International Journal for Modern Trends in Science and Technology, 6(8): 173-180, 2020

ISSN: 2455-3778 online
DOI: https://doi.org/10.46501/IJMTST060830
Available online at: http://www.ijmtst.com/vol6issue08.html

Study of Probability Applications in Science and Engineering

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To Cite this Article
Dr.Daruri Venugopal, "Study of Probability Applications in Science and Engineering", *International Journal for Modern Trends in Science and Technology*, 6(8): 173-180, 2020.

Article Info
Received on 13-July-2020, Revised on 03-August-2020, Accepted on 05-August-2020, Published on 13-August-2020.

ABSTRACT

The terminology 'possibility 'and'probability are most familiar to everyone. After going to this detailed manuscript we can understood that different cases of sample space, algebra of events, Probability of an event. We can easily understood about 'Odds in favor' and odds against an event. Applications of addition theorem, approaching methods of conditional probability concept in various sectors. It provides the clear objectives of probability mass function and probability density function. It also provides the basic knowledge of mean and variance, Joint and marginal probability mass function. Provides the extension of probability applications using two dimensional distribution function and stochastic independence.

KEYWORDS: *Algebra of events, addition theorem, density function, marginal probability, stochastic, two Dimensional.*

I. INTRODUCTION

The terminology Probability and Chance are quite familiar to everyone, many a times, we come across statements like Probably it may rain today, Chances of his visit to the university are very few, It is possible that'A' may pass the Cavils examination with good marks. In these statements the words probably, chance possible etc., convey the sense of uncertainty about the occurrence of some event. In general it may appear that there cannot be any perfect measurement for these uncertainties, but in Mathematics we do have methods for calculating the degree of certainty of evens in numerical values, most of the conditions are satisfied. To understand the concept of Probability as a set function, we need some basic terminology of various terms, like random experiment, discrete and continuous sample space, simple event, possible and impossible event etc., The random phenomena, past information no matter how voluminous will not allow to formulate a rule to determine uniquely what will happen in future. The concept and ideology of probability is the study of such random phenomena which are not deterministic. In analyzing and interpreting statistical data that involves an element of chance probability theory plays a vital role in the theory and application of statistics. The development of probability theory which finds application in engineering, biology, economics, computer science, politics, traffic control, medicine, meteorology, agriculture, psychology, geography and management of natural resources.

II. ELEMENTARY PROBABILITY

- The total number of all possible outcomes in any trial termed as Exhaustive Events.
- Counting Techniques – Factorial : The product of all the whole numbers from the given values to 1. Noted as n!

- ❖ Permutation – An arrangement of objects in a specific sequence. P(n,r)
- ❖ Circular Arrangement – If the objects are arranged on a circle like no starting position.
- ❖ Combinations – The arrangement of objects without regard to order: C(n,r)
- ❖ Favorable Events – The events which entail the happening of an event.
- ❖ Mutually Exclusive Events – If the happening of one of the events prevents the happening of all other events in the same trail.
- ❖ Equally Likely Events – If there is no reasonto expect any one in preference to any other.
- ❖ Independent and Dependent – Two or more events in a trial are said to be independent if the happening or non happening of any one does not depend or affect the happening or non-happening of any other otherwise they are said to be dependent.
- ❖ Simple Event – An event which cannot be further split up into events.
- ❖ Compound Event – An event which can be further be split up into simple events.
- ❖ Sample Space – The set of all simple events in a random experiment.
- ❖ Complementary Events – If the intersection of the events is Null set.

III. PROBABILITY SET FUNCTION

In real life situations when we perform experiments in science and engineering, repeatedly under very nearly identical conditions, we get almost the unique results.We call such type of experiments as deterministic experiments.

There are other experiments in which the outcomes may not be essentially unique, even if the experiment is performed under very closely identical conditions. We know such type of experiments as random experiments. A random experiment is also known as a non-deterministic experiment.

Sample space of a random experiment is prescribed as the set of all possible results of the experiment. The total favorable outcomes are known as sample points. The sample space generally denoted by the letter S (in capital form). The number of sample points in the sample space S is denoted by n(S).

