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Coupled Oscillators and Eigen Values

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ABSTRACT

In this paper, we study the oscillations of a single body subject to a Hooke's law force. We examine the oscillations of several bodies, each of which is connected by springs to one or more of the others. We will study coupled oscillations of a linear chain of identical non-interacting bodies connected to each other and to fixed end points by identical springs.

I. Introduction

A system is a molecule, which we can model as a system of masses connected by springs. If each of the masses were attached to a separate, fixed spring, with no connections between the masses, then each would oscillate independently. However, the forces that connect the masses to each other turn this into a coupled system. We shall find that a single oscillators exhibits a single natural oscillation frequency, two or more oscillators have a corresponding number of natural frequencies, and that general motion is a combination of vibrations at all the different normal frequencies.

Newton's second law of motion:
Force is equal to the change in momentum(mV) per charge in time. For a constant mass, force equals mass time acceleration.
F=ma

II. Hooke's law

Hooke's law stating that the strain in a solid is proportional to the applied stress with in the elastic limit of the solid.
F=-kx

Where *k*= Spring constant, *x*= Distance

In case of two masses and three springs
Consider the two identical bodies shown in the figure joined up with identical springs without friction on a horizontal track as follows.

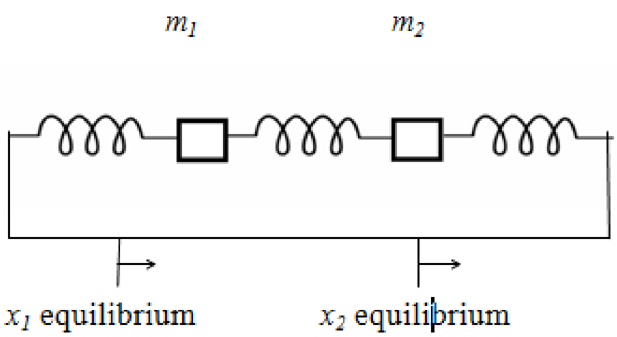

Figure 1

Two masses attached by springs to fixed walls and to each other. Here x_1 and x_2 represent the equilibrium position of the two masses. Let $x_1(t)$ and $x_2(t)$ be the distances from the equilibrium position of the two masses at time *t* and let *k* be the spring constant. Spring *1* is stretches by an amount x_1, so it exists a force *kx*, Spring *2* is more complicated since it is affected by the position of both masses it exists a force $k(x_2-x_1)$ due to the centre spring.

The net force acting on the first mass is

$$F_1 = -kx_1 + k(x_2 - x_1)$$
$$= -2kx_1 + kx_2$$

Similarly, the net force acting on the second mass is

$$F_2 = kx_1 - 2kx_2$$

Now applying the Newton's second law of motion given the following system of differential equation.

$$m\ddot{x}_1 = -2kx_1 + kx_2$$
$$m\ddot{x}_2 = kx_1 - 2kx_2$$

These above equations can be written in the matrix as becomes

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = -k/m \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (1)$$

Now find the Eigen value the matrix

Here $A=\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

The Eigen values of A are those values of λ satisfying the equation, $\det(A-\lambda I) = 0$,

Where I is the identity matrix of the same size of A.

Now $\det(A-\lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)^2 - 1 = 0$

$\Rightarrow \lambda = 1, 3$

Let us now find the corresponding Eigen vectors if $x = \begin{bmatrix} a \\ b \end{bmatrix}$ is an Eigen vectors of A corresponding to the Eigen value 1, then one would have

$$AX=X \text{ (or) } (A-I)X=0$$

$$\begin{bmatrix} 1 & -1 & . & . & 0 \\ -1 & 1 & . & . & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & . & . & 0 \\ 0 & 0 & . & . & 0 \end{bmatrix}$$

Which means that $a = b$ so $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is a basis for the Eigen space corresponding to 1

Similarly, one can show that the vectors $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is a basis for the Eigen space corresponding to 3.

Now let $P = 1/\sqrt{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Then $P^{-1}AP = D$, where $D=\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ is the diagonal form of A.

Here $A=PDP^{-1}$, so equation (1) becomes

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = -k/m \; 1/\sqrt{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \sqrt{2}/2 \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (2)$$

Now, let us consider the following change of variables

$$y_1 = x_1 + x_2/\sqrt{2}, \quad y_2 = -x_1 + x_2/\sqrt{2}$$

So, $x_1 = y_1 - y_2/\sqrt{2}$, $x_2 = y_1 + y_2/\sqrt{2}$

Taking the second derivative given

$$\ddot{x}_1 = \ddot{y}_1 - \ddot{y}_2/\sqrt{2}, \quad \ddot{x}_2 = \ddot{y}_1 + \ddot{y}_2/\sqrt{2}$$

So the equation (2) becomes

$$\begin{bmatrix} \ddot{y}_1 & -\ddot{y}_2 \\ \ddot{y}_1 & +\ddot{y}_2 \end{bmatrix} = -\omega_0^2 \begin{bmatrix} 1 & -3 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

After simplification

This given the following system of harmonic equation

$$\ddot{y}_1 = -\omega_0^2 y_1$$
$$\ddot{y}_2 = -3\omega_0^2 y_2$$

This implies that there are two possible frequencies ω at which the masses can oscillate.

III. Results and Discussion

Now we discuss above the physical interpretation of this problem.

It is not difficult to see that there are two special kinds of motion that one can easily describe.

1. Here in the first Eigen vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ corresponding to the Eigen value 1.

In this fact that the components are equal tells us that x_1 and x_2 are equal. Consequently the system oscillates back and forth but the middle spring is never stretched. If we had two masses, each attached to a spring constant k. In this frequency is given by

$$\omega_0 = \sqrt{k/m} = \sqrt{1}\,\omega_0$$

2. In this case of second Eigen vector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ x_1 and x_2 are always equal but have opposite directions. This is an 'in' and 'out' motion type. The frequency of the system is also predictable in thiscase, each mass is attached to a spring compressed by distance $x_1$and to another stretched by a distance $2x_1$. It is as if the mass is attached to one single spring of constant 3k.

The frequency in this case is $\sqrt{3k/m_0}$ = $\sqrt{3}\,\omega_0$

3. Also $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is Eigen vector corresponding to the Eigen value 3, $\sqrt{3}$ appears in the frequency above.

IV. Conclusion

If we had N coupled masses, we would have an N by N Eigen problem to solve which would provide us with N normal frequency. In this paper, involving two masses can be deals with taking about Eigen values and Eigen vectors. The benefit of using that algebraic technique is more apparent in the cases of more than two masses.

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