Three dimensional models of living human faces are presented with such a high resolution that single hairs are visible.

***KEYWORDS:** Image Restoration, Tychonoff Regularization, Smoothend solution, Gresh, norm of sum of gradients, Laplacian operator ,Hilbert space function, Holographic Image, Topological optimization, Adjoint method.*

1. INTRODUCTION

Digital image processing techniques now are used to solve a variety of problems. Although often unrelated, these problems commonly require methods capable of enhancing pictorial information for human interpretation and analysis. Geographers use the same or similar techniques to study pollution patterns from aerial and satellite imagery. Image enhancement and restoration procedures are used to process degraded images of unrecoverable objects or experimental results too expansive to duplicate. In archaeology, image processing methods have successfully restored blurred pictures that were the only available records of rare artifacts lost or damaged after being photo graphed.

The main idea of the topological gradient is to find an optimal solution design with poor information on the shape of the structure. The shape optimization problem describing minimization of function $j(\Omega)=j(\Omega,U\Omega)$. Where the function $U\Omega$ defined, on a variable open and bounded subset Ω of Rn. The p-Laplacian operator and the theoretical result obtained with an application in image restoration.

Examples of the type of information used in machine perception are statistical moments, Fourier coefficients, and multi-dimensional distance measures.

Typical problems in machine perception that routinely utilize image processing techniques are automatic character recognition, industrial machine vision for product assembly and inspection, military recognizance, automatic processing of fingerprints, screening of x-rays and blood samples, and machine processing of aerial and satellite imagery for weather prediction and crop assessment.

2. IMAGE RESTORATION

As an image enhancement, the ultimate goal of restoration techniques is to improve an image in some sense. For the purpose of differentiation, we consider restoration to be a process that attempts to reconstruct or recover an image that has been degraded by using some a priori knowledge of the degraded phenomenon. Thus restoration techniques are oriented toward modelling the degradation and applying the reverse process in order to recover the original image.

This approach usually formulating a criterion of goodness that will yield some optimal estimate of the desired result. By contrast, enhancement techniques basically are heuristic procedures designed to manipulate an image in order to take advantage of the psychophysical aspects of the human visual system. For example, contrast stretching is considered as enhancement technique because it is based primarily on the pleasing aspects it might present to the viewer, whereas removal of image blur by applying a de-blurring function is considered a restoration technique.

Early techniques for digital image restoration were derived mostly from frequency domain concepts. However this focuses on a more modern, algebraic approach, which has the advantage of allowing the derivation of numerous restoration techniques from the same basic principles. Although a direct solution by algebraic methods generally involves the manipulation of large systems of simultaneous equations, we show that, under certain conditions, computational complexity can be reduced to the same level at that required by traditional frequency domain restoration techniques.

2.1 Shape-from-focus

Shape from focus relies on surface texture for the computation of depth. In many real world applications ,surfaces can be smooth and lacking in detectable texture. In such cases, shape from focus generates inaccurate and sparse depth maps.

This paper presents a novel extension to previous shape from focus approaches. When h focus operator used is 2D Laplacian, the optimal illumination pattern is found to be a checkerboard, whose pitch equals the distance between adjacent weights in the discrete Laplacian kernel. This analysis also reveals the minimum number of images required for accurate shape recovery.

2.2. Surface interpolation

The interpolation method is based upon three focus values exactly met by the curve, while the first method takes up to twenty points into account and finds the curve by minimizing the overall deviation. That makes the fit more accurate and more stable in the presence of noise. however; its computational effect is much higher since an iterative optimization step is involved. in holographic facial measurement, where computational time is not important as in real time applications, the Gaussian fit proved to be an useful tool for surface refinement .the steps in the height map are much larger than the interslice distance .these steps are lessened considerably through the Gaussian fit .the height profile gained with the fit is shifted for better comparability.

