



# Evaluation of Mean Deviation Techniques to Solve Multi-Objective Fractional Programming Problem via Point Slopes Formula

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## ABSTRACT

New Mean Deviation Techniques for solving multi-objective fractional programming problems via point slope formula have been proposed recently. A modified equation for the formulation of multi-objective function was suggested. The method is not appropriate for multi-dimensional objective functions. The examples with unique solutions for all the objectives have been used in the study. The results have not been interpreted appropriately.

**Keywords:** Multi-Objective Linear Fractional Programming Problem, Multi-Objective Quadratic Fractional Programming Problem, Point-Slopes Formula, Mean Deviation.

## 1. INTRODUCTION

Most of the multi-objective optimization (MOO) problems are solved using multi-objective function. The first appropriate technique for formulation of multi-objective function was suggested by Sen in 1983 [1]. Since then several new techniques for the formulation of multi-objective function have been proposed [2],-----[14]. New mean deviation technique have been suggested [15] recently is evaluated in this study. The technique has been tested with examples. The results of the proposed technique has been compared with the existing techniques. The formulation of multi-objective function with various techniques is described below.

## 2. FORMULATION OF MULTI-OBJECTIVE FUNCTION

### 2.1 Sen's MOO Technique

Sen's multi objective function is mentioned below:

Optimize  $Z = \text{Max. } Z_1, \dots, \text{Max. } Z_m, \text{Min. } Z_{m+1}, \dots, \text{Min. } Z_n$

$$\text{Max. } Z = \sum_{i=1}^m \frac{Z_i}{|\theta_i|} - \sum_{j=m+1}^n \frac{Z_j}{|\theta_j|}$$

Subject to;

$$AX \geq, \leq, = b$$

$$X \geq 0$$

Where;

$Z$  is the multi- objective function,  $Z_i$  is the  $i^{\text{th}}$  maximization objective function and  $Z_j$

is the  $j^{\text{th}}$  minimization objective function.  $|\Theta_i|$  is the optimal value of  $i^{\text{th}}$  maximization objective function and  $|\Theta_j|$  is the optimal value of  $j^{\text{th}}$  minimization objective function.  $X$  are the decision variables,  $A$  is the coefficients matrix and  $b$  is the constraint vector.

## 2.2 Averaging Techniques

$$\text{Max. } Z = \sum_{i=1}^m \frac{Z_i}{\Theta_i} - \sum_{j=m+1}^n \frac{Z_j}{\Theta_j}$$

Where;

$\Theta_i$  are the mean of optimal values of maximization objective functions and  $\Theta_j$  are the mean of optimal values of minimization objective functions. The other things will be same as mentioned in Sen's MOO technique.

## 2.3 New Mean Deviation Technique

The new mean deviation factors  $MD_m$  and  $MD_n$  are estimated as detailed below:

For maximization objectives the  $MD_m$  is calculated as

$$MD_m = \sum_{i=1}^m \frac{Z_i}{Z_m}$$

And for minimization objectives the  $MD_n$  is estimate as

$$MD_n = \sum_{j=m+1}^n \frac{Z_j}{Z_n}$$

Where;

$MD_m$  and  $MD_n$  are mean deviation factor for maximization and minimization objectives respectively. The new mean deviation (NMD) is estimated as explained below:

$$NMD = (MD_m + MD_n) / R$$

Where  $R = \text{Number of objectives} / \text{Types of objectives}$

## 2.4 Advanced Mean Deviation Technique

The advanced mean deviation (AMD) is estimated as

$$AMD = (MD_m + MD_n) / S$$

Where ;

$S = \text{Number of objectives}$

Using all the above four factors the multi-objective function is formulated for solving the MOO problems. Two examples have been solved as detailed below.

## 3. EXAMPLES

Following two examples have solved using above mentioned techniques.

### Example 1:

$$\text{Max. } Z_1 = [(2x_1 + x_2 + 1)(2x_1 + x_2 + 2)] / (2x_1 + 2x_2 + 2),$$

$$\text{Max. } Z_2 = [(4x_1 + 2x_2 + 2)(6x_1 + 3x_2 + 6)] / (3x_1 + 3x_2 + 3)$$

$$\text{Max. } Z_3 = [(4x_1 + 2x_2 + 2)(6x_1 + 3x_2 + 6)] / (6x_1 + 3x_2 + 6)$$

$$\text{Min. } Z_4 = [(-8x_1 - 4x_2 - 4)(6x_1 + 3x_2 + 6)] / (5x_1 + 5x_2 + 5),$$

$$\text{Min. } Z_5 = [(-4x_1 - 2x_2 - 2)(10x_1 + 5x_2 + 10)] / (2x_1 + 2x_2 + 2)$$

