



Robust Estimation of Direction of Arrival (DOA) using Modified Music Algorithm

Srinivash Roula | K. Siva Shankar | M. Sessa Sai Vivek | V. Manikanta | B. PavanKumar

Department of Electronics and Communication Engineering, Godavari Institute of Engineering and Technology(A), JNTUK, Kakinada.

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ABSTRACT

The direction of arrival (DOA) estimation is a prime issue in Uniform Linear Array (ULA) in array signal processing for source localization and tracking. In literature, many works have been proposed for robust DOA estimation, when data gets contaminated with impulsive noise. But, performance of the DOA algorithms also fails to give desired result when one node in an array becomes faulty. In this paper, modified Multiple Signal Classification (MUSIC) algorithm in presence of faulty sensor node in ULA is discussed. The performance of the MUSIC algorithm depends on covariance matrix which degrades due to non-gaussian noise. Therefore, in the proposed robust MUSIC algorithm, the covariance matrix is estimated by using Tyler statistical method. The simulation results show that the performance of the proposed robust MUSIC algorithm out performs the conventional MUSIC by providing high probability of resolution (PR) and low root mean square error (RMSE).

KEYWORDS: Direction of arrival, Source localization, Robust technique, MUSIC algorithm.

1. INTRODUCTION

In array signal processing DOA estimation is a hot research area. Since last decades, it is a burning area of research because of real time remote sensing applications like environmental monitoring, source localization, tracking, and remote health monitoring *etc.*

(DOA) estimation is an important problem for source localization and tracking. It is a common problem in radar. In DOA estimation methods, the direction from which a propagating wave arrives at sensor array is estimated. In distributed ULA, the DOA of multiple sources can be estimated by assuming the sensor network as an array. The DOA estimation of multiple emitting sources is an important issue in array processing and has various applications in radar, sonar, wireless communications and source localization [2].

In the literature, many high-resolution sub optimal techniques have been proposed and analyzed, such as Multiple Signal Classification (MUSIC) [3], the minimum variance method of Capon, estimation of signal parameters via rotational invariance technique (ESPRIT) [4] and more. The MUSIC technique is used here because of its superior performance compared to others. The MUSIC algorithm is one of the most widely used high resolution DOA estimation techniques and it is based on subspace methodology [5]. It estimates the DOAs by exploiting the orthogonality between the noise subspace and array manifold [6], [7].

The existing DOA estimation algorithms [8]-[13] assume that all the sensor nodes in an array are fault free. However, in practice, the array elements are subjected to fault due to several reasons. In such case the

conventional algorithms fail to estimate the true DOA. In the literature robust algorithms are proposed to alleviate the effect of impulsive noise. The impulsive noise occurs occasionally and when occurs, the performance degrades. The authors in [14] have proposed a robust algorithm based on subspace decomposition in presence of impulsive noise.

When a sensor node in a sensor array becomes faulty, then it behaves abnormally. The faulty sensor provides erroneous data like impulsive noise always. Therefore, it is

challenging problem in array signal processing to improve the performance of the DOA estimation algorithms in presence of faulty sensors. The proposed robust DOA estimation algorithm is addressed when multiple sensor nodes in a sensor array becomes faulty. The proposed algorithm is different from the existing robust algorithms as the present algorithm deals with faulty sensor node whereas the existing algorithms consider impulsive and spatially non-white noise [15]-[18].

Robust parameter estimation techniques provide a good fit when the data contain impulsive noise [19]. The faulty sensor node in a sensor array belongs to one of the impulsive noise conditions, which needs to be taken care. Once the data corrupted due to faulty sensor node, the noise under goes high variance and its covariance matrix changes and the conventional MUSIC fails to work.

In the proposed algorithm, first the covariance matrix is estimated by following robust statistical methods and then conventional MUSIC algorithm is employed to estimate the DOA. The simulation results show that the modified MUSIC works best in presence of faulty sensor node in ULA, where the conventional MUSIC algorithm fails completely.

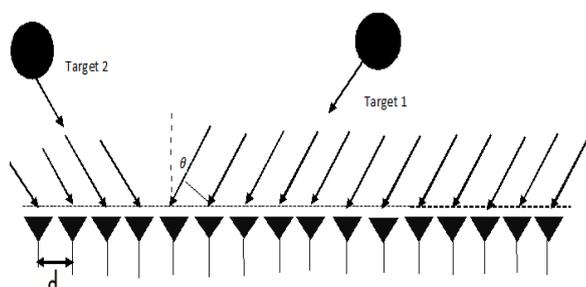


Fig.1: Uniform linear array of 16 sensor nodes and 2 targets

2. PROBLEM FORMULATION

A. System Model

Let us consider a ULA having M sensor nodes distributed in an arbitrary geometry and received signals form N narrow band far-field signal sources at unknown locations θ . The sensor nodes in the network form an arbitrary array to estimate the DOA. The output of sensor nodes is modelled as [2]

$$\mathbf{x}(i) = \mathbf{A}(\theta) \mathbf{s}(i) + \mathbf{n}(i), i=1,2, \dots, K \quad (1)$$