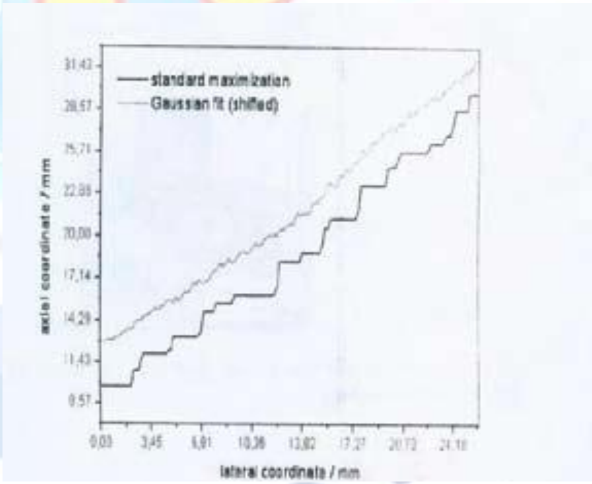


Fig: Exemplary height maps with and without Gaussian fit

2.3. Adaptive Selection of Neibourhood Size

The choice of the neighborhood size is a critical step in the procedure of surface extraction. The lateral resolution of the extracted surface is limited by the size of the neighborhood, since structures smaller than this size cannot be resolved. Even in the absence of noise, the minimal neighborhood size is limited to a certain extent by the feature size of the Texture of the recorded object.

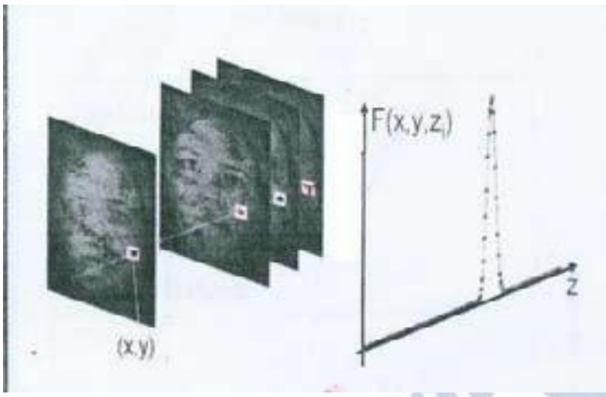


Fig: For each fixed lateral point (x, y), the focus values F(x,y, z) are maximized, which leads to 3-geometry

Information about the object. The normalized leads tom confidence values between zero and one, a higher value indicates a more reliable surface point. The confidence values of one surface point differ for different neighborhood sizes. The normalized leads from confidence values between zero and one, a higher value indicates a more reliable surface point. The confidence values of one surface point differ for different neighborhood sizes.

3. PROBLEM DESCRIPTION

3.1 Image Restoration

First we show that there are some similarities between image processing tools and topological optimization methods. Then we will apply topological asymptotic expansion to image restoration.

Let $\Omega \subset R^2, v \in L^2(\Omega)R^2,$ be the noisy image. Let $v = \mu + \eta$ where u is the reference image and η is the noise.

A classical restoration is obtained by solving the following boundary value problem.

$$\begin{cases} -div(C\Delta u) + u = v in\Omega, \\ \delta_n u = 0 on\Gamma = \delta\Omega \end{cases} \quad \text{--------(1.1)}$$

Where c is small. This can be derived by the considering the canonical embedding.

$$K : H^1((\Omega)/R) \rightarrow L^2(\Omega)/R \quad \text{----------(1.2)}$$

$$U \rightarrow K_u = u \quad \text{---------(1.3)}$$

in order to solve $K_u = v$, we consider the minimization problem:

$$\inf_{\mu \in H^1(\Omega)/R} \int_\Omega |v - K\mu|^2 dx \quad \text{--------(1.4)}$$

Which is equivalent to $K * K_u = K * v$

3.2 The Tychonoff Regularization

The Tychonoff Regularization equation is

$$\inf_{\mu \in H^1(\Omega)/R} \int_\Omega |v - K\mu|^2 dx + \int_\Omega c|\mu|^2 dx$$

---------(1.5)

This problem is equivalent to $(K * K + cI_d)u_c = K * v$

-----------(1.6)

The operator $(K * K + cI_d)$ has a good property.

