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Introduction to Finite Element Methods in Engineering

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ABSTRACT

Finite Element Method is very useful powerful technique. FEM is espescially used in civil and structural engineering disciplines, and other branch disciplines and applied sciences are also used. The main aim of FEM is reducing the time for large problem calculations. It is a numerical method, so easy to solve with sufficient time and accurately. By using FEM analyse the structural behaviour of structures and it is also useful for non-structural members also. It is one of the important numerical technique. It is used to solve problems in engineering disciplines in a mathematical way. It is used for structural analysis, fluid flow, heat transfer and mass transfer problems etc. there are number of softwares are available for FEM. In this some important are Ansys, Cosmos, Nisa, Nastran, Sap etc. there are most important principles are there. Which are really useful. In this paper, we discuss about introduction to FEM, dicretization, element and node, types of elements in FEM, some important equations.

KEYWORDS: *Finite Element Method, Finite Element Analysis, Discretization, Node and Elements.*

INTRODUCTION

Finite Element Method is a numerical method. It is used to solve problems in a hand calculations and mathematical calculations. It gives an exact solutions with more accuracy. It is used to solve elasticity problems and analysis of structures in mechanical, aero space and civil/structural engineering disciplines and it is also useful for applied sciences. Finite Element Method helps to calculate stress functions and strain functions and stress-strain functions and displacement functions, shape functions, stiffness functions, force functions. Also determine plane stress and plane strain matrixes. Finite Element Method helps in determining stiffness and strength parameters. In Finite Element Method, many of softwares are available to determine results properly and accurately.

Finite Element is nothing but a member is divided into number of several small members. So, it consists specific shape and geometry. Hence it is known as Finite Element. It is used for numerical solution of a problem with partial differential equations. Finite Element Method is simple and not complicated so much. It is used to divide the entire area of the object into small snall pieces and then calculated each and every one.

Finite Element Method is used for both structural and non structural engineering. It includes hand calculations, experiments and computer simulations.

Finite Element Method is involves some applications of CAD and CAM.

BACKGROUND OF FEM

- The first work done by Alexander Hrennikoff in the year of 1941. and then later the second work done by Richard Courant in the year of 1942.
- Framework method is introduced by Hrenikoff , In this Framework method plane is elastic medium, and it is represented as collections of bars and beams.
- Digital computer is used In the year of 1950, to solve large equations and tough problems.
- The first paper is published In the year of 1960. The First conference organized In the year of 1965,
- The First Book is Published In the year of 1967, the title of the book is "Finite Element Method" by Zienkiewicz and Chung.
- Finite Element is widely used In the year of 1960 and 1970, in this year this methods are most popular and mostly used. Finite Element Method is using widely for solving variety of problems in engineering.

Finite Element Softwares are started In the year of 1970, The most important Finite Element Method softwares are ANSYS, ABAQUS, NASTRAN, COSMOS, NISA, SAP etc. It leads to development of computer programmes.

- In Finite Element Method started developing algorithms on fluid flow, thermal flow and mass transfer problems in the year of 1980.
- Then later Engineers are solving structural and non-structural problems in the year of 1990. They are used to evaluate ways to control the vibrations by using Finite Element Method. Acuuracy also improved in this year. With this today we understand and analyse structural behaviour accurately and easily.

LITERATURE REVIEW

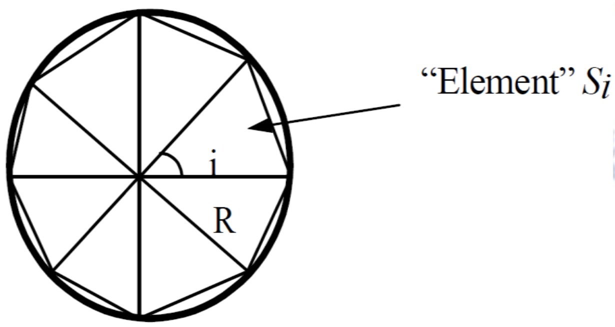

Fig. Approximation of the area of a circle

From the above figure we can see that, the whole circle is divided into number of small equal parts. Which is known as Finite Element. 'R' is the radius of the circle and 'i' is the angle of the circle. Element is represented as 'S_i'