Where $\mathbf{s}(i)$ is the unknown vector of signal waveforms, $\mathbf{n}(i)$ is unpredicted noise process, and K denotes the number of data samples (known as snapshots). The steering matrix $\mathbf{A}(\theta)$ has the following special structure defined as

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_N)] \quad (2)$$

Where, $\mathbf{a}(\theta)$ is called steering vector and $\theta = [\theta_1, \theta_2, \dots, \theta_N]^T$ are the parameters to be estimated. The exact form of $\mathbf{a}(\theta)$ depends on the array geometry and the position of the nodes in sensor network. Further, the signal and noise are assumed to be stationary, temporarily white, zero-mean complex Gaussian random variables with the following second-order moments

$$\begin{aligned} E[\mathbf{s}(i)\mathbf{s}(j)^H] &= S\delta_{ij} \\ E[\mathbf{s}(i)\mathbf{s}(j)^T] &= 0 \\ E[\mathbf{n}(i)\mathbf{n}(j)^H] &= \sigma^2 \\ E[\mathbf{n}(i)\mathbf{n}(j)^T] &= 0 \end{aligned} \quad (3)$$

Where, δ_{ij} is the Kronecker delta, $(\cdot)^H$ denotes complex conjugate transpose, $(\cdot)^T$ denotes transpose, $E(\cdot)$ stands for expectation operator.

Under the assumptions taken above, the observation process, $\mathbf{x}(i)$, constitutes a stationary, zero-mean Gaussian random process having second-order moments

$$\mathbf{R} = E[\mathbf{x}(i)\mathbf{x}(i)^H] = \mathbf{A}(\theta)\mathbf{S}\mathbf{A}^H(\theta) + \sigma^2\mathbf{I} \quad (4)$$

Where, \mathbf{S} is source covariance matrix and \mathbf{I} is $M \times M$ Identity matrix.

The problem addressed here in is the estimation of from a batch of K measured data $\mathbf{x}(1), \dots, \mathbf{x}(K)$.

A. Signal Model in Presence of Faulty Sensor Node

Let us consider F number of faulty sensor nodes are introduced in the sensor network. Let the faulty sensor nodes are numbered as i_1, i_2, \dots, i_F .

The data generated by faulty node is given by

$$\mathbf{x}(i) = \mathbf{A}(\theta)\mathbf{s}(i) + \mathbf{v}(i), i = 1, 2, \dots, K$$

where, $\mathbf{v}(i)$ is assumed as normal distributed Gaussian noise as defined in [1], but having different covariance matrix. It is because; few sensor nodes which are faulty have very high noise variance. The covariance matrix defined in (4), now can be redefined as

$$\mathbf{R}_F = \mathbf{A}(\theta)\mathbf{S}\mathbf{A}^H(\theta) + \mathbf{Q}$$

Where each diagonal elements of the noise covariance matrix \mathbf{Q} is defined as

$$\overline{\mathbf{R}}_{ii} = \sigma^2 + b_i \sigma_F^2 \quad (7)$$

Where, σ^2 is the variance of fault free sensor nodes; σ_F^2 is the variance of faulty sensor node which is much greater than σ^2 and b_i is the switching element which is defined as $b_i = 1$ for faulty sensor nodes i_1, i_2, \dots, i_r and 0 for faulty free sensor node

3. MODIFIED MUSIC ALGORITHM

Tyler's Estimator is a covariance estimator, we focus on Tyler's Covariance M-Estimator. The covariance of the Tyler's M-estimator is given as a fixed point solution to,

$$\hat{\Theta} = \frac{p}{n} \sum_{i=1}^n \frac{\mathbf{x}_i \mathbf{x}_i^T}{\mathbf{x}_i^T [\hat{\Theta}]^{-1} \mathbf{x}_i} = \frac{1}{n} \sum_{i=1}^n \frac{p}{\mathbf{x}_i^T [\hat{\Theta}]^{-1} \mathbf{x}_i} \mathbf{x}_i \mathbf{x}_i^T \quad (8)$$

$$\hat{\Theta}_{i+1} = \frac{p}{n} \sum_{i=1}^n \frac{\mathbf{x}_i \mathbf{x}_i^T}{\mathbf{x}_i^T [\hat{\Theta}_i]^{-1} \mathbf{x}_i} \quad (9)$$

The MUSIC algorithm completely failed to estimate the source DOA when data is corrupted by impulsive noise [14]. It has been observed from the MUSIC algorithm that the algorithm solely depends on the covariance matrix. When the data gets corrupted due to faulty node, the covariance matrix \mathbf{R}_F gets corrupted which results in failure of conventional MUSIC algorithm to provide better performance. Therefore, in this proposed method a modified covariance matrix is proposed using statistical methods.