Subject to:

$$x_1 + 2x_2 \leq 4$$

$$3x_1 + x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

### Example 2:

$$\text{Max. } Z_1 = (3x_1 - 2x_2) / (x_1 + x_2 + 1)$$

$$\text{Max. } Z_2 = (9x_1 + 3x_2) / (x_1 + x_2 + 1)$$

$$\text{Max. } Z_3 = (3x_1 + 5x_2) / (2x_1 + 2x_2 + 2)$$

$$\text{Min. } Z_4 = (-6x_1 + 2x_2) / (2x_1 + 2x_2 + 2)$$

$$\text{Min. } Z_5 = (-3x_1 - x_2) / (x_1 + x_2 + 1)$$

Subject to:

$$x_1 + x_2 \leq 2$$

$$9x_1 + x_2 \leq 9$$

$$x_1, x_2 \geq 0$$

## 4. SOLUTIONS

The solutions of both the examples have been presented in table 2 and 4. The results of individual optimizations have also been mentioned in table 1 and 2. It is very clear from individual optimizations that all the objectives in both the examples have unique solutions. This clearly indicated the absence of conflicts amongst objectives and multi-objective optimization is not required. The values of each objective is same in individual as well as in the multi-objective optimizations. However, the values of multi-objective functions were different for all the MOO techniques. The values of multi-objective functions are not good indicators for making any conclusion. The MOO techniques with higher values of multi-objective function are declared superior which is wrong conclusion.

**Table 1: Individual Optimization of example 1**

Item	Individual optimization				
	Max. Z <sub>1</sub>	Max. Z <sub>2</sub>	Max. Z <sub>3</sub>	Min. Z <sub>4</sub>	Min. Z <sub>5</sub>
X <sub>1</sub> , X <sub>2</sub>	2, 0	2, 0	2, 0	2, 0	2, 0
Z <sub>1</sub>	5	5	5	5	5
Z <sub>2</sub>	20	20	20	20	20
Z <sub>3</sub>	10	10	10	10	10
Z <sub>4</sub>	-24	-24	-24	-24	-24
Z <sub>5</sub>	-50	-50	-50	-50	-50

**Table 2: Individual Optimization of example 2**

Item	Individual optimization				
	Max. Z <sub>1</sub>	Max. Z <sub>2</sub>	Max. Z <sub>3</sub>	Min. Z <sub>4</sub>	Min. Z <sub>5</sub>
X <sub>1</sub> , X <sub>2</sub>	1, 0	1, 0	1, 0	1, 0	1, 0
Z <sub>1</sub>	1.5	1.5	1.5	1.5	1.5
Z <sub>2</sub>	4.5	4.5	4.5	4.5	4.5
Z <sub>3</sub>	0.75	0.75	0.75	0.75	0.75
Z <sub>4</sub>	-1.5	-1.5	-1.5	-1.5	-1.5
Z <sub>5</sub>	-1.5	-1.5	-1.5	-1.5	-1.5

**Table 3: Multi-Objective Optimization for Example 1**

Item	Sen's MOO	Advanced Optimal Average	Advanced Harmonic Average	New Mean Deviation	Advanced Mean Deviation
X <sub>1</sub> , X <sub>2</sub>	2, 0	2, 0	2, 0	2, 0	2, 0
Z'	5	18.793	13.1707	14.6855	29.3712
Z <sub>1</sub>	5	5	5	5	5
Z <sub>2</sub>	20	20	20	20	20
Z <sub>3</sub>	10	10	10	10	10
Z <sub>4</sub>	-24	-24	-24	-24	-24
Z <sub>5</sub>	-50	-50	-50	-50	-50

**Table 4: Multi-Objective Optimization for Example 2**

Item	Sen's MOO	Advanced Optimal Average	Advanced Harmonic Average	New Mean Deviation	Advanced Mean Deviation
X <sub>1</sub> , X <sub>2</sub>	1, 0	1, 0	1, 0	1, 0	1, 0
Z'	5	21.6666	9.75	16.25	32.5
Z <sub>1</sub>	1.5	1.5	1.5	1.5	1.5
Z <sub>2</sub>	4.5	4.5	4.5	4.5	4.5
Z <sub>3</sub>	0.75	0.75	0.75	0.75	0.75
Z <sub>4</sub>	-1.5	-1.5	-1.5	-1.5	-1.5
Z <sub>5</sub>	-1.5	-1.5	-1.5	-1.5	-1.5

## 5. CONCLUSION

The whole analysis reveals that the proposed mean deviation technique for solving MOO problems is not appropriate. The examples used for testing the technique were not suitable. The conclusions drawn with the results were not satisfactory.

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