A sample space is called discrete. If it contains only finitely many points and which can be arranged into a simple sequence $k_1, k_2, k_3, \ldots\ldots$

Examples :

- Experiment Name – Throwing of a fair die
  Sample Space : S = { 1,2,3,4,5,6}
- Experiment Name - A family of two children
  S = { BB.GB.BG.GG }
- Experiment Name - Tossing of an unbiased coin
  S = { H,T }

Problem-1:

A team contains 2 white and 3 red balls. What is the sample space if the random experiment consist of drawing two balls from the Team. ?

Sol : - No. of elements in the sample space
C(5,2)= 10

S = $\{R_1R_2, R_1R_3, R_1W_1, R_1W_2, R_2R_3, R_2W_1, R_2W_2, R_3W_1, R_3W_2, W_1W_2\}$

Problem-2:

A fair coin is tossed twice. If 'X'denotes the happening as " tossed result of coin heads is odd" and 'Y'denotes the happening "number of tails isminimum once". Then the favorable event X ∩ Y ?

S = { TT,TH,HT,HH }

X= { HT,TH}, Y = { HT,TT,TH }

X ∩ Y = { HT, TH }

IV. PROBABILITY OF AN EVENT

If in a random experiment, there are 'n' exhaustive, equally likely outcome and A be an event and there are 'm' outcomes favorable to the happening of it.

Then the probability Properties are as below :

- P(A) = [Total favorable events / Total Events]
  = m / n
- In Probability if A,B, are two events then P(A) + P(B) = 1

  Where $0 \le P(A) \le 1$ and

  $P(A^c) = 1 - P(A)$
- If Φ is the null set, then P(Φ) = 0.
- LetP, Q are any two events then :

  P(P) – P(P∩Q)= P(PQ) = and

  Considering that P(P) ≤ P(Q)
- If P,Q are any two events in a experiment

then

$P(P) + P(Q) - P(P \cap Q) = P(P \cup Q)$

- Let in a Experiment events are;P,Q and Rsuch that $P(P \cup Q \cup R)$ denotes the expansion

$= P(P)+P(Q)+P(R) - P(P \cap R)- P(Q \cap R) - P(P \cap Q) + P(P \cap Q \cap R)$

The probability of occurrence of the event A and P(A) must be considered as a function defined on all events.

Problem-3:

Three bikes K,L,M are in a race; K is twice as likely to win as L and L is twice as likely to win as M. What are their individual probabilities of winning ?

Find the probability that L or M wins the race.

Sol:- Let given relation $p + 2p + 4p = 1$

$\therefore$ $P = 1/7$.

$\rightarrow$ $4p = 4/7$; $2p = 2/7$;

Probability of Winning the race by L or M :

$P(L \cup M) = = P(L) + P(M) = 3/7$

Problem-4:

Let 2 items are picket at random from a bag containing 12 items of which 4 are defective. Let 'A' = Both items are defective.

'B' = Both items are non-defective.

'C' =At least one item is defective.

Find P(A) , P(B) and P(C).

Sol:- Total Events : C(12,2) = 66 ways.

A can Occur : C(4,2) = 6 ways.

$P(A) = 6 / 66$

B can Occur : C(8,2) = 28 ways.

$P(B) = 28 / 66$

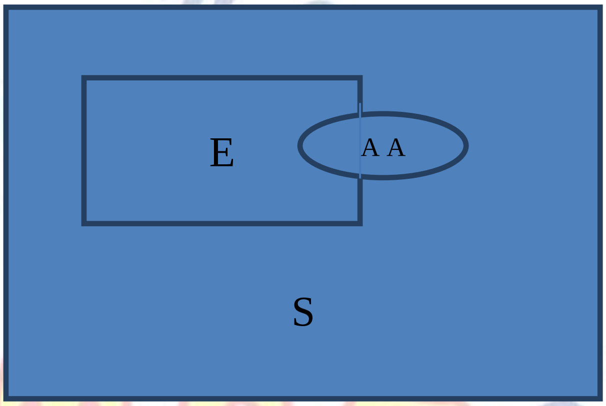
$P(C) = P(B^c)$ [Since by the given statements]

$P(C) = 1- P(B) = 1 - (28/66) = 19/33$.