Advantages of FEM

1. Solve any geometry problems with a good accuracy.
2. Handle any members subjected to loading conditions.
3. It can be useful for Homogeneous and Non Homogeneous materials easily.
4. Higher order partial differential equations and integrations are used for solving problems.
5. It is also used for non linear behaviour of materials.
6. It is also used for materials subjected to large deformations.
7. It is used for any kind of boundary conditions.
8. Finite Element Model easily and cheaply.

Disadvantages of FEM

1. It is a computer operations. Some times need it.
2. It is costly
3. It takes more time for calculations.
4. Output results may vary some times
5. Initial cost is high
6. It doesnot gives exact results but gives acceptable results
7. It involves differentiation and integration, matrix analysis are used.

METHODOLOGY

Methods of FEM

1. **Flexibility Method:** In this forces are considered as unknowns. It is also called as flexibility method.
2. **Displacement:** In this displacements are considered as unknowns. It is also called as stiffness method

There are various types of methods are available in Finite Element. Methods of Engineering Analysis. In this some of important are as follows;

1. **Experimental Method:** In this proto type, man power and materials is used. It is time consuming and costly.
2. **Analytical or Theoritical Method:** It gives quick and closed solutions. It is applicable for certain simplified situations.
3. **Numerical or Approximation Method:** It gives approximate solutions but acceptable.

For different engineering problems , we can use different Finite Elements. It depends on algebraic equations, boundary conditions, differential equations etc.

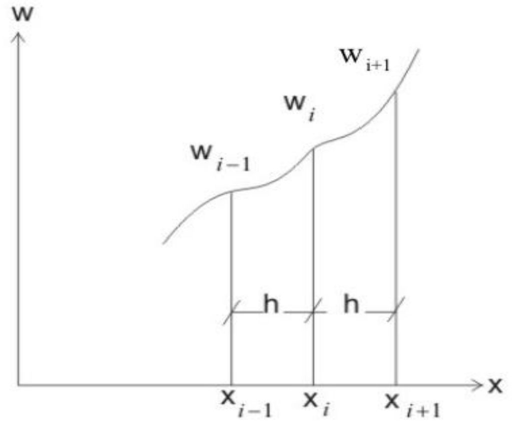

Fig. Displacement Function

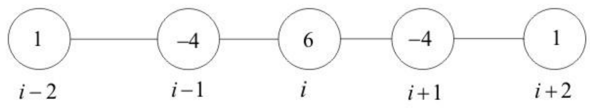

Fig. Finite difference equation at node i

Elements and Nodes

From above figure we can clearly understand about nodes and elements.

Nodes : Which are represented with i

Elements: Which are represented with numbers

From the above figure, nodes are connected to the members. Let the nodes are i-2, i-1, i, i+1, i+2. Similarly the elements are represented in the circles. Such as 1, -4, 6, -4, 1. In this small small elements are connected to each other to form a member and every element is connected to the two nodes. Two nodes make forms a one element and combination of elements called as members.

Every element has material properties and geometrical properties. Material properties are constant.

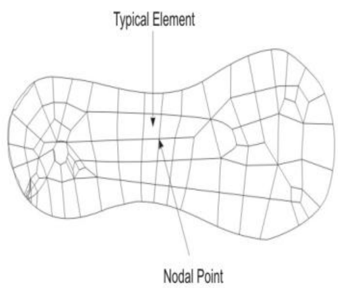

Fig. Finite element discretization of a domain

Degree of freedom

It is the number of independent co-ordinates in the system.

Let, take an truss members. the truss member is formed by connecting of all elements each other. It subjected to axial deformations only.

