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E-Inversive Semigroups with the identity abc = ac

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ABSTRACT

A semigroup S is called an E-inversive if for every a∈S there exists x in S Such that ax∈E(s), where E(s) is the set of all idempotents of S, introduced by G.Thierrin. The concept of sub direct product of two E-inversive semigroups introduced by H. Mitsch by using the concept of sub homomorphism of inverse semigroups introduced by Mc Alisterand N.R.Reilly. The semidirect of two E-inversive semigroups introduced by F.Catino and M.M.Miccoli. In this paper we study some special identities in an E-inversive semigroup and we present preliminaries and basic concepts of E-inversive semigroups.

Keywords: *E-inversivesemigroup, Idempotent and Regular*

Prelimanaries:

1.1. Definition: An element 'a' of a semigroup S is called an E-inversive if there is an element x in S such that $(ax)^2 = ax \Rightarrow ax \in E(S)$, where E(S) is set of all idempotent elements of S.

1.2. Definition: A semigroup S is called an E –inversive semigroup if every element of S is an E-inversive.

Examples:

1. Regular semigroups (a= axa implies that $ax \in E(S)$)
2. Eventually regular semigroups(a^n regular for some n >1 implies that $a^n x a^n = a^n$ and $a(a^{n-1}x) \in E(s)$ for some $x \in S$)

1.3. Definition: An E-inversive semigroup is said to be an E-dense if all elements of an E-inversive semigroup are commute.

1.4. Definition: A subset A is said to be right unitary if for any $a \in A$, $s \in S$ implies $(sa)^2 = sa \in A$, and $s \in A$

1.5. Definition : A subset A is said to be left unitary if for any $a \in A$, $s \in S$ implies that $(as)^2 = as \in A$, and $s \in A$.

1.6. Definition : A subset A of S is said to be unitary if it is both left and right unitary.

1.7. Remark: We have established the equivalence between the two identities aba = a and abc = ac, on idempotent semigroups thus either one of them define rectangularity.

1.8. Lemma : [9] An element a of a semigroup S is an E-inversive iff there exists

$y \in S$ such that y = yay

1.9.Lemma : Let S be an idempotent commutative semigroup then S is an E-inversive semigroup.

Proof: Let S be an idempotent commutative semigroup.

To prove that S is an E-inversive semigroup we have to prove that every element $a \in S$ is an E-inversive.

To show that a is an E-inversive implies there exist x in S such that $(ax)^2 = ax$.

Consider $(ax)^2 = ax.ax$

$= a(xa)x$

$= a(ax)x$

(since S is commutative)

$= (aa)(xx)$

$= (a^2)(x^2)$

$= ax$

(since S is an idempotent semigroup)

$(ax)^2 = ax$

∴ Every $a \in S$ is an E -inversive element.

Hence S is an E-inversive semigroup.

1.10.Theorem: A semigrouup S is an E-inversive semigroup iff it is an inverse semigroup

Proof: Let S be a semigroup.

Assume that S is an inverse semigroup then for any $a \in S$ there exists $a^1 \in S$ such that

$aa^1a = a$ and $a^1aa^1 = a^1$

To show that S is an E-inversive semigroup.

That is ; every element in S is an E-inversive

Let $a \in S$ then $(aa^1)^2 = aa^1\, aa^1$

$= (aa^1a)a^1$

$= aa^1$

(since S is inverse semigroup)

$(aa^1)^2 = aa^1 \Rightarrow$ a is an E-inversive.

∴ S is an E-inversive semigroup.

Conversely, let S be an E-inversive semigroup then

every element of S is an E-inversive

for $a \in S$ there exists an element $x \in S$ such that $(ax)^2 = ax$

Let $a \in S$ and $a^1 \in S$

Let $a = xa^1 x$ for any $x \in S$

Consider $aa^1a = (xa^1x)a^1(xa^1x)$

$= (xa^1)(xa^1)(xa^1)x$

$= (xa^1)(xa^1)^2x$

$= (xa^1)(xa^1)x$

$= (xa^1)^2x$

$= (xa^1)x$

$= xa^1x$

$aa^1a = a$ (since $xa^1x = a$)

Similarily we can prove that $a^1aa^1 = a^1$

Hence S is an inverse semigroup.