Let the corrupted covariance matrix due to non-uniform nature of the noise is represented by each row as

$$\mathbf{R}_F = \begin{bmatrix} r_1 \\ r_2 \\ \dots \\ r_M \end{bmatrix}^T \quad (10)$$

The data from i th sensor is given in the i th row of the covariance matrix. Therefore, the robust covariance matrix can be written in terms of their rows as

$$\mathbf{R}_B = [r_{b_1}, r_{b_2}, \dots, r_{b_M}]^T \quad (11)$$

where, r_{b_i} is the i th row of \mathbf{R}_B . using

$$\frac{1}{K} \sum_{i=1}^K \mathbf{x}(i)\mathbf{x}(i)^H = \frac{1}{K} \mathbf{X}\mathbf{X}^H \quad (12)$$

The Eigen value decomposition of is given by

$$\begin{aligned} &= \mathbf{U}\mathbf{U}^H \\ &= \mathbf{U}_s \mathbf{U}_s^H + \sigma^2 \mathbf{U}_n \mathbf{U}_n^H \end{aligned} \quad (13)$$

Where, $\mathbf{U} = [\mathbf{U}_s \mathbf{U}_n]$ and $\text{diag} \{\lambda_1, \dots, \lambda_M\}$ are diagonal matrix containing M principal Eigen values and \mathbf{U}_s containing corresponding or the normal Eigen vectors respectively. The range space spanned by \mathbf{U}_s is known as signal subspace and its orthogonal complement, spanned by \mathbf{U}_n , is called noise subspace. The MUSIC method searches for the peaks of the spatial spectrum, which is given by [3].

$$Z_{\text{MUSIC}}(\theta) = \frac{\mathbf{a}^H(\theta)\mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}(\theta)} \quad (14)$$

In the literature [7], it has been seen that the MUSIC algorithm provides best performance for uncorrelated data and at higher SNR. When a sensor node becomes faulty, then the data from the data from faulty sensor node acts as impulsive noise. In this scenario the MUSIC algorithm fails to find the DOA (observed in the simulation results). Therefore, a robust MUSIC algorithm is proposed here which is described in the following subsection.

4. SIMULATION RESULTS

In this section, we have simulated proposed robust algorithm and compared the performance with that of conventional MUSIC algorithm. The performances of these methods are compared in two ways: (a) the DOA estimation root-mean squared error (RMSE), which is calculated as [13]

$$\sqrt{\frac{1}{NQ} \sum_{l=1}^Q \sum_{n=1}^N (\hat{\theta}_n(l) - \theta_n)^2} \quad (15)$$

Where, N is the number of sources, $\hat{\theta}_n(l)$ the estimate of the n th DOA achieved in the l th run, θ_n is the true DOA of the n th source; and (b) the ability to resolve closely spaced sources known as probability of resolution (PR). By definition, two sources are said to be resolved in a given run if both $|\hat{\theta}_1 - \hat{\theta}_2|$ and $|\hat{\theta}_2 - \theta_2|$ are less than $|\theta_1 - \theta_2|/2\hat{\theta}_1$

We have taken 16 sensor nodes in a sensor network.

The position of the sensor nodes are given as

The arbitrary network topology of the sensor nodes is shown in Fig.1. The steering matrix A is formulated as per the position of the sensor nodes in the network.

Let us assume that two equal-power, uncorrelated signals impinge on distributed wireless sensor network from 150° and 158° . The SNR varies from -20 dB to 20 dB with the step size of 2 dB taken for simulation. We repeated each experiment 100 times and the average result are shown in the figures.

In order to show that the MUSIC algorithm is not robust to faulty nodes, first the performance of the MUSIC algorithm is compared with and without faulty nodes. In Fig.2 and 3 the RMSE and PR performance are compared when 20% sensor nodes are faulty (out of 16 nodes 4 nodes are faulty) respectively. The faulty nodes are $3, 5, 9$ and 15 . From the figures, it is clear that the conventional non-robust MUSIC algorithm works fine in non-faulty environment, but fails completely in presence of faulty sensor nodes.

In Fig. 4 and 5, the PR and RMSE performance of the MUSIC and the proposed robust MUSIC algorithms are compared in presence of $20\%, 30\%, 40\%$ faulty node.