3.3 Variation Formulation

$$((K * K + cI_d)\mu, \omega)H^1(\Omega)/R = (K * v, \omega)H^1(\Omega)/R \forall \omega \in H^1(\Omega)R$$

-----------------(1.7)

$$(\mu, \omega)L^2(\Omega)/R + (cu, \omega)H^1(\Omega)/R = (v, \omega)L^2(\Omega)/R \forall \omega \in H^1(\Omega)/R$$

--------(1.8)

$$\int_\Omega u(x)\omega(x)dx + \int_\Omega c\nabla u(x).\nabla \omega(x)dx = \int_\Omega v(x)\omega(x)dx \forall \omega$$

--------)1.9)

$$u - div(c\nabla u) = v \quad \text{------(1.10)}$$

This is exactly and the boundary condition $\delta_n u = 0$ is obtained in a natural way. They the classical restoration method is exactly the Tikhonov regularization applied to the canonical embedding of H1 into L2.

3.3.1 A Nonlinear Approach

$$E(u) = \int_\Omega |v - u|^2 dx + \int_\Omega \Psi(|\nabla u|)dx$$

----(1.11)

Is equivalent to:

$$\begin{cases} -div(C\Delta u) + u = v in\Omega, \\ \delta_n u = 0 on\Gamma = \delta\Omega \end{cases} \quad \text{------------------(1.12)}$$

where $c = \Psi^1(|\nabla u|)/|\nabla u|$

And

$\Psi(s)=\sqrt{(1+s^2)}$ Is a convenient function

That is $c=\phi(|\nabla u|)$ with $\phi(s)=(1+s^2)^{-1/2}$

The algorithm:

$1-c_0=1; k=0;$

2- Compute uk;

3- Compute $c_k=\phi(|\nabla u_k|);$ 4- Go to 2.

This algorithm recalls the homogenization approach for topological optimization.

3.3.2 Application of the Topological Asymptotic Expansion

$\omega_p=x_0+\rho\omega$

$\Omega_\rho=\Omega/\varpi\rho$

$c=\{_{\in in\omega_{\rho,\in>0}}^{lin\Omega_p}$ ----------(1.13)

To find c that minimizes:

$j_c(u_c)=\int_{\Omega\rho}|\nabla u_c|^2 dx,$ ------------(1.14)

$c=\chi\Omega_\rho.1+(1-\chi\Omega_\rho)\in.$ -------------(1.15)

Where uc is the solution to

$$-div(C\Delta u)+u=v in\Omega, \quad \{ \quad \delta_n u=0 on\Gamma=\delta\Omega$$ -----------(1.16)

3.4 The Topological Asymptotic Expansion

We have that

$j(p)-j(o)=j(u_c)=f(\rho)G(x_0)+0(f(\rho)),$

------------------------(1.17)

$f(\rho)>0$ At $\lim_{\rho\to 0+f(\rho)}=0$

----------------------(1.18)

If ω is a ball we have

$G(x_0)=-2\phi(\nabla u(x_0).n)(\nabla p(x_0).n-2\phi|\nabla u(x_0).n)|^2$

---------------------------(1.19)

Where P is the solution to the ad joint problem

$$-div(c\nabla p)+p=-\delta J(u)in\Omega, \quad \{ \quad \delta_n p=0on\Gamma$$ ---------------------(1.20)

The main idea of the topological gradient is to find an optimal solution design with poor information on the shape of the structure. The shape optimization problem describing minimization of function

$j(\Omega)=j(\Omega,u_\Omega)$

Where the function u_Ω is defined, on variable open and bounded subset Ω of Rn.

The p-Laplacian operator and the theoretical result obtained with an application in image restoration.

The paper is organized as follows in topic 1 we recall image restoration models in topic2 we present general ad joint method

Topic 3 is devoted to the topological optimization problem in topic 4. Topological optimization algorithm and numerical application in image restoration.

Ad joint method: in this topic we gave an adaptation of the ad joint Method introduced into a nonlinear problem.

Let υ be a Hilbert space and let aε(u, υ) = Lε(υ), υ ∈V -----(1) be a variation formulation associated to a partial differential equation.

Suppose that there exists forms δa(u, v), δl and a function f (ε)>0 which goes to 0 when ε goes to 0.

