Available online at: http://www.ijmtst.com/vol5issue01.html
International Journal for Modern Trends in Science and Technology
ISSN: 2455-3778 :: Volume: 05, Issue No: 01, January 2019

Application to Markov Chain

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To Cite this Article
Dhanalakshmi M, V Jyothi, Ch Jeevanalakshmi, B Jyothi and K Ramalakshmi, "Application to Markov Chain", *International Journal for Modern Trends in Science and Technology*, Vol. 05, Issue 01, January 2019, pp.-18-19.

Article Info
Received on 13-Dec-2018, Revised on 20-Jan-2019, Accepted on 24-Jan-2019.

I. Introduction

Suppose there is a physical or mathematical system that has 'n' possible states and at any one time, the system is in one and only one of its 'n' states. As well, assume that at a given observation period. K^{th} period, the probability of the system being in a particular state depends only on its status at the K-Istperiod such system is called markov chain or markov process.

Main Result:

Suppose a bus rental agency has three locations in Vijayawada: Down town location (labeled A),East end location (labeled B) and a west end location (labeled C).The agency has a group of delivery drivers to serve all three locations.
The agency's statistician has determined the following:

1. Of the calls to the downtown location,30% are delivered in downtown area,30% are delivered in the east end and 40% are delivered in the west end.
2. Of the calls to the east end location,40% area delivered in downtown area,40% are delivered in the east end and 20% are delivered in the west end.
3. Of the calls to the west end location,50% are delivered in downtown area,30% are delivered in the east end, and 20% are delivered in the west end .

After making a delivery, a driver goes to the nearest location to make the next delivery. This way, the location of a specific driver is determined only by his or her previous location.

The Matrix is

$$T=\begin{bmatrix}0.3 & 0.4 & 0.5\\ 0.3 & 0.4 & 0.3\\ 0.4 & 0.2 & 0.2\end{bmatrix}$$

T is called the transition matrix.

In our example, a state is the location of a particular driver in the system at a particular time.

Let's assume that it takes each delivery person the same time to make a delivery and then to get to their next location.
According to the statistician's data, after 15 minutes of the drivers that began in A,30% will again be in A,30% will be in B,and 40% will be in C.Since all drivers are in one of those three locations after their delivery,each column sums to 1. Because we are dealing with probabilities, each entry must be between 0 and 1.Let us model this situation as a markov chain is that the next location for delivery depends only on the current location.

If you begin at location C, the probability P, that you will be in area B after 2 deliveries. you can get to B in two steps.

We can go from C to C, then from C to B, we can go from C to B, then from B to B, or we can go from C to A, then from A to B.

Let P(XY) denote the probability of going from X to Y in one delivery. If two (or more) independent events must both.To obtain the probability of them both happening , we multiply their probability together. To obtain the probability of either happening,we add the probability of those events together.

P=P(CA)P(AB)+P(CB)P(BB)+P(CC)P(CB) for the probability that a delivery person goes C to B in 2 deliveries.

P=(0.5)(0.3)+(0.3)(0.4)+(0.2)(0.3)

=0.33

This tells us that if we begin at location C, we have a 33% chance of begin in location B after 2 deliveries.

Let's try this for another pair, if we begain at location B, what is the probability of being at location B after 2 deliveries?

The probability of going from location B to location B in two deliveries.

P(BA)P(AB)+P(BB)P(BB)+P(BC)P(CB)

=(0.4)(0.3)+(0.4)(0.4)+(0.2)(0.3)

=0.34

Going from CtoB in deliveries is the same as taking the inner product of row 2 and column 3.

Going from B to B in deliveries is the same as taking the inner product of row 2 and column 2.

If you multiply T but ,the (2,3) and (2,2) entries are respectively

$$T^2=\begin{bmatrix}0.41 & 0.38 & 0.37\\ 0.33 & 0.34 & 0.33\\ 0.26 & 0.28 & 0.3\end{bmatrix}$$

You will notice that the elements an each column still add to 1 and each element is between 0 and 1.this matrix indicates the probabilities of going from location i to location j in exactly 2 deliveries.

This matrix, to find where we will be after 3 deliveries.

Let's find the probability of going from C to B in 3 deliveries.

P(CA)P(AB)+P(CB)P(BB)+P(CC)P(CB)

=(0.37)(0.3)+(0.33)(0.4)+(0.3)(0.3)

=0.333

You will see that this probability is the inner product of row 2 of T^2 and column 3 of T.

Therefore, if we multiply $T^2$by T.

Will get the probability matrix for 3 deliveries.

$$T^3=\begin{bmatrix}0.385 & 0.39 & 0.393\\ 0.333 & 0.334 & 0.333\\ 0.282 & 0.276 & 0.244\end{bmatrix}$$

We find the matrix of probabilities for 4,5 or more deliveries. the elements on each column still add to 1.

$$T^4=\begin{bmatrix}0.3897 & 0.3886 & 0.3881\\ 0.3333 & 0.3334 & 0.3333\\ 0.2770 & 0.2780 & 0.2786\end{bmatrix}$$

$$T^5=\begin{bmatrix}0.38873 & 0.38899 & 0.38905\\ 0.33333 & 0.33334 & 0.33333\\ 0.27794 & 0.27772 & 0.27762\end{bmatrix}$$

$$T^6=\begin{bmatrix}0.388921 & 0.388878 & 0.388857\\ 0.333333 & 0.333334 & 0.333333\\ 0.277746 & 0.277788 & 0.277810\end{bmatrix}$$

$$T^7=\begin{bmatrix}0.3888825 & 0.3888910 & 0.3888953\\ 0.3333333 & 0.3333334 & 0.3333333\\ 0.2777842 & 0.2777765 & 0.2777714\end{bmatrix}$$

The numbers in each row seems to be converging to a particular number.

At the end of the week, we have a 38.9% chance of being at location A, a 33.3% chance of being at location B, and a 27.8% chance of being in location C.

Remark:

If all the entries of the transition matrix are between 0 and 1 exclusive the convergence is generated take place the convergence may take place when 0 and 1 are in the transition matrix, but convergence is no longer generated.

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