If there is only two independent co-ordinates are present, then the degree of freedom of the truss member is two.

Let Displacements are represented by u,v,w with respect to horizontal, vertical and inclined directions. Similarly, the rotations are represented by θx , θy, θz with respect to horizontal, vertical and inclined directions.

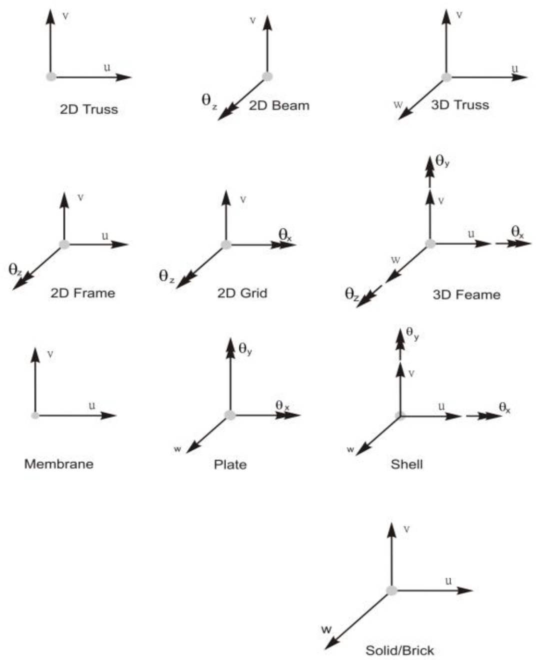

Fig. Degrees of Freedom for Various Elements

Any member can be discretized in different ways. It depends on geometry and shape of the members respectively. Based on this we can decide the discretization.