V. CONDITIONAL PROBABILITY INDEPENDENCE

In a Sample space S assume that E be an arbitrary event in S such that Prob(E) > 0
The probability that an event A occurs once E has occurred then the conditional form is

Written $P(A/E) = [\ P(A \cap E) \ / \ P(E)\ ]$



In corresponding Venn diagram P(A/E) defines the conditional probability such that

$P(A \cap E) = |A \cap B| / |S|$

$$P(A/E) = \frac{P(A \cap E)}{P(E)}$$

Problem-5:

Let a pair of fair dice be tossed. If the total is 6, find the probability that one of the dice is such that to appear '2' on it. Find P(A/E) ?[Considering that E = Sum 6, and A is 2 appears on at least one die.]

Sol:- E = Sum is 6.
A = { Value 2reflects on at least one die.}

P(E) = { (1,5),(5,1),(2,4),(4,2),(3,3) }

$(A \cap E)$ = {(2,4),(4,2)} ; P(A/E) = 2/5

VI. CONDITIONAL PROBABILITY MULTIPLICATION

Considering the Conditional probability and cross multiply the equation using the condition $(A \cap E) = (E \cap A)$

Then, $P(E \cap A) = P(E).P(A/E)$

Problem-6:

A basket contains 12 items of which 4 are defective.
There items are drawn at random from the basket one after the other. Find p such that all the three are non defective items.

Sol:- The probability that the first item is nondefective is 8/12.
If the first item is nondefective, then the probability that the next item is nondefective is 7/11. If the first two items are nondefective, then the probability that the last item is nondefective is 6/10.
P(All the 3 are defective items in the problem)

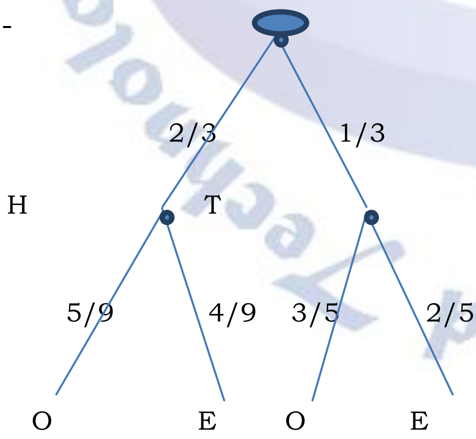
$= (8/12).(7/11).(6/10)=14/55$

VII. STOCHASTIC PROCESSES – TREE DIAGRAMS

A sequence of experiments in which each experiment has a limited/fixed number of outcomes with given probabilities is called a finite stochastic process. A convenient way of describing such a process and computing the probability of any event is by a tree diagram as illustrated below example the multiplication Theorem is applicable.

Problem-7:
A coin weighted so that probability of Head(H) is = 2/3 and Probability of Tails(T) is = 1/3 is tossed. If Heads appears then a number is picked at random from the numbers from 1to 9. if tails (T) appears, then a number is picked at random from the numbers 1 to 5. Find the probability 'such that the selected number is Even.

Sol:-



The probability of selecting an even number from the numbers 1 through 9 is 4/9 since there are 4 even numbers out of the 9 numbers, whereas the probability of selecting an even number fro the numbers 1 through 5 in 2/5 since there are 2 even numbers out of the 5 numbers. Hence lead to an even number :

$P = P(E) = [(2/3).(4/9) + (1/3).2(5)] = 58/135$

VIII. PARATITITONS AND BAYES' CONDITION

The Evens A1,A2,A3,...An form a partition of samples space S; that is the events Ai are mutually exclusive and their union is S.

