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Applications of Laplace Transform in Mechanical Engineering

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ABSTRACT

Laplace transform is a very useful tool to solve the differential equation of higher order as well as very useful to solve integral equations. The mathematical modulation of most of the engineering problems such as the deflection of the beam, equation of motion, modeling of the mechanical system for Transfer Function, and vibrating system is in the terms of differential equations. The solution to such an equation can be found by different methods. In this paper, we used the Laplace Transform method to find solutions to the modulation of recent engineering problems with examples.

KEYWORDS: Differential Equation, Transforms, Laplace Transform (LT), Inverse Laplace Transform, Transfer Function, Dirac delta function

1. INTRODUCTION

The L.T. is a significant field of Mathematical Analysis. It has broad applications in various fields of engineering technology, basic sciences, mathematics, and econom-ics. L.T. is the method applied to solve the differential equations at boundary-value. Mathematical formulations of most engineering problems are in the form of differ-ential equations. These equations can easily be solved by using Laplace transform.

In day-to-day life, mathematical applications and models are commonly used. In 2016, Daci [4] used LT in a Mathematical model on population projection in Albania and he experimentally verified how LT is used in population growth [4]. Some of the simple applications of LT in engineering fields related to the transfer function of mechanical systems, nuclear physics as well as Automation engineering, Control engineering, and Signal processing are discussed in by Sawant[9]. Laplace transform methods have a key role to play in the modern approach to the analysis and design of engineering systems. Patil [1] showed how to present discounted value in finance related to Laplace transforms.

Das [3] presented the applications of LT in control systems using Finite Laplace transform (see. [2]). Laplace transform is also used in the theory of Partial fraction (see Thakur et al [5]). Laplace transforms theory is related to other transforms also (see. Rani and Devi [6], Anumaka [8], Aleksandar [13]). Many authors namely Ananda and Gangadharaiah[9], Artem [10], Duz, [14], Sedletskii [11], Stankovi[12], Ramamna [16], Parashar and ELzaki [18], Chandramouli[19], Bhullar [15], Subramanian [20] done work on applications of Laplace transform in different fields. Also, see [21-23].

In this article, we will see the applications of LT for finding the solution of the equation of motion for a forced elastic string with damping and without damping, the equation for motion with two masses with one and three springs, in deflection of beam and modeling of the mechanical system. Before going to see applications we first present some definitions.

1.1 Definition:

Let f(t) be a function of t defined for all t >0. Then the Laplace Transform of f(t), denoted by L[f(t)], is defined by

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt = F(s)$$

Where s is a parameter, which may be real of the complex. The symbol L is the Laplace transform operator. L [f(t)] defines a function of s and is denoted by F(s).

1.2 Definition:

If the Laplace transform of f(t) is F(s). i.e. L[f(t)]=F(s), then f(t) is called the inverse Laplace transform of F(s) and we write $L^{-1}[F(s)] = f(t)$.

II. APPLICATIONS OF LAPLACE TRANSFORM IN THE EQUATION

In this section, we shall now see the applications of Laplace Transforms to solve the equation of motion.

2.1 Forced oscillation without damping (Example):

Let mass m be loaded to the bottom end side of an elastic spring whose top side is fixed and whose stiffness is k, when the driving force is $\mathbf{F_0 sinat}$ (see Fig. 1). This example comes under the forced oscillation without damping. Taking the external periodic force to be $\mathbf{F_0 sinat}$ the equation of motion is

$$m\frac{d^2x}{dt^2} = mg - k(e+x) + F_0 \sin at \qquad (2.1.1)$$

Where x is the length of the stretched position of the spring (displacement) after the time t, k is the restoring force per unit stretch of the spring due to the elasticity, e is the elongation produced in the spring by mass m, a is arbitrary constant and p is any scalar. We solve this equation by using LT when $a^2 \neq k/m$ and velocity and displacement are zero when t=0. In this particular example, mg=ke, from equation (2.1.1)

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = \frac{F_0 \sin at}{m}$$

or

$$\frac{d^2x}{dt^2} + n^2 x = \frac{F_0 \sin at}{m}, \quad \text{where} \quad n^2 = \frac{k}{m}$$