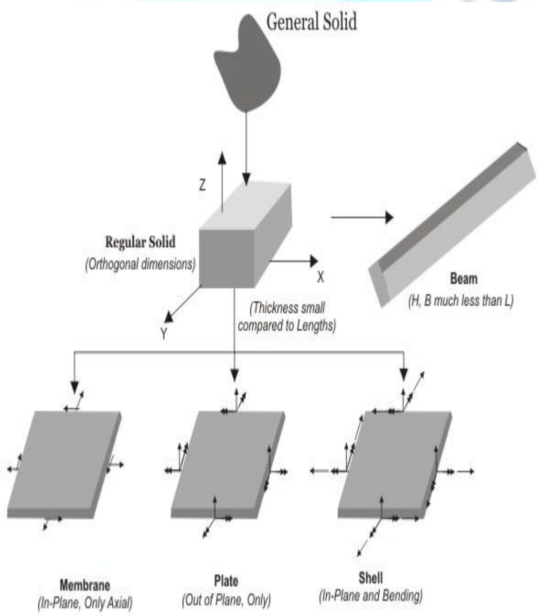

Fig. Various ways of Idealization of a Continuum

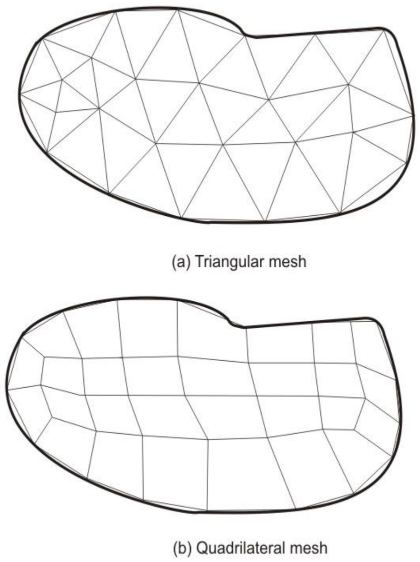

Fig. Discretization of a continuum

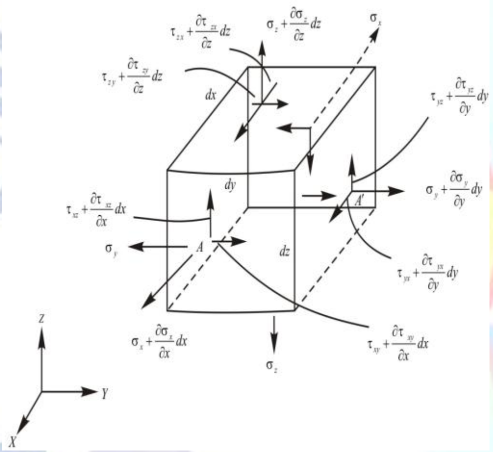

Fig. Graphical Representation of stress-strains

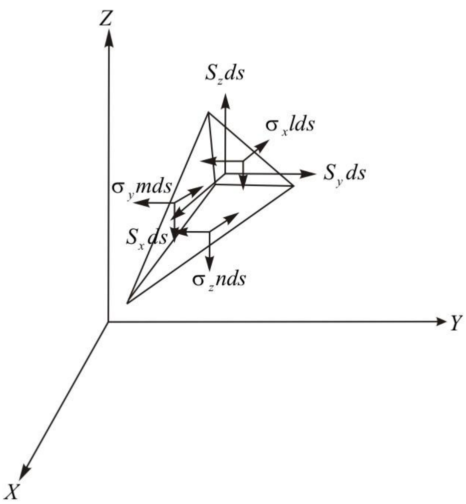

Fig. Forces acting on the element boundary

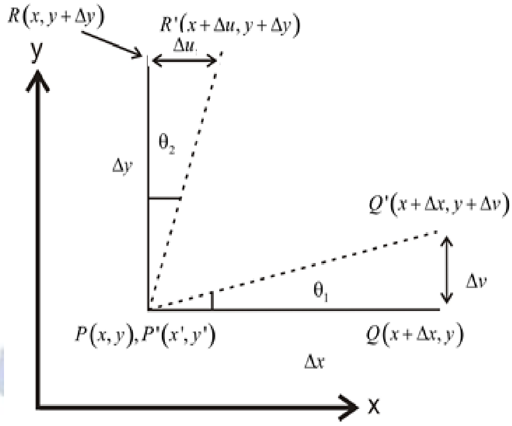

Fig. Representation of Shear strain

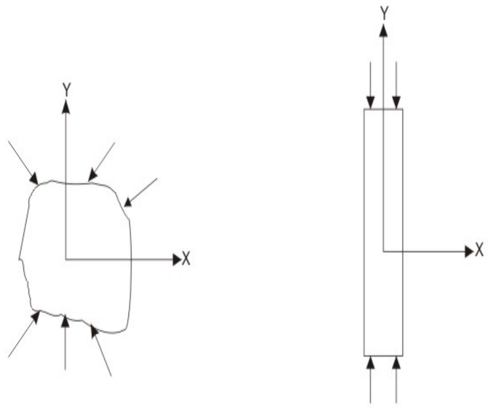

Fig. Plane stress; thin plate with bending

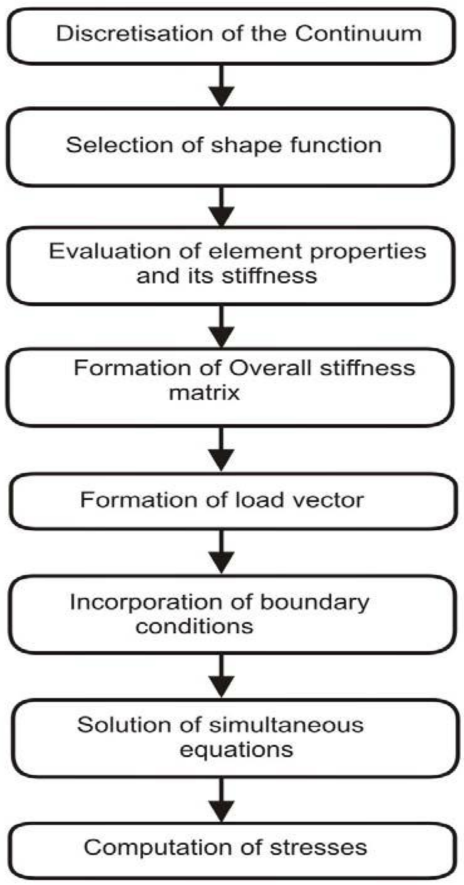

Fig. Flow chart of FEM

Types of elements used in FEM

1. Truss
2. Beam
3. Frame
4. Plate/Shell

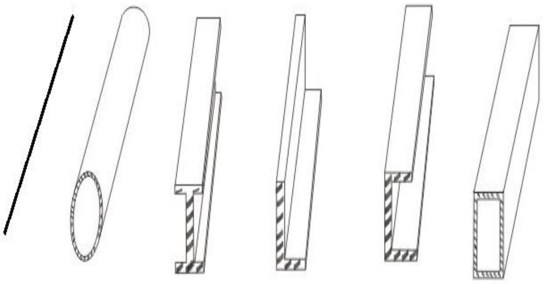