Let uε be the solution of the equation (1). Suppose that the following hypothesis holds

| | uε -u0 | | υ = o(f (ε)),

| | aε - a0 - f (ε) δa | | υ = O(f (ε)),

| | lε - l0 - f (ε) δl | | υ = O(f (ε))

Let j(ε) = Jε(uε) be the cost function. We suppose that for ε=0, J0(u0) = J(u0) is differentiable with respect to u0 and we denote by DJ(u0) the derivative of J at u0.

We suppose that there exists a function δJ such that jε(uε) - J0(u0)= Dj(u0(uε - u0) + f(ε)δJ(u0) + O(f(ε).

Under the information hypothesis we have the following theorem.

Theorem:

let υ ∈ L2 (Ω) problem (1) has a unique solution moreover, one has

| | uε -u0 | | υ = O(f (ε)) -------- (1.1) where

||u||v=||v||L2(Ω)+ || ∇ U||Lp(Ω) =

$$\left(\int_{\Omega}|U|^{2}dx\right)^{1/2}+\left(\int_{\Omega}|U|^{p}dx\right)^{1/p}$$ ------------(1.2)

In order to prove the lemma, we need the following result. Let Ω be a bounded open domain of Rn (No smoothness is assumed on ∂Q) and p, p` be real numbers such that

1<p, p`< + ∞, 1/p +1/p` =1 ---- (1.3)

Consider the operator A defined on W1, p (Ω) by

A (u) = - div (a(x, u(x), ∇ u(x)), --- (1.4)

Where a: Ω × R × RN → R is a caratheodory function satisfying the classical laray-lions hypothesis in the sense of [20], described in what follows | a(x,s,ξ) | ≤ c(x) + k1| s |p-1 + k2| ξ |p-1

[a(x,s,ξ) - a(x,s,ξ*)] (ξ - ξ*) >0,

$$\frac{a(X,S,\xi)\xi}{|\xi|+|\,\xi|^{p-1}}\rightarrow_{|\xi|}\rightarrow+\infty$$,

Almost every x ∈ Ω, for all s∈ R,

ξ, ξ* ∈ RN, ξ ≠ ξ*

Consider the nonlinear elliptic equation -div(a(x,un, ∇ un)) = fn + gn in D`(Ω ---- (1.5)

We assume that un converges to u on

Wl,p(Ω) weakly and strongly in Lploc(Ω) ----- (1.6)

fn → f strongly in W-1,p`(Ω), and almost everywhere in Ω -----(1.7)

The hypothesis (1.5) - (1.7) imply that gn ∈ W-1,p`(Ω) and is bounded on this set. We suppose either gn is bounded in Mb(Ω) (space on random measures), that is

$$|(g_{n},\Psi)|\leq C_{k}\|\Psi\|L^{\infty}(K),$$ for all $\Psi\in D(K),$

With supp(ψ) subset of K, Ck is a constant which depends only on the compact k.

4. METHODOLOGY

4.1 Implementation

Input image can be noisy, Excessive smoothing, edges are seemed out, smoothing where gradients are high (edges), reduce C where gradients are high.

To Choose the Right C

- ✓ Strategy
- ✓ Drill holes in the domain and define C to be very small inside the holes.
- ✓ Set C equal to some suitable constant outside the holes.

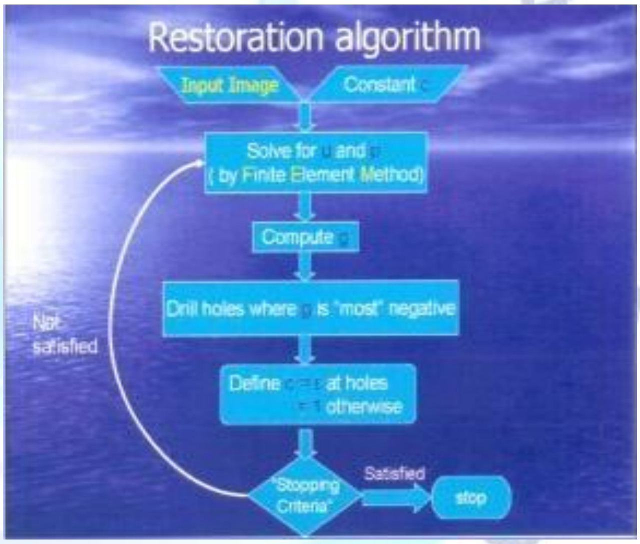
New Idea: Topological Gradient

Tool from Shape Optimization is following:

- ✓ a domain Ω,
- ✓ a cost functional J(u),
- ✓ With u solutions of a PDE.