Taking LT on both sides we get,

$$\left[s^2X(s) - sx(0) - x'(0)\right] + n^2X(s) = \frac{F_0}{m}\frac{a}{s^2+a^2}$$

Using x(0)=0, x'(0)=0 we get

$$\left(s^2+n^2\right)X(s) = \frac{F_0}{m}\left(\frac{a}{s^2+a^2}\right)$$

i.e.

$$X(s) = \frac{aF_0}{m}\frac{1}{\left(s^2+n^2\right)\left(s^2+a^2\right)}$$

Now by partial fraction

$$\frac{1}{\left(s^2+n^2\right)\left(s^2+a^2\right)} = \frac{As+B}{s^2+n^2} + \frac{Cs+D}{s^2+a^2}$$

i.e. $1 = s^3(A+C) + s^2(C+D) + s(n^2A + a^2C) + (n^2B + a^2D)$

Solving $\mathbf{A = 0, B = \frac{1}{n^2-a^2}, C = 0, D = -\frac{1}{n^2-a^2}}$ and hence

$$X(s) = \frac{aF_0}{m(n^2-a^2)}\left[\frac{1}{s^2+a^2} - \frac{1}{s^2+n^2}\right]$$

Applying Inverse LT we get,

$$x(t) = \frac{aF_0}{m(n^2-a^2)}\left[\frac{1}{a}\sin at - n\sin nt\right] = \frac{aF_0}{mn(n^2-a^2)}\left[n\sin at - a\sin nt\right],$$

Where, $n = \sqrt{\frac{k}{m}}$ and $\frac{k}{m} \neq a^2$

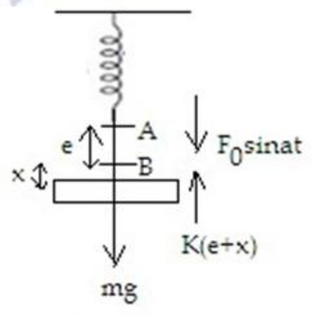

Fig. 1

2.2 Forced oscillation (Example):

The forced oscillations of an elastic spring whose one end is fixed and the other end is hung a mass m (see fig. 2) is governed by the differential equation

$$m\frac{d^2y}{dt^2}+ky=F_0\sin pt\,,$$

Where k is spring constant and F_0sinpt is the driving force. Initially, if the mass is at rest in its equilibrium position then we have $y(0) = 0, y'(0) = 0$. Apply the LT of the governing equation we get,

$$s^2Y(s)+w^2Y(s)=kF_0\frac{p}{s^2+p^2},$$

Where $w=\sqrt{k/m}$ and $k=F_0/m$.

Hence,
$$Y(s)=\frac{kp}{(s^2+w^2)(s^2+p^2)}.$$

If $w^2\neq p$ the Inverse LT gives

$$y(t)=\frac{kp}{p^2-w^2}\left(\frac{1}{w}\sin wt-\frac{1}{p}\sin pt\right).$$

This case correspondence to no resonance

If $w^2=p$ the inverse Laplace transform gives

$$y(t)=\frac{k}{2w^2}(\sin wt-w\sin wt).$$

This correspondence to resonance.

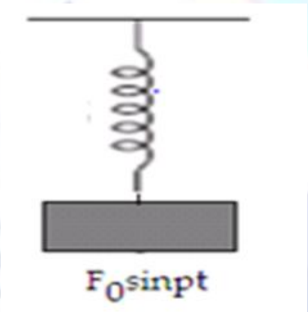

Fig. 2

2.3 Example

The 2 masses m and M free to move in a straight line are joined by a spring of stiffness λ. At $\mathbf{t = 0}$ when they are both rest and the spring unstrained, a blow of impulse p is given to M in the way towards m. Now we obtain the motion of m and M. Let "a" denotes the natural length of the spring and x and y are the displacements of M and m at any time t from their original position at $\mathbf{t = 0}$. Then by Hook's law, the compression in the spring shown in figure 3 is given by

$$T = \lambda(x - y)$$

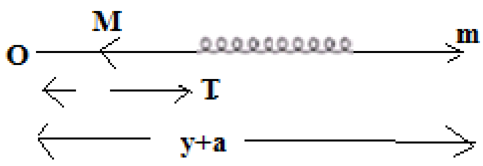

Fig. 3

The equation of motion for M and m are

$$M\frac{d^2x}{dt^2}=p\delta(t)-T \quad \text{or} \quad M\frac{d^2x}{dt^2}+\lambda(x-y)=p\delta(t)$$