1.11. Theorem: An E-inversive semigroup is commutative band (Semilattice) iff it is both left and right regular.

Proof: Let S be an E-inversive semigroup.

Assume that S be a commutative band (Semilattice) .

Now we show that S is left regular and right regular

Let $a,b \in S \Rightarrow a^2 = a$ and $b^2 = b$

Now $ab = (ab)^2$

$ab = ab.ab$

$= ab(ab)$

$= ab(ba)$

$= a(bb)a$

$= ab^2a$

$= aba$

$ab = aba$

∴ S is left regular

$ab = (ab)^2$

$= (ab)(ab)$

$= (ba)(ab)$

$= b(aa)b$

$= ba^2b$

$= bab$

$ab = bab$

∴ S is right regular

Hence S is both left and right regular band.

Conversely, assume that S is both left and right regular then

Let $a,b \in S \Rightarrow ab = (ab)^2$

(Since S is an E-inversive semigroup)

$= ab.ab$

$= a(bab)$

$= a(ba)$

(S is left regular)

$= aba$

$ab = ba$

(S is right regular)

∴ S is Commutative.

Now we prove that S is a band

We have $a,b \in S \Rightarrow (ab)^2 = ab$

(Since S is an E-inversive)

put $b = a$

$(a.a)^2 = a.a$

$(a.a)^2 = a^2$

$a.a = a$ for any $a \in S$

∴ S is ba and.

Hence S is Semilattice

1.12. Note: In the above theorem S is an E-inversive semigroup and is commutative so S is an E-dense semigroup.

1.13. Theorem: Let S be a semigroup. Assume that S is left(right) singular the S is an E-inversive semigroup.

Proof: Let S be a semigroup with left singular property $ab = a$ for any $a,b \in S$

To prove that S is an E-inversive semigroup we have to prove that every element of S is an E-inversive.

Consider $(ab)^2 = ab.ab$
$= a(ba)b$
$= a.b.b$
$= a(bb)$
$= ab$
$(ab)^2 = ab$

That is for any a∈S there exists an element b∈S such that $(ab)^2 = ab \Rightarrow ab \in E(S)$

∴' a' is an E-inversive in S

Hence S is an E-inversive semigroup.

1.14. Lemma: An idempotent semigroup S with an identity abc = ac, for any a,b,c∈S is an E-inversive semigroup.

Proof: Let S be an idempotent semigroup with an identity abc = ac where a,b,c∈S.

$abc = ac \Rightarrow abcb = acb$
put c=a
$abab = a.ab$
$(ab)^2 = a^2b$
$(ab)^2 = ab$ (since $a^2 = a$)

∴ Every element of S is an E-inversive semigroup.

Hence S is an E-inversive semigroup.

1.15. Lemma: An E-inversive semigroup S with an identity abc = ac is normal.

Proof: Let S be an idempotent semigroup with an identity abc=ac for all a,b,c∈S.

Now we have to show that S is normal

Consider $abca = a(bc)a$
$= a(bc)^2a$
(S is an E-inversive)
$= abcbca$
$= abc(bca)$
$= abcba$ (bca = ba)
$= (abc)ba$
$= acba$ (abc = ac)
$abca = acba.$

∴S is normal.

1.16. Lemma: An E-inversive semigroup S with an identity abc = ac is regular.

Proof: Let S be an E-inversive semigroup with an identity abc = ac

We have to prove that S is regular.

Let a,b,c ∈S then $abca = ab(ca)$
$= ab(ca)^2$
(S is an E-inversive semigroup)
$= abcaca$
$= a(bca)ca$
$= a(ba)ca$ (bca = ba)
$abca = abaca$

∴ S is regular.

1.17. Lemma :An E-inversive semigroup S with an identity abc = ac is right(left) Semi regular.

Proof: Let S be an E-inversive semigroup with an identity abc = ac Now we show that S is right semi-regular.Let a,b,c∈S, then

$abca = a(bc)a$
$= abcbca$
(S is an E-inversive)
$= ab(cb)ca$
$= abcabca$ (cb = cab)
$= abca(bc)a$
$=abcabaca$ (bc = bac)
$abca = abcabaca$

Hence S is right semi-regular.

1.18. Note: Similarily we prove that an E-inversive semigroup S with an identity abc = ac for any a,b,c∈S is

1) left(right) quasi –normal
2) left(right) semi-normal
3) left(right) normal.

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