Fig. 1-D Elements

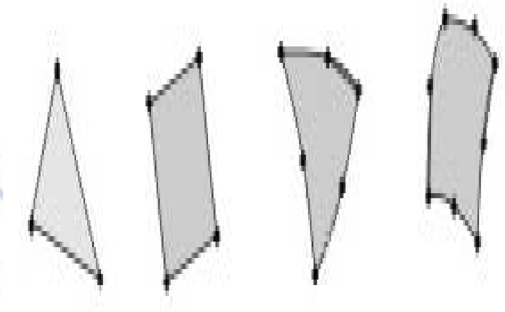

Fig. 2-D Elements

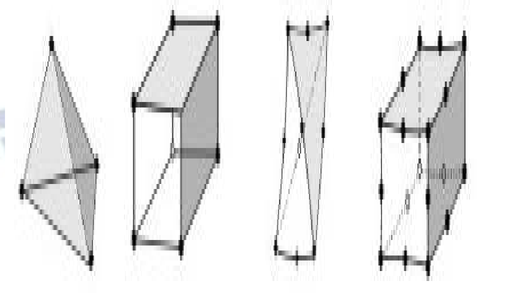

Fig. 3-D Elements

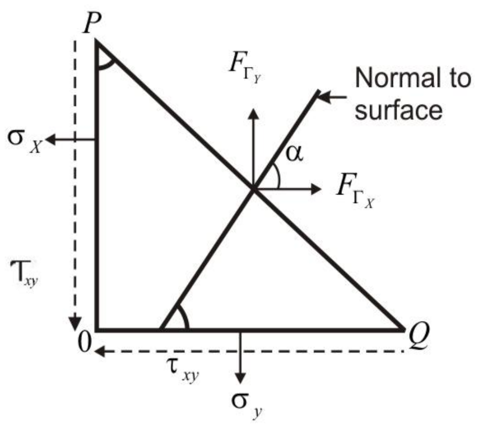

Fig. Elemental stresses in 2-D

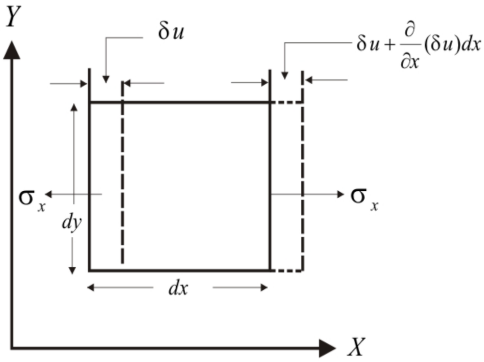

Fig. Element subjected to stresses

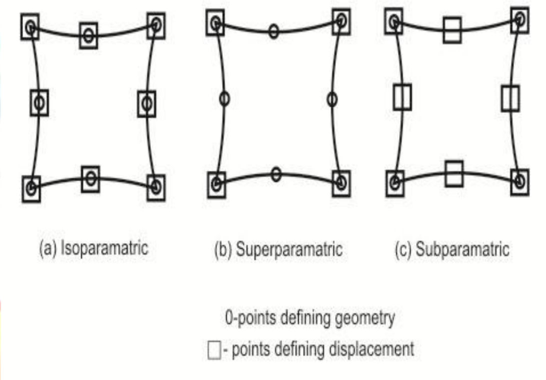

Fig. Shape functions for geometry and displacements

Important types of Finite Element Methods

1. 1-D FEM
2. 2-D FEM
3. 3-D FEM

1.1-D FEM

From below figure (a) we can understand, a member subjected to two nodes at each end and another figure (b) is subjected to 3 nodes , in this two nodes at each ends and one is at the