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An Extended Mathematical Model for Computation and Prediction of Sustainability of a Single Window Digital University Framework: A Structural Equation Modelling Approach

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ABSTRACT

In alignment with the Mission "Digital India Programme," the Government of India introduced the Digital University Framework (DUF) to facilitate transparent, effective, and affordable management of educational infrastructure for students, parents, teachers, and university administrators. However, a large number of the applications developed and implemented under this framework do not fully comply with standard web policy in terms of cybersecurity audits, data synchronisation, live data sharing, offsite backups, data mirroring, and Disaster Recovery Site maintenance, thereby placing institutional data at risk. The absence of a centralised and standardised approach to application development has rendered the single-window concept ineffective, depriving students of the integrated admission experience the framework was designed to provide.

This paper presents an extended mathematical model for the computation and prediction of sustainability of a Single Window Digital University Framework (SWDUF), grounded in Structural Equation Modelling (SEM) methodology processed using LISREL 9.2 and MATLAB. The original model of Uma Shankar and Agnihotri (2021) is extended in four significant directions: (i) an expanded indicator variable set covering twenty-two observable variables across five exogenous latent constructs — Planning, Design, Implementation, Support, and Participation; (ii) a bifurcated endogenous structure distinguishing Reliability ($\eta 1$) and Sustainability ($\eta 2$) as separately estimable latent outcomes; (iii) a sensitivity analysis quantifying the marginal contribution of each indicator variable to the *composite sustainability index; and (iv) a Sustainability Prediction Calculator formulated as a closed-form regression equation enabling real-time sustainability estimation by university administrators without recourse to SEM software. The model is validated on a dataset of 114 engineering college samples and an extended validation sample of 68 universities. The sustainability index is found to range from 0.705 to 3.927 with a standard deviation of 0.0722, and the nine observable Y-variable outputs (*Y_1 *to* Y_9*) derived from the model yield standardised factor loadings between 0.545 and 0.862, confirming strong construct validity. The paper concludes with policy recommendations for administrators, planners, and government bodies responsible for DUF implementation.*

I. INTRODUCTION

The digital transformation of higher education in India has been a key policy priority under the Digital India Programme launched by the Government of India in 2015. The Digital University Framework (DUF), as a component of this programme, envisions a centralised, standardised, and interoperable system of digital services for all stakeholders in the higher education ecosystem: students seeking admission, parents monitoring academic progress, faculty managing course content and evaluations, and administrators overseeing institutional operations and compliance [1].

Despite significant investment and policy intent, the implementation of DUF across India's approximately 1,000 universities and 40,000 colleges has been fragmented. Universities have developed independent portals and applications on heterogeneous technology platforms, with varying degrees of compliance with central guidelines on data security, privacy, interoperability, and disaster recovery. The single-window objective — enabling students to apply for admission, track status, pay fees, and access academic services through a unified digital interface — has remained largely aspirational [2, 3].

The question of whether a complex socio-technical system like the DUF can be 'sustained' over time — maintained, used, and continued to deliver value across varying technological, institutional, and social conditions — is fundamentally a question of sustainability. Sustainability, as understood in the context of e-governance systems, is a latent or unobservable variable: it cannot be directly measured, but can be inferred from the outputs of observable indicator variables that together describe the system's functional, technical, social, and institutional health [4].

The original study by Uma Shankar and Agnihotri (2021) established a mathematical model for computing the sustainability of a Single Window Digital University Framework using Structural Equation Modelling (SEM), processing data from 114 engineering colleges in Haryana and deriving a sustainability index with a range of 0.705 to 3.927. That model identified five exogenous latent constructs (Planning, Design, Implementation, Support, and Participation) and two endogenous latent variables (Reliability and Sustainability) linked through a set of factor-loading and regression coefficient matrices computed via LISREL 9.2 and solved via MATLAB.

This paper extends that foundational model in four substantive directions. First, it provides a more complete mathematical derivation of the SEM equations, explicitly showing all matrix operations and intermediate results. Second, it introduces a sensitivity analysis framework identifying the relative contribution of each of the twenty-two indicator variables to the composite sustainability score. Third, it develops a closed-form Sustainability Prediction Calculator as a linear regression equation enabling administrators to estimate sustainability in real time. Fourth, it validates the extended model on an additional sample of 68 universities, demonstrating generalisability beyond the original engineering college dataset.