Considering B as another event such that
$B = S \cap B = (A_1 U A_2 U A_3 U \ldots U A_n) \cap B$

$P(B) = P (A_1 \cap B) \cup P(A_2 \cap B) \cup \ldots \cup P(A_n \cap B).$

Problem-8:

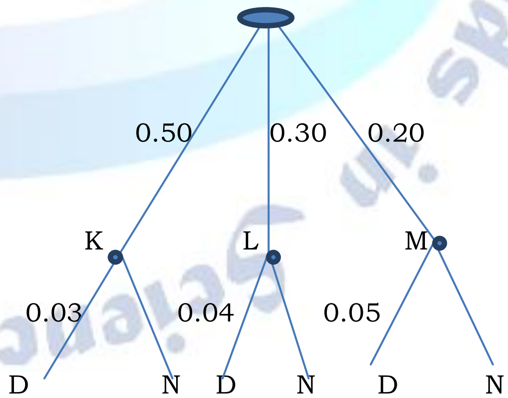
Three machines K,L,M produce respectively 50%, 30% and 20% of the total number of items of a factory. The percentages of defective result of these machines are (3/100),(4/100) and (5/100). If an item is selected at random, find the probability that the items is defective.

Sol :- Let X be the event that an item is defective .

$P(X) = P(K)P(X/K) + P(L)P(X/L) + P(M)P(X/M)$

$= (0.50)(0.03) + (0.30)(0.04) + (0.20)(0.05)=0.037$

In Tree Representation form it can be expressed as below :



IX. INDEPENDENCE APPLICATION

An event in probability test 'B' is said to be independent of an event A in its probability experiment and also if the probability that B occurs is not influenced by condition such that whether A has or has not occurred.

In Multiplication Theorem P(A∩B) = P(A)P(B/A)

∴ P(A∩B) = P(A).P(B)

Problem-9:

Three vehicles a,b,c, in a bike race together their respective probabilities of winning are 1/2/1/3/1/6. If the bikes race twice, then the sample space of the 2 repeated trails is T = {aa,ab,ac,ba,bb,bc,ca,cb,cc}. The probability of each point notation is :

p(aa) =p(a)p(a) =(1/2)(1/2) = 1/4

p(ab) =p(a)p(b) =(1/2)(1/3) = 1/6

p(ac) =p(a)p(c) =(1/2)(1/6) = 1/12

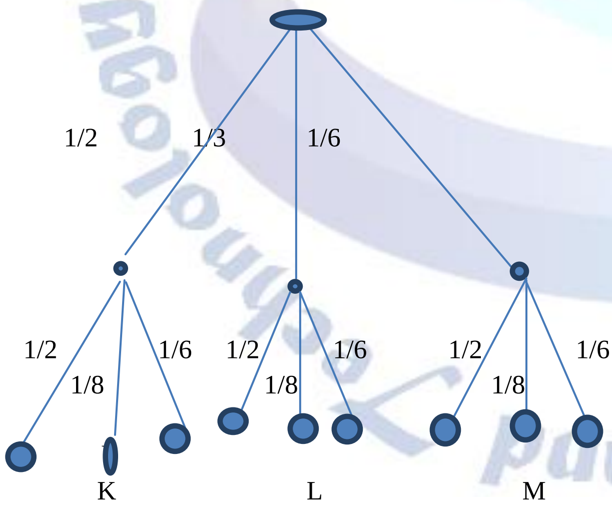
p(ba) =p(b)p(a) = 1/6 ; p(bb) =p(b)p(b) = 1/9 ;

p(bc) =p(b)p(c) = 1/18 ;

p(ca) = p(c)p(a) = 1/12 ;

p(cb) =p(c)p(b) = 1/18;

p(cc) =p(c)p(c) = 1/36;



A repeated trials process is a stochastic process whose tree diagram has the above properties.

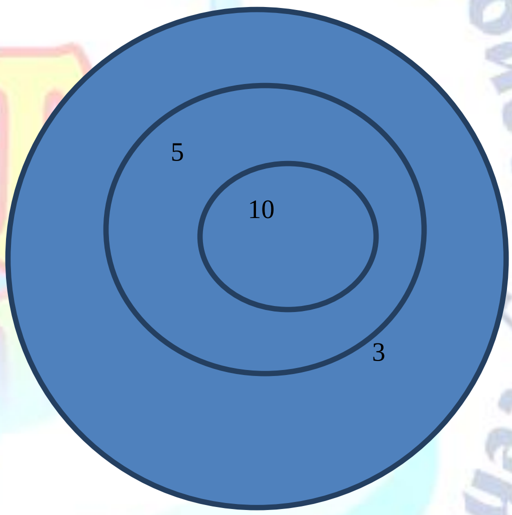
Every branch point has the outcomes a, b, c, and each branch leading to outcome a has probability ½, each branch leading to b has probability 1/3, and each leading to c has probability 1/6.