centre of the member. So, 1-D FEM we can categorized into two types based on the node on the members.

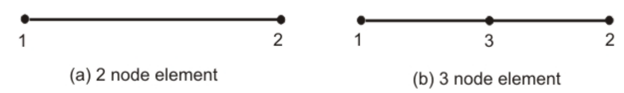

Fig. 1-D Elements

2. 2-D FEM

From the below figures we can clearly understand about 2-D FEM. In this most important common types of 2-D FEM are four noded rectangular and six noded triangular and eight noded quadrilateral and quadrilateral formed by two triangles and quadrilaterals are formed by four triangular elements.

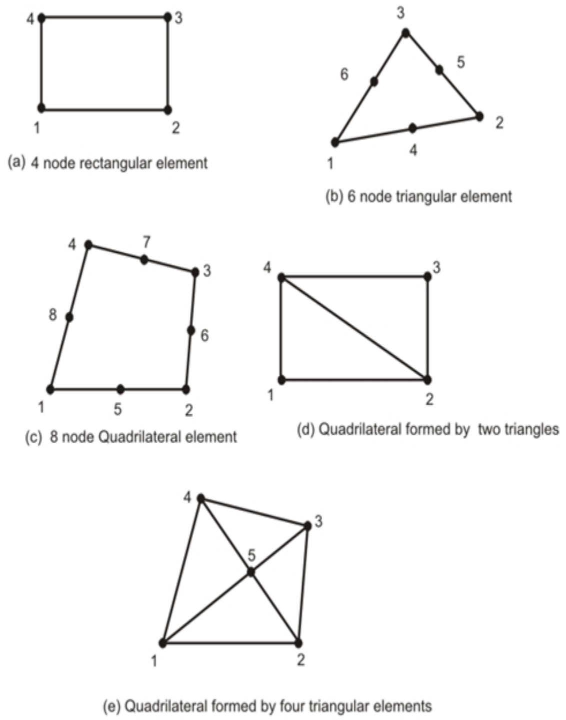

Fig. 2-D Elements

3. 3-D FEM

From the below figures we can understand about 3-D FEM. In this some of important are tetrahedron, rectangular brick and hexahedron.