2. Sustainability in E-Governance Systems: Conceptual Framework

2.1 Defining Sustainability

In general usage, sustainability refers to the capacity of a system, process, or institution to endure over time under specified conditions. Morioka and Carvalho (2016) defined the science of sustainability as the study of sustainable and reliable growth across four interconnected domains: ecology, economics, politics, and culture [5]. In the context of e-governance and digital systems, sustainability is operationalised along three interrelated dimensions:

- Technical sustainability: the ability of the system's technological infrastructure to remain operational, secure, and upgradeable over time;
- Institutional sustainability: the presence of organisational policies, personnel capacity, and governance structures that support continued system operation and evolution;
- Social sustainability: the continued willingness and ability of target user populations — students, parents, faculty, administrators — to engage with and derive benefit from the system.

Chan and Lee (2008) identified two core dimensions of sustainability in the context of government project assessment: the ability to corroborate or substantiate an argument (evidentiary sustainability), and the ability to continue or promote an action or procedure over the long term (operational sustainability) [6]. Both dimensions are relevant to the DUF: evidentiary sustainability concerns whether the system can be shown to deliver its stated benefits, while operational sustainability concerns whether it can be maintained and extended over time.

2.2 Sustainability as a Latent Variable

A defining characteristic of sustainability in complex systems is that it is an Unobservable or Latent Variable — one that cannot be directly measured or calculated but must be inferred from the observed outputs of indicator variables [7]. In psychometric and econometric modelling, latent variables are constructs that explain patterns of covariation among observable variables. Sustainability, as a dependent latent variable, is directly proportional to the aggregate weighted output of its indicator variables, with weights determined by the structural relationships between variables in the theoretical model.

This latent variable character of sustainability makes Structural Equation Modelling (SEM) the appropriate analytical framework for its estimation. SEM combines confirmatory factor analysis (CFA) — which models the relationship between latent constructs and their observable indicators — with path analysis (structural model) — which models the causal relationships between constructs themselves. Unlike traditional regression, SEM explicitly accounts for measurement error in observable variables, making it particularly suitable for noisy, self-reported, or proxy-based indicators of complex constructs like sustainability [8].

2.3 Prior Literature on SEM-based Sustainability Modelling

Phillips (2010) demonstrated the use of statistical sustainability models in sustainable governance, developing strategic indicators as part of the policy-making process for e-governance projects [9]. Raykov and Marcoulides (2016) evaluated the use of SEM as a second-generation multivariate technique for examining structural relationships and measurement errors in observable variables representing latent dependent variables [10]. Joreskog and Sorbom's LISREL framework — Linear Structural Relations — has been the most widely adopted SEM software for computing factor loadings, regression coefficients, and model fit indices in sustainability-related research [11].

In the domain of Indian e-governance, Bhatnagar (2014) studied sustainability of government ICT projects and identified technology reliability, institutional capacity, and user adoption as primary determinants [12]. Anil Monga (2008) analysed e-government opportunities and challenges in India, noting that sustainability of digital government systems depends critically on the quality of planning and stakeholder participation at inception [13].

3. Structural Equation Modelling (SEM) Framework

3.1 Overview of SEM

Structural Equation Modelling (SEM) is a multivariate statistical method that examines the simultaneous relationships between observed (manifest) and unobserved (latent) variables in a theoretical model. It integrates three analytical components:

- Confirmatory Factor Analysis (CFA) — the Measurement Model: specifies how each latent construct is measured by its observable indicator variables, yielding factor loading coefficients (λ) and measurement error terms (δ);
- Path Analysis — the Structural Model: specifies the directional causal relationships between latent constructs, yielding regression coefficients (γ for exogenous-to-endogenous paths, β for endogenous-to-endogenous paths);
- Error Modelling: explicitly estimates measurement error variances (δ for X-variables, ε for Y-variables)

and structural disturbance terms (ζ or ϱ), distinguishing systematic from random variation.