Problem-10:

In a Statistical Survey it is observed that Concentric circles with radius 1cm and 3 cm are drawn on a circular target of radius 5 cm.One PlayerMr.Dheeraj Akhil receives 3,5 or 10 points if he hits the target inside the smaller circle, inside the middle annular region or inside the outer annular region respectively. Suppose he hits the target with probability 1/2 and then is just as likely to hit one point of the target as the other.

Find the expected number(E) of points that Mr.Dheeraj Akhil Scores in his each fire.

Sol:- The probability of Scoring 10,5,3 or 0 points follows:



F(10) = (1/2) [Area of 10 Points / Area of Target)]

(1/2) [Λ $(1)^2$ / Λ $(5)^2$] = 1/ 50

F(5) = (1/2) [Area of 5 Points / Area of Target)]

(1/2) [{ Λ $(3)^2$ - Λ $(1)^2$} / Λ $(5)^2$] = 8/ 50

F(3) = (1/2) [Area of 3 Points / Area of Target)]

(1/2) [{ Λ $(5)^2$ - Λ $(3)^2$ } / Λ $(5)^2$] = 16/ 50

F(0) = 1/2

Thus E = 10(1/50) + 5(8/50) +3((16/50) + 0(1/2)

= 98 / 50 =1.96

Problem-11:

In a University 4 percent of the men and 1percent of the women are taller than 1.8m. In total strength 60 percent of the students are women. Now if a student is selected such that she is taller than 1.8m. Find the probability that the Selected student must be a woman?

Sol:- A = { Sudeten's taller than 1.8m}

We consider P(W/A) as the probability that a student is a women given that the student is taller than 1.8m.

By Bayes principal P(W/A) need to be apply

$$P(W/A) = \frac{P(W)P(X/A)}{P(A)P(X/A)+ P(B)P(X/B)+}$$

$$P(W/A) = \frac{(0.60)(0.01)}{(0.60)(0.01)+ (0.40)(0.04)}$$

= 3/11

X. DISTRIBUTION FUNCTION

- Let S be the sample space of given random experiment. A random variable is nothing but a real valued function 'x' defined on the sample space S.
- A random variables is called a discrete random variable if it can take only finitely many values.
- If a random variable take any value between certain limits then the variable is known as continues random variable.
- Let X be a random variable, F(x)=P(X≤x) is called the distribution function of the random variable X. A distribution function is also known as cumulative distribution function.
- The range of of F(x) is (0,1).

Problem-12:
Find the probability distribution of the random variable " number of heads" a) when two coins are tossed
b)one coin is tossed twice

Let S be the sample space { HH,HT,TT,TH }

Favorable outcomes of x are : {0, 1, 2}

P(X=0) = P(TT) = 1/4

P(X=1) = P(HT,TH) = 2/4 = 1/2

P(X=2) = P(HH) = 1/4

x	0	1	2
P(x)	1/4	1/2	1/4

b) Let H be the event of getting a head.

For Number of heads in two tosses possible values : 0,1,2.

$P(x=0) = P(H_1' H_2') = (1/2)(1/2) = 1/4$

$P(x=1) = P(H_1 H_2') + P(H_1' H_2)$

$= (1/2)(1/2)(1/2)(1/2)$

$= 1/2$

$P(x = 2) = P(H_1 H_2) = (1/2)(1/2) = 1/4.$

Distribution table for the data :

x	0	1	2
P(x)	1/4	1/2	1/4

XI. MEAN AND VARIANCE

Let x be a discrete random variable assuming values $x_1, x_2, x_3 \ldots x_n$ with respective to probabilities $p_1, p_2, p_3, \ldots$ Pn with $p_1+p_2+..p_n=1$.