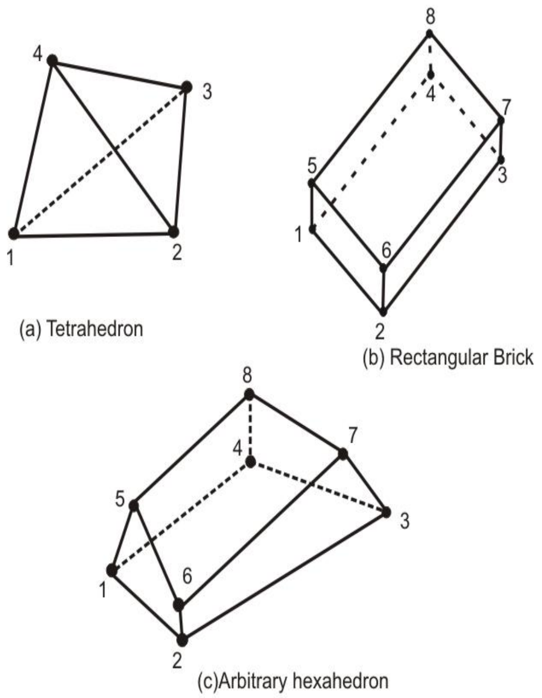

Fig. 3-D Elements

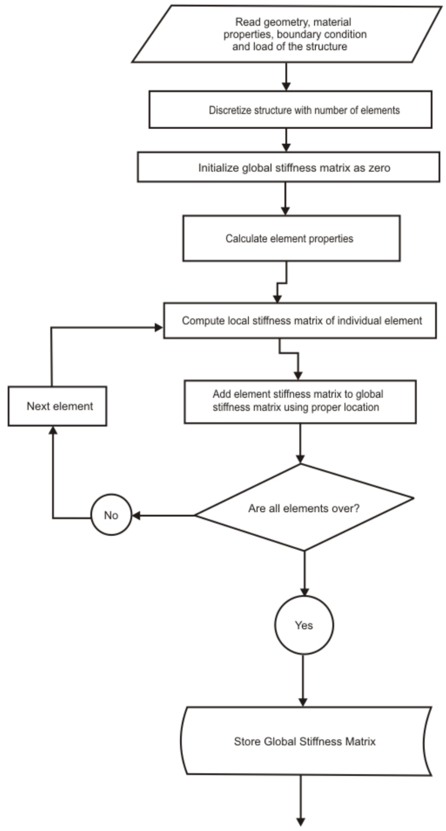

Fig. Stiffness matrix from local to global

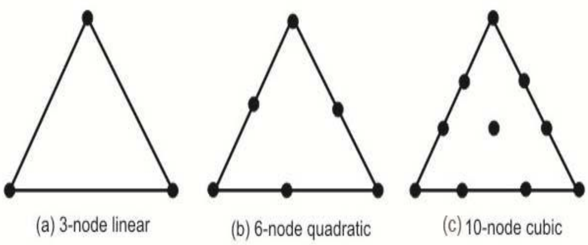

Fig. Triangular Elements

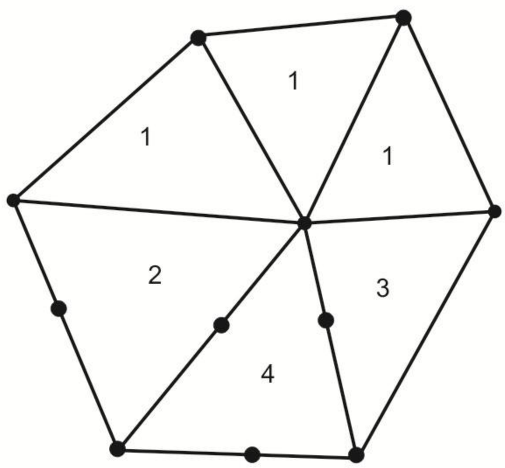

Fig. Group of Triangular elements

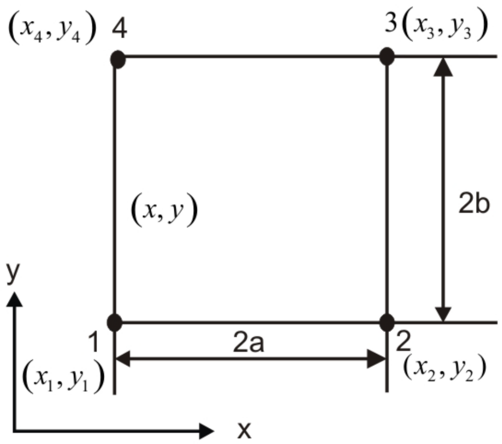

Fig. Rectangular elements in cartesian system

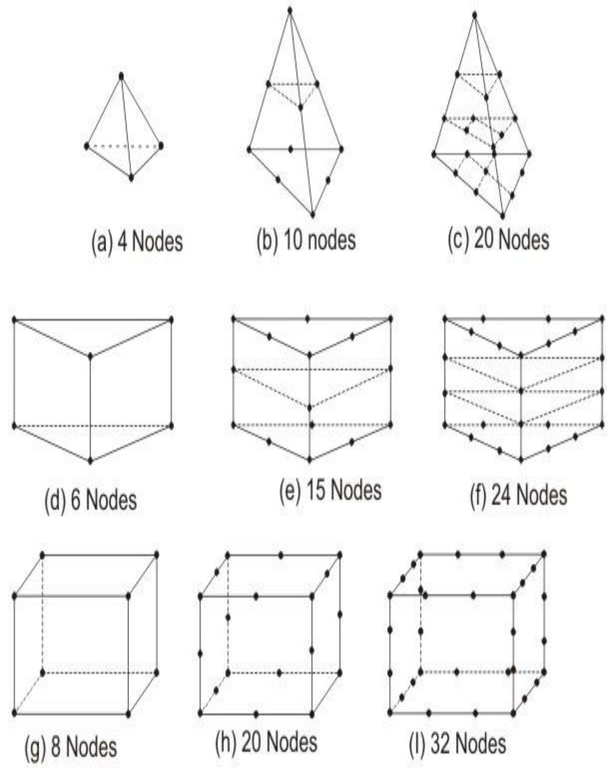

Fig. Three-dimensional solid elements

Equilibrium equations;

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial_z} + X = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0$$

$\tau_{xy} = \tau_{yx}$, $\tau_{yz} = \tau_{zy}$ and $\tau_{xz} = \tau_{zx}$

Strain;

$$\{\varepsilon\}^T = [\varepsilon_x \; \varepsilon_y \; \varepsilon_z \; \gamma_{xy} \; \gamma_{yz} \; \gamma_{yx}]$$

Strain Displacement;

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right]$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

$$\varepsilon_z = \frac{\partial w}{\partial z} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} + \frac{\partial u}{\partial y}\cdot\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y}\cdot\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\cdot\frac{\partial w}{\partial z}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial z} + \frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial z} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial z}$$

Strain-Displacement Relations;

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \gamma_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \qquad \gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