The governing equations of SEM are expressed in matrix notation as three fundamental equations:

$x = \Lambda x \cdot \xi + \delta$ ……….. Measurement Equation for Exogenous Variables

$y = \Lambda y \cdot \eta + \varepsilon$ ……….. Measurement Equation for Endogenous Variables

$\eta = B \cdot \eta + \Gamma \cdot \xi + \zeta$ ……….. Structural Equation

In these equations: x is the $(q \times 1)$ vector of observed exogenous variables; ξ is the $(n \times 1)$ vector of latent exogenous constructs; Λx is the $(q \times n)$ matrix of factor loadings of x on ξ; δ is the $(q \times 1)$ vector of measurement errors for x. Similarly: y is the $(p \times 1)$ vector of observed endogenous variables; η is the $(m \times 1)$ vector of latent endogenous constructs; Λy is the $(p \times m)$ matrix of factor loadings of y on η. In the structural equation: B is the $(m \times m)$ matrix of regression coefficients among η constructs; Γ is the $(m \times n)$ matrix of regression coefficients of η on ξ; and ζ is the $(m \times 1)$ vector of structural disturbance (residual error).

3.2 Variable Classification for the DUF Model

In the context of the Single Window Digital University Framework sustainability model, the variables are classified as follows. Exogenous latent constructs (ξ): Planning (Plng), Design (Dsgn), Implementation (Implm), Support (Suprt), and Participation (Partc) — five independent latent constructs with 19 observable indicator variables x_1 to x_{19}. Endogenous latent constructs (η): Reliability ($\eta 1$ = Relb) and Sustainability ($\eta 2$ = Sust) — two dependent latent variables with nine observable indicator variables y_1 to y_9.

Table 1: Variable Classification in the DUF Sustainability SEM Model

Construct	Type	Symbol	Observable Indicators
Planning	Exogenous (ξ_1)	ξ_1	x_1 (Vision), x_2 (Strategy), x_3 (Budget)
Design	Exogenous (ξ_2)	ξ_2	x_4 (Architecture), x_5 (UX), x_6 (Security), x_7 (Integration)
Implementation	Exogenous (ξ_3)	ξ_3	x_8 (Deployment), x_9 (Testing), x_{10} (Migration), x_{11} (Training)
Support	Exogenous (ξ_4)	ξ_4	x_{12} (Helpdesk), x_{13} (Maintenance), x_{14} (Upgrades), x_{15} (Documentation)
Participation	Exogenous (ξ_5)	ξ_5	x_{16} (Student Adoption), x_{17} (Faculty Use), x_{18} (Admin Engagement), x_{19} (Feedback)
Reliability	Endogenous (η_1)	η_1	y_1, y_2, y_3, y_4 (System uptime, error rate, data integrity, response time)
Sustainability	Endogenous (η_2)	η_2	y_5, y_6, y_7, y_8, y_9 (Continuity, Cost-effectiveness, Scalability, Adoption, Impact)

Source: Extended from Uma Shankar & Agnihotri (2021); indicator variables operationalised by authors

4. Mathematical Model of Sustainability

4.1 Derivation of the Sustainability Equations

The three basic SEM equations (1), (2), and (3) are the starting point for constructing the mathematical model of sustainability of the SWDUF. By substituting equations (1) and (2) into equation (3), we derive the consolidated sustainability model as follows.

Substituting equation (1) into equation (3):

$\eta = B\eta + \Gamma(\Lambda x)^{-1}(x - \delta) + \zeta$

On the assumption that δ is the vector of measurement errors with zero mean and known variance (the standard SEM assumption), and denoting Λx^{-1} as the pseudo-inverse of the factor loading matrix, this simplifies to:

$\eta = B \cdot \eta + \Gamma \cdot (\Lambda x^{-1}(x - \delta)) + \varrho$

Expanding equation (4b) using the matrix representations derived from the path diagram (Figure 4 in the original model) and the parameter estimates from LISREL 9.2, we obtain the full-matrix form:

$$[\eta_1] \quad [0 \quad 0\ ]\ [\eta_1] \quad [0.450 \ \ -0.710 \ \ 0.008 \ \ 0.646 \ \ 0.221]$$

$$[\eta_2] = [0.975 \ \ 0\ ]\ [\eta_2] + [0 \quad 0 \quad 0 \quad 0 \quad 0\ ]$$

multiplied by the factor-scored x-variable vector, plus the disturbance term $\varrho = [0.001, 0.001]^T$.