and

$$m\frac{d^2y}{dt^2}=T \quad \text{or} \quad m\frac{d^2y}{dt^2}+\lambda(y-x)=0$$

Initial conditions are $\mathbf{x = y = x' = y' = 0}$ at $\mathbf{t = 0}$. Taking LT we get

$$(Ms^2+\lambda)X(s)-\lambda Y(s)=p \qquad (2.3.1)$$

and
$$(ms^2+\lambda)Y(s)-\lambda X(s)=0 \qquad (2.3.2)$$

Multiply equation (2.3.1) by $(ms^2+\lambda)$ and equation (2.3.2) by λ and adding we get

$$\left\{(Ms^2+\lambda)(ms^2+\lambda)-\lambda^2\right\}X(s)=p(ms^2+\lambda)$$

i.e.
$$X(s)=\frac{(ms^2+\lambda)}{s^2\left\{s^2+\lambda(m^{-1}+M^{-1})\right\}}\frac{p}{mM} \qquad (2.3.3)$$

Or
$$X(s)=\frac{p}{M+m}\left\{\frac{1}{s^2}+\frac{mM^{-1}}{s^2+\lambda(m^{-1}+M^{-1})}\right\}$$
and

$$x=\frac{p}{M+m}\left\{t+\frac{m}{Mp}\sin pt\right\}$$
, where $p^2=\lambda(m^{-1}+M^{-1})$.

Now from result (2.3.1) and (2.3.2)

$$Y(s)=\frac{\lambda}{s^2\left\{s^2+\lambda(m^{-1}+M^{-1})\right\}}\frac{p}{mM}$$

$$=\frac{p}{Mm(m^{-1}+M^{-1})}\left\{\frac{1}{s^2}+\frac{1}{s^2+\lambda(m^{-1}+M^{-1})}\right\}$$

By inverse LT we get
$$y(t)=\frac{p}{M+m}\left\{t-\frac{1}{p}\sin pt\right\}$$

We now consider another example of mechanics; two masses are connected to three springs.

2.4 Two Masses Three springs (Example)

Let's consider three spring and two masses. Here all the three springs have same spring constant k. The mechanical system as shown in figure 4 is governed by the differential equations.

$$\frac{d^2y_1}{dt} = -ky_1 + k(y_2 - y_1) \quad \text{and}$$

$$\frac{d^2y_2}{dt^2} = -k(y_2 - y_1) - ky_2,$$

Where k is spring modulus of each of the springs, y_1 and y_2 are the displacements of the masses from their position of elastic equilibrium.

Assuming the masses of the springs and the damping are neglected. Let the initial conditions are

$$y_1(0) = 0, y_2(0) = 1, y_1'(0) = \sqrt{3k}, \ y_2'(0) = -\sqrt{3k}$$

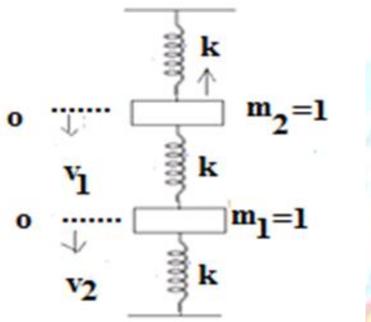

Fig. 4

Taking Laplace Transform of above both equations we get

$$s^2Y_1(s) - s - \sqrt{3k} = -kY_1(s) + k[Y_2(s) - Y_1(s)]$$

and $s^2Y_2(s) - s + \sqrt{3k} = -k[Y_2(s) - Y_1(s)] - kY_2(s)$

Elimination yields

$$Y_1(s) = \frac{(s+\sqrt{3k})(s^2+2k) + k(s-\sqrt{3k})}{(s^2+2k)^2 - k^2}$$

and $Y_2(s) = \dfrac{(s-\sqrt{3k})(s^2+2k) + k(s+\sqrt{3k})}{(s^2+2k)^2 - k^2}$

Breaking into partial fractions we have

$$Y_1(s) = \frac{s}{s^2+k} + \frac{\sqrt{3k}}{s^2+3k}$$

and $Y_2(s) = \dfrac{s}{s^2+k} - \dfrac{\sqrt{3k}}{s^2+3k}$.