Mean $= \sum_{i=1}^{n} pixi$

Variance $= \sum_{i=1}^{n} pi(xi - □)2$

S.D. = √ ∑px²- □2

Problem 13:

From Well shuffled pack of 52 cards two cards are selected randomly. Find the mean and standard deviationfor the number of ace cards..

Sol:- Let A be the event of getting an ACE card.

P(A) = 4/52 = 1/13; P(A') – 1- (1/13) = 12/13

Let 'x' be no. of aces then possibilities: x=0,1,2.

P(x=0) = P(non ace cards) = (12/13).(12/13)= 144/169

P(x=1) = P(One ace card) = $P(A_1)P(A'_2) + P(A'_1)P(A_2)$

= [(1/13)X(12/13) + (12/13)X(1/13)]

= 24/ 169

x	p	px	px²
0	144/169	0	0
1	24/169	24/169	24/169
2	1/169	2/169	4/169
	∑ P=1	∑ Px = (2/13)	∑px2 = (28/169)

Mean = ∑ px = 2/13

Standard Deviation = $\sqrt{\sum px^2 - ▯2}$

$\sqrt{(28/169) - (2/13)^2}$ = (2√6) / 13

Problem 14:

If a book is drawn from a shelf containing 10 books numbered 1 to 10 inclusive, Compute theprobabilitysuch that the numberX drawn is

(a) Less than 4

(b) Evzn number

(c) Prime Number

(d) Find the probability mean and variance of the random variable X.

Sol:- (a) Each ticket has the same probability for being drawn, the probability distribution is discrete uniform distribution given by f(x) = 1/10 for x=1,2,3,…10.

$P(x < 4) = \sum_{X=0}^{3} P(x) = \sum (1/10) = (1/10)\,[1+1+1] = 3/10$

(b) 2,4,6,8,10 are the favorable with probability 10.

∴ The probability of even number = 5/10 = 1/2

(c) favorable events : 2,3,5,7

∴ The probability of Prime Numbers = 4/10=2/5

(d) *Mean : E(x) =* $= \sum_{i=1}^{10} pixi = \sum_{i=1}^{10} x.(\frac{1}{10})$

(1/10) [1+2+3+….10) = 5.5

Variance = $\sum_{i=1}^{10}((x -$ ▯2 $)p(x)$

(1/10) [$(1-5.5)^2+(2-5.5)^2+\ldots(10-5.5)^2$] = 8.25

Variance = $E(x^2) - [E(x)]^2 = \sum x^2.p(x) - (5.5)^2$

(1/10) { [n(n+1)(2n+1)] /6 }

= 38.5 – 30.5 = 8.25

XII. Conclusion

As we observe that in a sequence of random experiments, it is not possible to predict individual results. These individual results are subjected to irregular random fluctuations which cannot be used for exact calculations. If we proceed from individual experiments to the whole sequence of experiments, We observe an important phenomenon. The random experiment is conducted a large number of items under identical conditions, it will be seen that the relative frequency of an event stabilizes to a certain value. This phenomenon we observe often in our real life and is termed as statistical regularity of chance phenomena.

Suppose A is an out come of a random experiment, A occurs 'f' times when the experiment is repeated ',' times under identical conditions. The ratio (f/n) is called relative frequency of A in 'n' trials. Most of the situation in nature are having some sort of relationship either directly or indirectly, resulting a sort of dependence. In order to understand this we discussed about the concept and computation stochastic independence and mutual independence. Finally to formulate a probability model to a random experiment in this process we applied the discrete and continuous and its probability distribution.

ACKNOWLEDGMENT

I wish to show my gratitude to my Academic Guru Dr.N.Goutham Rao Garu and Dr.G.Manohar Reddy Garu Management of Swamy Vivekananda Educational Trust, Hyderabad for their motivational thoughts and Support in Academics & Research Activities.

I am very much thankful to the Vice-Chancellor, Prof.Dr.Dalel Singh,Dean School of Science, and The Registrar of OPJS University,Churu,Rajasthan for their contribution in Publication work. I am extremely grateful to Prof.V.Kannan Former Pro-Vice-Chancellor of University of Hyderabad, Who unconditionally gave me his valuable time and guidance in Research Activities.

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