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Alternative Averaging Techniques for Solving Multi-Objective Optimization Problems

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ABSTRACT

Several Averaging Techniques of Multi-Objective Optimization (MOO) have been proposed during the past two decades. Most of these techniques have not been formulated appropriately. The examples used to illustrate the techniques were not suitable. The results have also not been interpreted correctly. The present paper suggests an alternative averaging techniques for solving multi-objective optimization problems.

Keywords: Multi-Objective Optimization, Sen's Multi-Objective Optimization, Averaging Techniques of Multi-Objective Optimization.

1. INTRODUCTION

The weighting technique for obtaining non inferior solution of MOO problems was introduced by Zadeh (1963) [1]. Marglin (1967) [2] and Major (1969) [3] used of weighting technique in multi-objective public investment problems. An appropriate technique for solving MOO problems was first introduced by Sen in the year 1983[4]. Sen.s MOO technique have been widely applied for improving the resources use planning in agriculture [5]-----[9].Several new MOO techniques have been proposed [10]-------[17] using mean and median of optimal values of individual optimization. Harmonic means, advanced averaging techniques using various kind of means [18], [19], [20] have been used for solving MOO problems. An advanced transformation technique has been proposed recently [21] by Yesmin and Alim.An improved averaging technique has been suggested [22] by Sen.The objective functions have been scalarized by the mean values of optimal and sub optimal values of the respective objective function. An alternative averaging technique have been proposed in the present study. The alternative averaging techniques have been tested by solving suitable example and compared with the results of existing averaging MOO techniques.

2. MULTI-OBJECTIVE OPTIMIZATION TECHNIQUES

A. Sen's MOO Technique

The structure of multiple objective optimization is explained below:

$$\text{Let Max./Min. } Zi = \sum_{i=1}^{m} CijXj$$

Subject to:

$$AX \leq/=/\geq b$$

Where,

C_{ij} = Coefficient of J[th] decision variable X for i[th] Objective function

A = coefficient matrix of constraints, b= upper/lower limit of the constraints.

Optimize Z= [Max. Z_1, Max. Z_2..........Max. Z_r, Min.Z_{r+1}......Min. Z_s]

Subject to:

$$AX \leq/=/\geq b$$

All the objection functions are optimized (max./min.) individually for the formulation of Multi-Objective function. The values of individual optima are given as:

$$Z_{\text{optima}} = [\Theta_1, \Theta_2 \ldots\ldots\ldots\ldots \Theta_r, \Theta_{r+1} \ldots\ldots\ldots\ldots \Theta_s]$$

The multi-objective functionformulated by Sen is detailed below:

$$\text{Max. } Z = \sum_{i=1}^{r} \frac{z_i}{\theta_i} - \sum_{J=r+1}^{s} \frac{Z_j}{\theta_j}$$

Subject to common constraints mentioned as above.

B. Existing Averaging MOO Techniques

The multi-objective function in the existing MOO techniques is formulated as explained below:

$$\text{Max. } Z = \frac{\sum_{i=1}^{r} Z_i}{Am} - \frac{\sum_{j=r+1}^{s} Z_j}{An}$$

Where,

Am = Averages of coefficients of decision variables or averages of all the individual optima of all the maximization objective functions.

An=Averages of coefficients of decision variables or averages of all the individual optima of all the minimization objective functions.

C. Alternative Averaging MOO Techniques

The multi-objective function in the alternative MOO techniques is formulated as detailed below:

$$\text{Max. } Z = \frac{\sum_{i=1}^{r} Z_i}{Ai} - \frac{\sum_{j=r+1}^{s} Z_j}{Aj}$$

Where,

Ai = Averages of coefficients of decision variables or averages of all the individual optimal and suboptimal values of i[th] maximization objective function.

Aj = Averages of coefficients of decision variables or averages of all the individual optimal and suboptimal values of i[th] minimization objective function.

3. EXAMPLE

Examples play an important role in understanding the mathematical theory. Most of the studies on MOO [23], the suitable examples have not been used. The results have also not interpreted appropriately. The present study presents a comparative analysis of existing and proposed averaging MOO techniques using following example.

Example

Max. $Z_1 = 2000X_1+3900X_2+9000X_3+4300X_4+5000X_5$

Max. $Z_2 = 20X_1+7X_2+18X_3+8X_4+2X_5$

Min. $Z_3 = 600X_1+100X_2+700X_3+150X_4+250X_5$

Min. $Z_4 = 120X_1+50X_2+90X_3+40X_4+70X_5$

Subject To:

$$2X_1+X_2+3X_3+X_4+X_5=6$$

$$X_4 \geq 1$$

$$X_1, X_2, X_3, X_4, X_5 \geq 0$$

4. RESULTS

The example has been solved for optimization of each objective function individually. The solution is presented in table1.