Hence taking inverse Laplace transform the solution is obtained as

$$y_1(t) = \cos\sqrt{k}\ t + \sin\sqrt{3k}\ t$$

and $y_2(t) = \cos\sqrt{k}\ t - \sin\sqrt{3k}\ t$.

3. LAPLACE TRANSFORM IN DEFLECTION OF BEAM

Suppose a beam is kept along horizontal x-axis and its one end is kept at x=0 and other at x=L. Suppose the beam suffers a transverse deflection y(x) which is produced by applying a vertical load w(x) per unit length, the deflection is given by

$$\frac{d^4y}{dx^4} = \frac{w(x)}{EI}, \ 0 < x < L,$$

Where E is the Young's modulus of elasticity for the beam, I is the M.I. of the beam about x axis in a cross section. The boundary conditions are:

1) If beam is longed or has a simply supported ends then $y = y'' = 0$
2) If the beam is clamped or has a fixed ends then $y = y' = 0$
3) If beam has a free ends then $y'' = y''' = 0$

3.1 Example

A beam of length L is clamped horizontally at both ends and loaded at x=L/4 by weight w then to find the equation for deflection y of the beam at any point and maximum deflection, consider the equation of the deflection:

$$EI\frac{d^4y}{dx^4} = w\delta\left(x - \frac{L}{4}\right).$$

The boundary conditions are $\mathbf{y = \frac{dy}{dx} = 0}$ **at x = 0 and x = L**. Taking Laplace transform on both sides we get

$$s^4Y(s) = \frac{w}{EI}e^{-LS/4} + sy_2 + y_3.$$

Since $\mathbf{y_0 = y_1 = 0}$, Taking Inverse LT we get

$$y = \frac{1}{6}\frac{w}{EI}\left(x - \frac{L}{4}\right)^3 u\left(x - \frac{L}{4}\right) + \frac{1}{2}y_2x^2 + \frac{1}{6}y_3x^3$$

For $\mathbf{x} > L/4$,

we have $y = \dfrac{1}{6}\dfrac{w}{EI}\left(x - \dfrac{L}{4}\right)^3 + \dfrac{1}{2}y_2x^2 + \dfrac{1}{6}y_3x^3$

and $y' = \dfrac{1}{2}\dfrac{w}{EI}\left(x - \dfrac{L}{4}\right)^2 + y_2x + \dfrac{1}{2}y_3x^2$

Puttin $\mathbf{y = y' = 0}$ **at x = L we get**,

$$y = \frac{1}{6}\frac{w}{EI}\left(\frac{3L}{4}\right)^3 + \frac{1}{2}y_2L^2 + \frac{1}{6}y_3L^3$$

and $y' = \dfrac{1}{2}\dfrac{w}{EI}\left(\dfrac{3L}{4}\right)^2 + y_2L + \dfrac{1}{2}y_3L^2$

This gives $y_2 = \dfrac{9}{64}\dfrac{w}{EI}L$ and $y_3 = \dfrac{27}{32}\dfrac{w}{EI}$.

Therefore, $y = \dfrac{1}{6}\dfrac{w}{EI}\left(x - \dfrac{L}{4}\right)^3 u\left(x - \dfrac{L}{4}\right) + \dfrac{1}{2}y_2x^2 + \dfrac{1}{6}y_3x^3$

At the point of maximum deflection $\mathbf{y'}$ must be zero.

Now for $\mathbf{x} < L/4$, $$y' = y_2x + \frac{1}{2}y_3x^2 = \frac{9}{64}\frac{w}{EI}x(L-3x).$$

So $\boldsymbol{y'}$ is never zero for $\mathbf{0} < \mathbf{x} < L/4$.