An indication on j(u) will vary by drilling a small hole in the domain at each point x0. These conditions are applying to the topological gradient.

The flow chart below gives the methodology adopted for restoration. Stopping criteria.



Region homogeneity can be used as stopping criteria: A filling algorithm is started from a point outside edges. If the filled region pixels are homogeneous, it is accepted as a properly segmented region, and the process is continued with remaining regions. If any region is not acceptable, the whole creation process is continued.

5. RESULTS AND SUMMARY

5.1 Summary

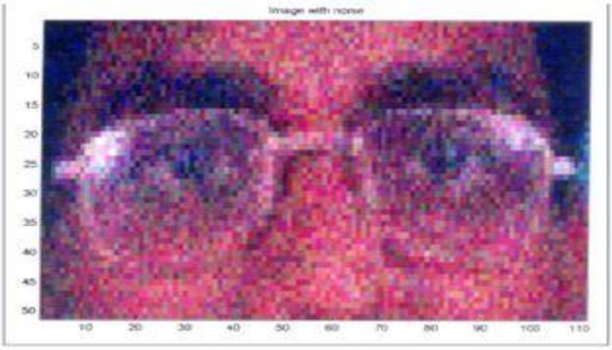
The topological gradient method has been adapted to solve restoration and segmentation problems in image processing.

Mat lab routines were developed that can handle both gray and RGB color images. Region filling and merging algorithms
were devised and implemented several images with different levels of complexity were processed.

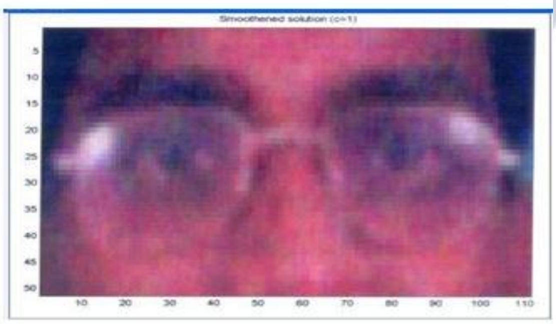
5.2 Results

The results obtained by the Image Restoration method in different noise levels. Input Image can be noisy, Excessive smoothing, edges are seemed out, smoothing where gradients are high(edges), An indication on J(u) will vary by drilling a small hole in the domain at each point x_0. These conditions are applying to the topological gradient. Additionally, we presented an algorithm for automated adoption of the neighborhood size to the condition of the data. This is chosen as small as possible while preventing corruption through the noise.

