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Properties of E-Inversive Semigroups

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ABSTRACT

A semigroup S is called an E-inversive if for every a∈S there exists x in S Such that ax∈E(s), where E(s) is the set of all idempotents of S, introduced by G.Thierrin. In this paper we contribute some theorems of E-inversive semigroups with some properties. we discussed the left , right and regular identities On E-inversive semigroups.

Prelimanaries:

1.1. Definition: A semigroup (S, .) is called left(right) regular if it satisfies the identity $xa^2 = a$ ($a^2x = a$) for all a,x in S.

1.2. Definition: A semigroup (S, .) is called regular if it satisfies the identity a = axa for all a,x in S.

1**.3. Theorem:** A semilattice S is an E-inversive semigroup if and only if it satisfy the identity aba = ab (aba = ba) for any a,b∈S.

Proof: Let S be Semilattice, assume that S satisfies the identity aba = ab for all a,b∈S.

aba = ab ⇒ a(ba) = ab

a(bab) = ab

(ab)(ab) = ab

$(ab)^2 = ab$

i.e., for every a∈S there exists an element b∈S such that $(ab)^2 = ab$ ⇒ 'a' is an E-inversive.

Hence S is an E-inversive semigroup

Conversely, let S be an E-inversive semigroup.

Let a∈S, then there exists b∈S such that

$ab = (ab)^2$

ab = (ab)(ab)

= (ab)(ba) (S is commutative band)

= a(bb)a

$= a(b)^2a$

= aba

ab = aba

∴S satisfies the identity aba = ab. for all a,b∈S.

1.4. Theorem: A Semilattice S is an E-inversive semigroup if and only if it satisfies the identity ab = a (ba = a) for any a,b∈S.

Proof: Let S be a semilattice. Assume that S satisfies the identity ab = a (ba = a) for any a,b∈S.

Let ab = a ⇒ (a)b = (a)

abb = ab (since ab = a)

a(b)b = ab

abab = ab (b = ba)

(ab)(ab) = ab

$(ab)^2 = ab$

That is, for every $a \in S$ there exists an element $b \in S$ such that $(ab)^2 = ab \Rightarrow a$ is an E-inversive. Every element of S is an E-inversive

Hence S is an E-inversive semigroup.
Conversely, let S be a semilattice.
Assume that S is an E-inversive semigroup then

$ab = (ab)^2$ for any $a,b \in S$.
$= (ab)(ab)$
$= (ab)(ba)$ (S is commutative)
$= a(bb)a$
$= ab^2a$ (S is band)
$= aba$
$= a$
$ab = a$

Hence S satisfies the identity $ab = a$ for any $a,b \in S$. Similarly we prove the same with the identity $ab = b$ for all $a,b \in S$.

1.5. Theorem: A commutative total semigroup S is left (right) regular if and only if S is an E-inversive semigroup
Proof: Let S be a commutative total semigroup

Assume that S is left regular then

$aba = ab$
$a(ba) = ab$
$a(bab) = ab$ (left regular)
$(ab)(ab) = ab$
$(ab)^2 = ab$

That is, for every $a \in S$ there exists an element $b \in S$ such that $(ab)^2 = ab \Rightarrow a$ is an

E-inversive.

Hence S is an E-inversive semigroup.
Conversely, let S be an E-inversive semigroup.
Since S is total every element of S can be written as product of two elements in S

i.e, $S^2 = S$.

Let $x \in S$ then $x = ab$ for some $a,b \in S$.

Since S is an E-inversive semigroup and $x \in S$ implies by lemma 3.1.8 there exists

$c \in S$ such that $xcx = x$

$(ab)c(ab) = ab$
$(ab)c(ba) = ab$
$a(bcb)a = ab$
$a(b)a = ab$
$(bcb = b)$

$aba = ab$

Hence S is left regular.

1.6. Note: For any total semigroup S which is commutative is an E-inversive semigroup if and only if S is left(right) singular.

1.7. Theorem: An E-inversive semigroup S is regular if and only if it is left(right) seni-normal.

Proof: Let S be an E-inversive semigroup.
Assume that S be a regular semigroup then $abca = abaca$ for any $a,b,c \in S$
$= (aba)ca$ $(aba = a)$
$= aca$
$= a(c\,)a$
$= acbca$ $(c = cbc)$
$abca = acbca$.