For $\frac{\mathbf{L}}{\mathbf{4}} < \boldsymbol{x} < L$, $\mathbf{y'}$ is given by

$$y' = \frac{1}{2}\frac{w}{EI}\left(x - \frac{L}{4}\right)^2 + y_2x + \frac{1}{2}y_3x^2$$

Equation to zero we get

$$\frac{1}{2}\frac{w}{EI}\left(x - \frac{L}{4}\right)^2 + \frac{9}{64}Lx - \frac{27}{64}\frac{w}{EI}x^2 = 0$$

$$\Rightarrow 5x^2 - 7Lx + 2L^2 = 0 \Rightarrow (5x - 2L)(x - L) = 0$$

So maximum deflection occurs at $\mathbf{x} = \mathbf{2L/5}$ and its value is

$$= w\left[\frac{1}{2}\left(\frac{2L}{5} - \frac{L}{4}\right)^3 + \frac{9}{128}L\left(\frac{2L}{5}\right)^2 - \frac{9}{64}\left(\frac{2L}{5}\right)^3\right](EI)^{-1}$$

$$= wL^3\left[\frac{9}{1000} - \frac{9}{80} - \frac{9}{1000}\right](EI)^{-1} = \frac{63wL^3}{8000EI}$$

3.2 Example

A beam of stiffness EI is simply supported at its end $\mathbf{x} = \mathbf{0}$ and $\mathbf{x} = \mathbf{L}$. It carries a uniform load w per unit length from $\mathbf{x} = \mathbf{L/4}$ to $\mathbf{x} = \mathbf{3L/4}$. To find deflection at any point consider the equation of the deflection:

$$EI\frac{d^4y}{dx^4} = w(x)$$, Where w(x) is load per unit length.

$$w(x) = w\left\{u\left(x - \frac{L}{4}\right) - u\left(x - \frac{3L}{4}\right)\right\}.$$

In this case:

The boundary conditions are $y = 0$, and $y'' = 0$ at $x = 0$ and $x = L$. Taking LT on both sides we have,

$$s^4Y(s) = \frac{w}{EI}\left(e^{-LS/4} - e^{-3Ls/4}\right)/s + s^2y_1 + y_3$$

i.e. $$Y(s) = \frac{w}{EI}\left(\frac{e^{-LS/4}}{s^5} - \frac{e^{-3Ls/4}}{s^5}\right) + \frac{y_1}{s^2} + \frac{y_3}{s^4}$$

Now taking inverse LT on both sides we get

$$y(x) = \frac{1}{24}\frac{w}{EI}\left\{\left(x - \frac{L}{4}\right)^4 u\left(x - \frac{L}{4}\right) - \left(x - \frac{3L}{4}\right)^4 u\left(x - \frac{3L}{4}\right)\right\} + y_1x + \frac{1}{6}y_3x^3$$

For $\mathbf{x} > 3\mathbf{L/4}$ we have

$$y(x) = \frac{1}{24}\frac{w}{EI}\left\{\left(x - \frac{L}{4}\right)^4 - \left(x - \frac{3L}{4}\right)^4\right\} + y_1x + \frac{1}{6}y_3x^3$$

and $$y'(x) = \frac{1}{2}\frac{w}{EI}\left\{\left(x - \frac{L}{4}\right)^2 - \left(x - \frac{3L}{4}\right)^2\right\} + y_3x$$

Putting boundary conditions

$$0 = \frac{1}{24}\frac{w}{EI}\left\{\left(\frac{3L}{4}\right)^4 - \left(\frac{L}{4}\right)^4\right\} + y_1L + \frac{1}{6}y_3L^3$$

and $$0 = \frac{1}{2}\frac{w}{EI}\left\{\left(\frac{3L}{4}\right)^2 - \left(\frac{L}{4}\right)^2\right\} + y_3L$$

Therefore, $y_3 = -wL/(4EI)$ and

$y_1 = (11/384)4wL^3/(EI)$ Putting these values in

$$y(x) = \frac{1}{24}\frac{w}{EI}\left\{\left(x - \frac{L}{4}\right)^4 u\left(x - \frac{L}{4}\right) - \left(x - \frac{3L}{4}\right)^4 u\left(x - \frac{3L}{4}\right)\right\} + y_1x + \frac{1}{6}y_3x^3$$

We get deflection at any point.

3.3 Example

When the loading is non-uniform, the use of LT methods has a distinct advantage, since by making use of Heaviside unit functions and impulse functions. Figure 5 illustrates a uniform beam of length l, freely supported at both ends, bending under uniformly distributed weight W.