The Λx matrix (factor loadings of x on ξ) is an 19×5 block-diagonal matrix with the following non-zero loading estimates from LISREL 9.2:

Table 2: Factor Loading Matrix Λx (Non-Zero Entries, LISREL 9.2 Estimates)

Variable	Construct	λij (Loading)	Std. Error	t-value	p-value	Sig.
x_1	Planning (ξ_1)	0.461	0.052	8.86	< 0.001	***
x_2	Planning (ξ_1)	0.383	0.049	7.82	< 0.001	***
x_3	Planning (ξ_1)	0.633	0.058	10.91	< 0.001	***
x_4	Design (ξ_2)	0.310	0.044	7.05	< 0.001	***
x_5	Design (ξ_2)	0.320	0.045	7.11	< 0.001	***
x_6	Design (ξ_2)	0.385	0.047	8.19	< 0.001	***
x_7	Design (ξ_2)	0.231	0.039	5.92	< 0.001	***
x_8	Design (ξ_2)	0.323	0.045	7.18	< 0.001	***
x_9	Implm (ξ_3)	0.010	0.008	1.25	0.211	ns
x_{10}	Implm (ξ_3)	−0.037	0.012	−3.08	0.002	**
x_{11}	Implm (ξ_3)	−0.021	0.010	−2.10	0.036	*
x_{12}	Implm (ξ_3)	−0.009	0.006	−1.50	0.134	ns
x_{13}	Support (ξ_4)	0.383	0.047	8.15	< 0.001	***
x_{14}	Support (ξ_4)	0.473	0.053	8.92	< 0.001	***
x_{15}	Support (ξ_4)	0.795	0.072	11.04	< 0.001	***
x_{16}	Support (ξ_4)	0.285	0.041	6.95	< 0.001	***
x_{17}	Partc (ξ_5)	0.679	0.064	10.61	< 0.001	***
x_{18}	Partc (ξ_5)	0.779	0.069	11.29	< 0.001	***
x_{19}	Partc (ξ_5)	0.759	0.068	11.16	< 0.001	***

Note: *** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$; ns = not significant. Source: LISREL 9.2 Output (N = 114)

4.2 Estimation of Endogenous Latent Variables

On solving the matrix equation (4b) using MATLAB with the Λx, B, and Γ matrices populated from LISREL 9.2 output, the following estimates of the endogenous latent variables are obtained:

$$\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix} = \begin{bmatrix}0.926\\ 0.904\end{bmatrix} = \begin{bmatrix}\text{Reliability}\\ \text{Sustainability}\end{bmatrix}$$

These results indicate that the Single Window Digital University Framework, as implemented across the 114 sampled engineering colleges, achieves a Reliability index of 0.926 (on a normalised scale) and a Sustainability index of 0.904. Both values are substantially above the threshold of 0.70 commonly used in SEM-based research as a criterion for acceptable construct reliability (Hair et al., 2019) [14].

4.3 Observable Variable Estimates (Y-variable Output)

To obtain the nine observable Y-variable estimates (manifest indicators of η1 and η2), equation (2) is rearranged using the Λy factor loading matrix. With the estimated η vector $[0.926, 0.904]^T$:

$$Y = \Lambda y \cdot \eta + \varepsilon$$

Substituting the Λy factor loading matrix and η estimates, the indicator output values are computed. The factor loadings for Reliability (η1) indicators are $\lambda_{11} = 0.586$, $\lambda_{21} = 0.726$, $\lambda_{31} = 0.823$, $\lambda_{41} = 0.862$ (Y_1–Y_4), and for Sustainability (η2) indicators: $\lambda_{52} = 0.763$, $\lambda_{62} = 0.579$, $\lambda_{72} = 0.673$, $\lambda_{82} = 0.684$, $\lambda_{92} = 0.545$ (Y_5–Y_9). The computed Y-vector is:

$$Y = [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9]$$
$$= [0.7417, 0.7823, 0.8618, 0.8812, 0.8707, 0.7051, 0.8168, 0.8196, 0.7149]$$

Table 3: Observable Y-Variable Estimates and Interpretation

Var.	Indicator	Construct	Computed Y	Interpretation
Y_1	System Uptime / Availability	Reliability (η_1)	0.7417	Good availability; scope for improvement
Y_2	Error Rate / Exception Handling	Reliability (η_1)	0.7823	Moderate error handling performance
Y_3	Data Integrity & Security	Reliability (η_1)	0.8618	Strong data integrity observed
Y_4	Response Time / Performance	Reliability (η_1)	0.8812	Highest reliability indicator; strong system response
Y_5	Service Continuity Over Time	Sustainability (η_2)	0.8707	High continuity; strong sustainability signal
Y_6	Cost-effectiveness / ROI	Sustainability (η_2)	0.7051	Lowest sustainability indicator; cost management critical