Table 1: Individual and Sen's Multi-Objective Optimization

Item	Individual Optimization				Sen's MOO
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4	
X1,X2,X3,X4,X5	0, 0, 0, 1, 5	2.5, 0, 0, 1, 0	0, 5, 0, 1, 0	0, 0, 1.67, 1, 0	0, 0, 0, 6, 0
Z_1	29300	9300	23800	19330	25800
Z_2	18	58	43	38	48
Z_3	1400	1650	650	1319	900
Z_4	390	340	290	190	240
Z*	Multi-Objective Function				3.95

The Value of first objective function was 29300 which is highest amongst all other objective functions. Similarly, when the second objective function was maximized, the highest value of 58 was achieved. When objective function third was minimized, its value was 650 only which is the lowest in comparison to remaining objective functions. The minimization of fourth objective function achieved its lowest value of 190 with non optimal values of the remaining objective functions. The values of all the objective functions are different in each individual optimization. This clearly indicates the conflicts amongst the objective functions. However the Sen,s MOO technique has achieved all the four objectives simultaneously. The value of first objective function is 25800 which lower than its individual optimal value of 29300 but greater than remaining three objective functions. The remaining three objective functions have also achieved the similar values. The value of multi-objective function was 3.95 which has less importance in the analysis. It can be concluded that Sen's MOO technique is efficient in generating the acceptable compromise solutions of the MOO problems. The example has been further solved by optimizing the combined objective function formulated by scalarizing the individual objective functions with the averages of coefficients of decision variables X. The formulation of combined objective function in both existing and alternative techniques was different as explained earlier. The arithmetic, geometric and harmonic averages have been used in formulation of combined objective functions.Table 2 presents the results of averaging MOO techniques using coefficients of decision variables.

Table 2: Solution of Averaging MOO Techniques Using Coefficients of decision Variables.

Item	Averaging Techniques (Averages of coefficients of decision variables)					
	Existing Averaging Techniques			Alternative Averaging Techniques		
	AA	GA	HA	AA	GA	HA
X1,X2,X3,X4,X5	0, 5, 0, 1, 0	0, 5, 0, 1, 0	0, 5, 0, 1, 0	0, 0, 0, 6, 0	0, 0, 0, 6, 0	0, 0, 0, 6, 0
Z_1	23800	23800	23800	25800	25800	25800
Z_2	43	43	43	48	48	48
Z_3	650	650	650	900	900	900
Z_4	290	290	290	240	240	240
Z*	5.50	5.53	5.50	3.96	5.94	7.68

Z*= Multi-Objective Function

AA= Arithmetic Average, GA= Geometric Average, HA= Harmonic Average

All the three solutions under existing averaging MOO techniques were same. These techniques have achieved the third objective function only ignoring the remaining three objective functions. However, the alternative averaging MOO techniques have achieved all the four objectives at a time. This result is similar as obtained in the Sen's MOO technique. The values of two multi-objective functions using arithmetic average and harmonic average under existing averaging MOO technique were equal. The third value of multi-objective function was highest in spite of same values of all the four objective functions under all the three averaging situations. Similarly, the values of multi-objective functions using alternative averaging MOO techniques were all different in spite of same values of all the four individual objective functions. The value of multi-objective function using arithmetic mean under alternative averaging MOO technique was lowest in spite of superior values of individual objective functions. Hence, the values of multi-objective functions cannot be used for any conclusion. Most of the studies [10], [11]--------[17] of existing averaging MOO techniques have evaluated the superiority of the technique on the basis of the highest value of multi-objective function which was not logical.

The example has been solved third time again with existing and alternative averaging MOO techniques using the values of the objective functions obtained under individual optimization. The results are given in table 3.

Table 3: Solution of Alternative Averaging MOO Techniques using values of individual optimization.

Item	Averaging Techniques (Averages of the values of in Individual optimization)					
	Existing Averaging Techniques			Alternative Averaging Techniques		
	AA	GA	HA	AA	GA	HA
X1,X2,X3,X4,X5	0, 5, 0, 1, 0	0, 5, 0, 1, 0	0, 5, 0, 1, 0	0, 0, 0, 6, 0	0, 0, 0, 6, 0	0, 0, 0, 6, 0
Z_1	23800	23800	23800	25800	25800	25800
Z_2	43	43	43	48	48	48
Z_3	650	650	650	900	900	900
Z_4	290	290	290	240	240	240
Z*	1.12	1.25	1.43	0.96	1.08	1.32

Z*= Multi-Objective Function

AA= Arithmetic Average, GA= Geometric Average, HA= Harmonic Average

It has been again observed that existing averaging techniques have achieved single objective only, whereas the alternative averaging techniques have achieved all the four objectives simultaneously. The values of all the multi-0bjective functions under both existing and alternative averaging MOO techniques were different. The values of all the individual objective functions were all equal using arithmetic, geometric and harmonic averaging MOO techniques.The alternative averaging MOO techniques have also resulted the equal values of all the individual objective functions. The value of multi-objective function is highest using harmonic average under existing averaging MOO techniques. However, the achievements of all the four objectives are better in spite of lower values of multi-objective functions under alternative averaging MOO techniques.

5. CONCLUSION

The present study proposed analternative averaging techniques for solving MOO problems. A suitable example has been solved with the both existing and the proposed averaging MOO techniques. The comparative analysis of the results of existing and alternative MOO techniques clearly indicates the superiority of the alternative averaging techniques over the existing averaging MOO techniques.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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