Our aim is to determine the transverse deflection y(x) of the beam. From the elementary theory of beams, we have

$$EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = W(x) \qquad (3.3.1)$$

Where W(x) is the transverse force per unit length, by means of a downwards force taken to be positive, and EI is the flexural rigidity of the beam (E is Young's modulus of elasticity and I is the moment of inertia of the beam about its central axis). It is supposed that the beam has uniform elastic property and a uniform cross-section over its length, so that both E and I are taken to be constants. Using the Heaviside step and the Dirac delta function the force W(x) can be expressed as

$$W(x) = w[1 - u(x - a)] + W[\delta(x - b)] \qquad (3.1.2)$$

Hence, from (3.1.1) and (3.1.2) we have

$$EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = w[1 - u(x - a)] + W[\delta(x - b)]$$

Where $$a^2 = \frac{P}{EI}, \bar{w} = \frac{w}{EI}, \bar{W} = \frac{W}{EI}$$

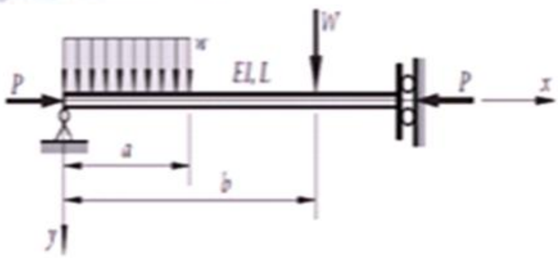

Fig. 5

Since the left end is a hinge support and the right end is a sliding support and the boundary conditions are:

At x = 0, deflection is zero, $y(0) = 0$, and bending moments is zero , $y''(0) = 0$

At x = L, slope is zero, $y'(L) = 0$ and share force is zero, $y''(L) = 0$. Applying Laplace transform

$$L\left[\frac{d^4y}{dx^4}\right]+a^2L\left[\frac{d^2y}{dx^2}\right]=\bar{w}L[1-u(x-a)]+\bar{W}L\delta(x-b)$$

Incorporating properties and Laplace transform of impulse and step function, we get

$$L[y(x)]=\frac{y'(0)}{s^2+a^2}+\frac{y''(0)+a^2y'(0)}{s^2(s^2+a^2)}+\bar{W}\frac{e^{-bx}}{s^2(s^2+a^2)}+\bar{w}\frac{1-e^{-ax}}{s^3(s^2+a^2)}$$

Taking Inverse LT and making uses of second shift theorem we get

$$\begin{aligned}y(x) = {} & \frac{y'(0)}{a}\sin at+[y''(0)+a^2y'(0)]\frac{1}{a^2}\left(t-\frac{\sin at}{a}\right)\\ & +\bar{W}\left[\frac{t-b}{a^2}-\frac{1}{a^2}\sin a(t-b)\right]u(t-b)\\ & +\bar{w}\left[\frac{1}{2a^2}(a^2t^2+2(\cos at-1))\right]\\ & +\left[\frac{1}{2a^4}(a^2(t-1)^2\right.\\ & \left.+2(\cos a(t-a)-1))\right]\end{aligned}$$

To find undetermined constants $y'(0)$ and $y''(0)$ we employ the unused boundary conditions at x = L , namely , $y'(L) = 0$ and $y''(L) = 0$.

4. ANALYSIS AND MODELING OF MECHANICAL SYSTEM

Laplace Transform in Transfer Function:

For calculating the transfer function of that certain system. Let's consider a big Pot as shown in figure 6. Initially the pot is empty at t = 0. Let F_i be the constant rate of flow added for $t > 0$ and $F_0 = BH$ be the rate at which flow leaves the tank. Let A be the cross sectional area of the tank. Now we determine the differential equation for the head H, identify the time constant and we find the transfer function of system by Laplace transform.

We know that $F_0 = BH$ (4.1.1)

Let's consider the fluid of mass m and fluid density σ. Since Mass is product of velocity and density.

Hence Mass = Velocity × density

i.e. $M = V\times\sigma = HA\times\sigma$

Now mass flow rate is given by $\bar{M}=\frac{dM}{dt}=A\sigma\frac{dH}{dt}$ and {Mass flow rate into tank} = {the mass flow rate} – {the mass out flow rate}

$\sigma A\frac{dH}{dt}=\sigma F_i-\sigma F_0$, which gives, $A\frac{dH}{dt}=F_i-F_0$.