As it can be seen from the Figs. 3 and 4, the proposed robust MUSIC yields significantly superior performance over conventional MUSIC for faulty sensor nodes in WSN by providing high PR and low RMSE values. The robustness of the algorithm is studied up to 40% faulty sensor nodes here

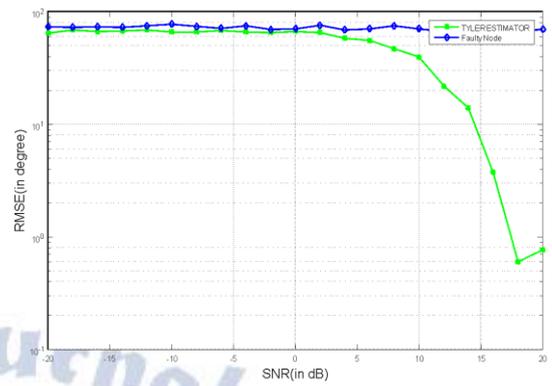


Fig. 2: PR Vs SNR for Tyler MUSIC for 40% fault at 150° and 158°

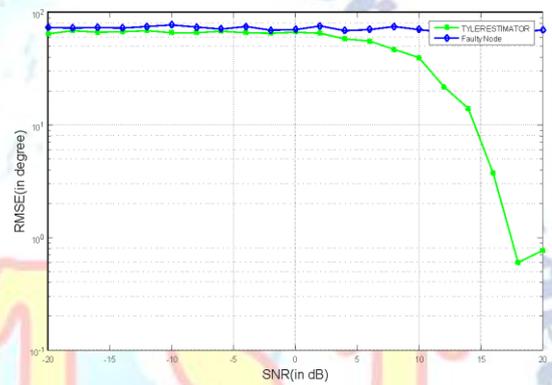


Fig.3: RMSE Vs SNR for Tyler MUSIC for 40% fault at 150° and 158° .

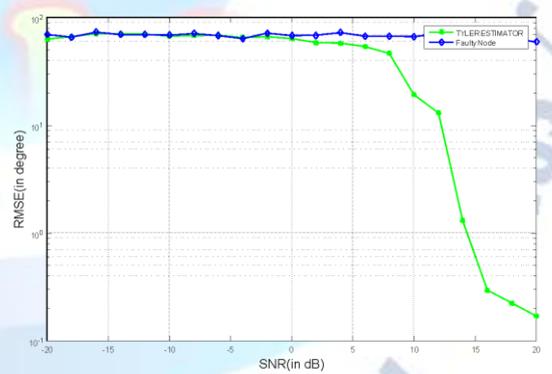


Fig.4: PR Vs SNR for Tyler MUSIC for 30% fault at 150° and 158°

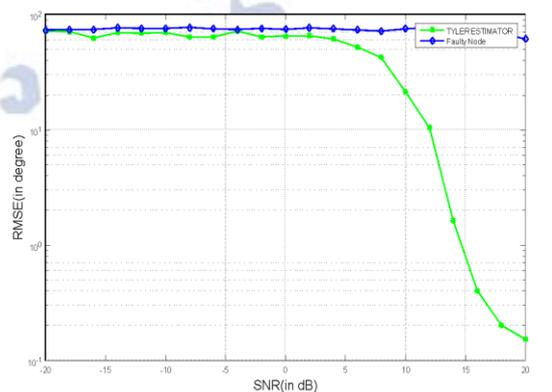


Fig. 5: RMSE Vs SNR for Tyler MUSIC for 30% fault at 150° and 158°

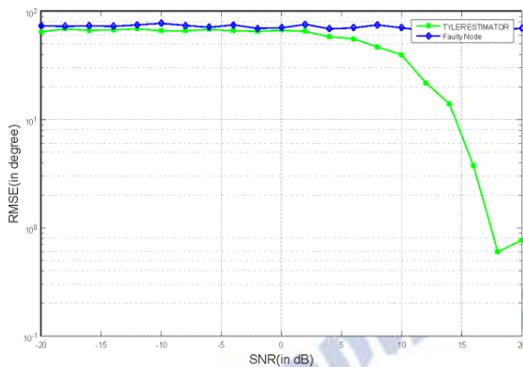


Fig.6: RMSE Vs SNR for Tyler MUSIC for 20% fault at 150° and 158°

5. CONCLUSION

In this paper, a robust MUSIC algorithm using Tyler's Estimator is proposed to estimate the source DOA in ULA with faulty sensor nodes. A new method is introduced here to determine the robust covariance matrix that mitigate the effect erroneous data. The PR and RMSE performance of the robust algorithm are compared with MUSIC with different percentage of faulty nodes. The simulation results demonstrate that, the proposed robust MUSIC algorithm outperform that of the existing algorithm in presence of faulty sensor nodes in Uniform linear array.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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