$\therefore$ S is left semi-normal.
Conversely, let S be left semi-normal then
$abca = acbca$
$= a(cbc)a$
$= aca$ $(cbc = c)$
$= (a)ca$
, $= abaca$ $(a = aba)$
$abca = abaca$
Hence S is regular.

1.8. Theorem: Let S be an E-inversive regular semigroup then S is left(right) semi-regular

Proof: et S be an E-inversive regular semigroup, then
$abca = abaca$ for any $a,b,c \in S$
$abca = aba(ca)$ (S is regular)
$= aba(ca)^2$ (S is an E-inversive)
$= abacaca$
$= abac(a)ca$
$= abacabaca$ $(a = aba)$
$= abac(abaca)$
$= abacabca$ (S is regular)
$abca = abacabca$
$\therefore$ S is left semi-regular

1.9. Lemma: An E-inversive semigroup S is left(right) semi-normal if and only if it is left(right) quasi-normal.

Proof: let S be an E-inversive semigroup S.

Assume that S is left semi-normal then

abca = acbca for any $a,b,c \in S$

abcac = acbcac

ab(cac) = acb(cac)

abc = acbc (by lemma 3.1.8)

Therefore S is left quasi-normal.

Conversely, let S be left quasi-normal then

abc = acbc

abca = acbca.

$\therefore$S is left semi-normal.

1.10. Theorem: Let S be an E-inversive semigroup and E(S) is set of all idempotent elements of S then E(S) is unitary.

Proof: Let S be an E-inversive semigroup and E(S) is set of all idempotent elements of S.

Now we show that E(S) is unitary

Let $a \in E(S)$. Since S is an E-inversive semigroup there exists an element $b \in S$ such that $(ab)^2 = ab \Rightarrow ab \in E(S)$.

Now we show that, $b \in E(S)$

$(a.b)^2 = a.b$

put a = b then

$(b.b)^2 = b.b$

$(b.b)^2 = b^2$

b.b = b

$b^2 = b$

$\therefore\ b \in E(S)$

Hence E(S) is left unitary.

Similarly we can prove that, E(S) is right unitary.

Therefore E(S) is unitary.

2. E-inversive Semigroups with Identity $ax^2 = a$

2.1. Theorem: Left regular semigroup is regular semigroup

Proof: Let (S .) be a left regular semigroup then

$a = xa^2$ for any a, x in S

$a = a\,x^2\,a^2$

$\therefore$S is left regular

$a = axxa^2$

a = axa

$\therefore$S is left regular

Therefore (S .) is regular semigroup

2.2. Theorem: Right regular semigroup is regular semigroup

Proof: Let (S .) be a right regular semigroup then

$a = a^2x$ for any a, x in S

$a = a^2x$

$a = a^2\,x^2\,a$

$a = a^2x\,xa$

a = axa

$\therefore$S is right regular

Therefore (S .) is regular semigroup.

2.3. Theorem: Regular semigroup is both left and right regular semigroup

Proof: Let (S .) be a regular semigroup

$a^2x = a^2x$ for all a,x in S

= aax

$= a^2x^2a$

$= (ax)^2a$

= axa

$\therefore$S is an E-inversive

$a^2x = a$

Therefore (S .) is right regular semigroup

$xa^2 = xa^2$

$= ax^2a^2$

$= a(xa)^2$

= a(xa)

= axa

$\therefore$S is an E-inversive =

$xa^2 = a$

Therefore (S .) is left regular semigroup

2.4. Theorem: Left (Right) regular semigroup is completely regular semigroup

Proof: Let (S .) be a left regular semigroup then from the above theorem (S .) is both

Right regular semigroup and regular semigroup

Now we have to prove that ax = xa

$a = xa^2$

$ax = xa^2x$

$= xa^2ax^2$

$\therefore$ S is right regular

$= xaaax^2$

$= xaa^2xx$

$\therefore$ S is right regular

$= xaax$

$= xa^2x$

$= xa$

$ax = xa$

Therefore (S .) is completely regular semigroup

2.5. Theorem: A regular semigroup is an E-inversive semigroup

Proof: Let (S .)be a regular semigroup then

$a = axa$ for any a,x in S

$ax = axax$

$ax = (ax)^2$

(S .)is an E-inversive semigroup

2.6. Theorem: S is both left and right regular semigroup then S is an E-inversive semigroup

The proof of the theorem is clear from the above theorem.

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