Hence , $F_i = A\frac{dH}{dt}+BH$.

This is differential equation for heat flow and its solution is easily obtained using Laplace Transform method. Taking Laplace transform on both sides we get

$$L(F_i)=A_sL\left(\frac{dH}{dt}\right)+B_sL(H)$$

or

$$F_i(s)=A_s\{H(s)-H(0)\}+B_sH(s)$$

$$F_i(s)=A_s\{H(s)\}+B_sH(s),\quad H(0)=0$$

This can be written as,

$$\frac{H(s)}{F_i(s)}=\frac{1}{A_s+B_s} \qquad (4.1.2)$$

Now taking LT of equation (4.1.1) we get

$$F_0(s)=B_sH(s) \qquad (4.1.3)$$

From (4.1.2) and (4.1.3) we get $\frac{F_0(s)}{F_i(s)}=\frac{1}{1+\left(\frac{A_s}{B_s}\right)s}$ and this is equation of transfer function of the system, where the time constant is given by $\tau=\frac{A_s}{B_s}$.

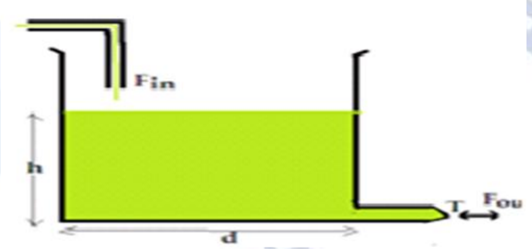

Fig. 6

Laplace Transform in Vibrating Mechanical System

Let's consider the vehicle of mass m. The important elements in the investigating the suspension system of vehicle are the mass of the vehicle, the springs and damper which are used to connect to the body of the vehicle to the suspension links. Mechanical translational systems involve three basic elements: mass (M kg), springs (having spring stiffness K, measured in Nm−1) and dampers (having damping coefficient D, measured inNsm-1. The associated variables are:

1) Displacement y(t) which is measured in m
2) Force F(t) (measured in N).

Figure 7 shows the Mass–spring–damper system. By Newton's and Hooke's law, we can develop mathematical model for Mass–spring–damper system. The represented system shown in side figure is governed by the differential equation

$$\frac{d^2y}{dt^2}+D\frac{dy}{dt}+Ky=F(t)$$

or
$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=2sinwt \qquad (4.2.1)$$

Taking Laplace transforms throughout in (4.2.1), we get

$$L\left[\frac{d^2y}{dt^2}\right]+2L\left[\frac{dy}{dt}\right]+5L(x)=2L(sinwt)$$

i.e.
$$(s^2+2s+5)L\left[y(t)\right]=[sy(0)+y'(0)+6y(0)]\frac{2w}{s^2+w^2}$$

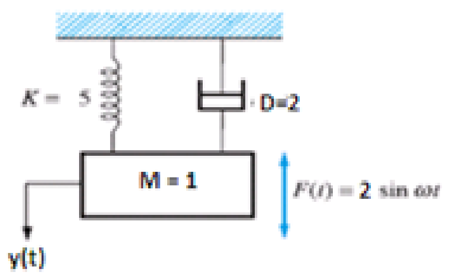

Fig. 7

Taking initial conditions $y(0)=y'(0)=0$ we can write

$$Y(s)=L\left[y(t)\right]=\frac{2w}{(s^2+2s+5)\left(s^2+w^2\right)}$$

On resolving into partial fractions for different values of w (say w = 2) we get

$$Y(s)=\frac{2}{(s^2+2s+5)\left(s^2+1\right)}=\frac{As+B}{\left(s^2+1\right)}+\frac{Cs+D}{(s^2+2s+5)}$$

On Taking inverse Laplace transforms

$$y(t)=\frac{1}{4}\left[\cos t-\frac{3}{2}\sin t\right]+e^{-t}\left(\frac{-1}{4}cos2t+\frac{23}{8}sin2t\right)$$

5. Conclusion

Throughout the paper, we have discussed some applications of Laplace Transform in various fields of Engineering.

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Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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