Constitutive Law;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

$$\{\sigma\} = [D]\{\varepsilon\},$$

Relationship between stress, strain, poisson ratio and modulus elasticity;

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \mu_{yx}\frac{\sigma_y}{E_y} - \mu_{zx}\frac{\sigma_z}{E_z}$$

$$\varepsilon_y = -\mu_{xy}\frac{\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \mu_{zy}\frac{\sigma_z}{E_z}$$

$$\varepsilon_z = -\mu_{xz}\frac{\sigma_x}{E_x} - \mu_{yz}\frac{\sigma_y}{E_y} + \frac{\sigma_z}{E_z}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G_{xy}},\ \gamma_{yz} = \frac{\tau_{yz}}{G_{yz}},\ \gamma_{zx} = \frac{\tau_{zx}}{G_{zx}}$$

Stress-Strain Relationships in 3-D Elements;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{Bmatrix} 1-\mu & \mu & \mu & 0 & 0 & 0 \\ & 1-\mu & \mu & 0 & 0 & 0 \\ & & 1-\mu & 0 & 0 & 0 \\ & & & \frac{1-2\mu}{2} & 0 & 0 \\ & & & & \frac{1-2\mu}{2} & 0 \\ & & & & & \frac{1-2\mu}{2} \end{Bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

The Final Constitutive Law ;

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

CONCLUSION

Finite Element Method is very useful for engineering disciplines including civil, mechanical, aerospace and structural engineering. It plays an important role to dividing entire domain into small domains. It is analytical method. Which gives an approximate solutions. Which will be near to original ones by using partial differential equations and stress-strain relationships. We can determine shape functions and stiffness functions accurately.

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