1.1 Original Image

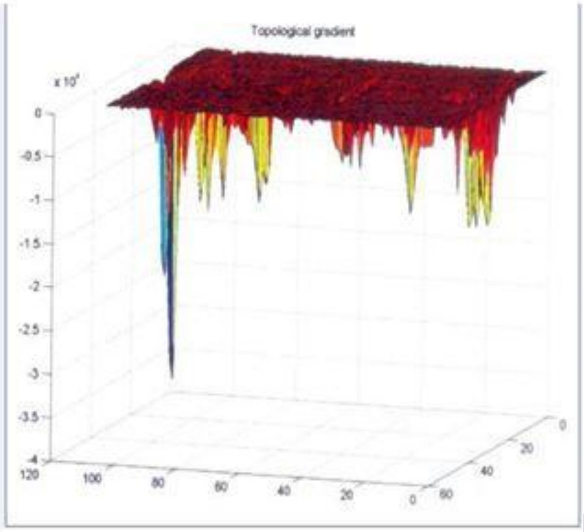


1.2 Image with noise

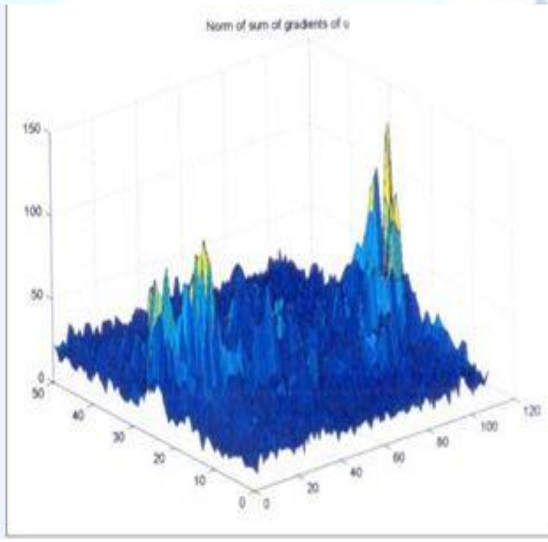


1.3 Smoothened solution(c=1)

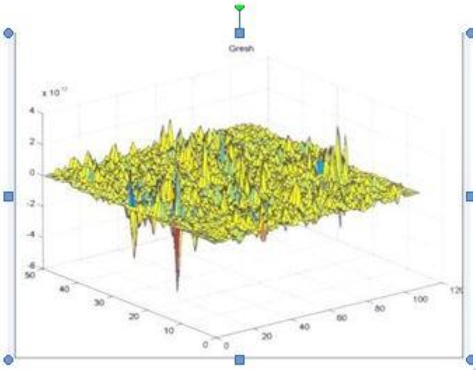
1.4 Modified Image



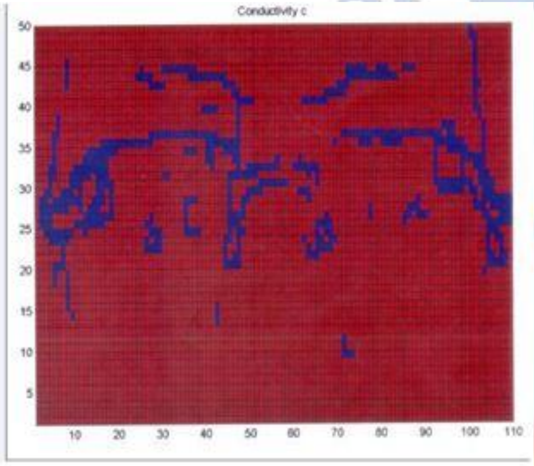
1.5 Topological gradient



1.6 norm of sum of gradients of u



1.7 Gresh



1.8 conductivity c

Image Dimensions Width: 350 Height:287 color Depth:3

The Corresponding iteration values:

Iteration 1 J=2976660.8691
Iteration 2 J=2792881.8622
Iteration 3 J=2691705.8464
Iteration 4 J=2615838.555
Iteration 5 J=2555238.409
Iteration 6 J=2497211.5443
Iteration 7 J=2447517.0983
Iteration 8 J=2401412.3462
Iteration 9 J=2357630.3415
Iteration 10 J=2313760.0754
Iteration 11 J=2275262.3946
Iteration 12 J=2237846.6716
Iteration 13 J=2202590.2468
Iteration 14 J=2164576.3671
Iteration 15 J=2133058.8258
Iteration 16 J=2100687.2751
Iteration 17 J=2067939.0246
Iteration 18 J=2037216.6449
Iteration 19 J=2008226.5865
Iteration 20 J=1977003.9915

6. CONCLUSION

The topological gradient method has been very successful for image restoration and segmentation. Image restoration and segmentation are achieved simultaneously the value of the diffusivity C in the smooth regions, the number of holes drilled in each iteration and the total number of holes are the main controlling parameters. If the filled region pixels are homogeneous, it is accepted as a properly segmented region, and the process is continued with remaining regions. If ant region is not acceptable, the whole creation process is continued. It was demonstrated that in facial measurement single hairs can be visualized through this procedure.

6.1 Future Work

Different cost functional, Use cracks and anisotropic diffusivity, Develop new stopping criteria, Implement morphological filters to narrow edges. It was also shown that a Gaussian fit to the focus profile improves the quality of the surface tremendously. Articrafts are eliminated and influence of noise is reduced, while at the same time a continuous surface not restricted to the digitization positions is created.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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