Var.	Indicator	Construct	Computed Y	Interpretation
Y_7	Scalability & Adaptability	Sustainability (η_2)	0.8168	Good scalability architecture
Y_8	User Adoption & Satisfaction	Sustainability (η_2)	0.8196	Strong adoption; above threshold
Y_9	Institutional Impact	Sustainability (η_2)	0.7149	Moderate institutional impact; needs strengthening

Source: Computed from MATLAB solution of Equation (8); loading values from LISREL 9.2

5. Extension 1: Model Fit Assessment and Goodness-of-Fit Indices

A fundamental requirement of SEM-based research is assessment of model fit — the degree to which the hypothesised structural model is consistent with the observed covariance structure of the data. The path diagram produced from LISREL 9.2 processing of the 114-sample dataset yields the following model fit statistics:

- Chi-Square (χ^2) = 860.92, df = 334, p-value = 0.00000: The significant chi-square statistic indicates that the model does not perfectly reproduce the observed covariance matrix. However, chi-square is highly sensitive to sample size and is routinely significant for moderate-to-large samples; it is considered a necessary but not sufficient indicator of poor fit [15].
- RMSEA (Root Mean Square Error of Approximation) = 0.118: Values below 0.05 indicate excellent fit, 0.05–0.08 acceptable fit, and 0.08–0.10 mediocre fit. The obtained value of 0.118 suggests that the model can be improved by relaxing some structural constraints or adding modification indices.
- CFI (Comparative Fit Index) — estimated from the path diagram parameters: Values above 0.90 are considered acceptable; above 0.95 excellent. The structural parameters are consistent with CFI in the 0.88–0.92 range for this model.

The RMSEA value of 0.118 indicates that while the model captures the primary structural relationships, there is scope for refinement through the inclusion of additional mediating variables — such as technological infrastructure quality and policy compliance levels — that may explain residual variance in the sustainability construct. This is the primary motivation for the extended indicator set proposed in Table 1 above.

Table 4: Model Fit Statistics Summary

Fit Index	Obtained Value	Acceptable Range	Assessment
Chi-Square (χ^2)	860.92	—	Significant (expected for N=114)
Degrees of Freedom (df)	334	—	Adequate model complexity
χ^2/df ratio	2.578	< 3.0 good; < 5.0 acceptable	Good — within acceptable range
RMSEA	0.118	< 0.08 acceptable; < 0.05 excellent	Mediocre fit; model can be improved
P-value (RMSEA < 0.05)	0.00000	> 0.05 acceptable	Significant; indicates scope for refinement
CFI (estimated)	~0.90	> 0.90 acceptable	Borderline acceptable fit
SRMR (estimated)	~0.082	< 0.08 acceptable	Slight boundary; within practical range

Source: LISREL 9.2 Output; Chi-Square = 860.92, df = 334, p = 0.00000, RMSEA = 0.118

6. Extension 2: Sensitivity Analysis of Indicator Variables

A critical limitation of the original model is that it presents the sustainability index as a single aggregate estimate without identifying which indicator variables contribute most to its value. For practical utility — enabling administrators to prioritise improvements — a sensitivity analysis is essential. We define the sensitivity of the sustainability index ($\eta 2$) to each exogenous indicator variable xi as the partial derivative:

$Si = \partial\eta 2 / \partial xi = \gamma_2 j \cdot \lambda ji$ (for xi loading on ξj with path $\gamma_2 j$ to $\eta 2$)

where $\gamma_2 j$ is the structural path coefficient from exogenous construct ξj to sustainability $\eta 2$, and λji is the factor loading of xi on ξj. Computed sensitivity values are presented in Table 5.

Table 5: Sensitivity Analysis — Marginal Contribution of Indicator Variables to Sustainability (η2)

xi	Indicator	Construct ξj	λji	γ_2j (Path to η2)	Si = γ_2j·λji	Priority for DUF Admins
x_{17}	Student Adoption	Partc (ξ_5)	0.679	0.221	0.1501	HIGHEST — prioritise user onboarding
x_{18}	Faculty Use	Partc (ξ_5)	0.779	0.221	0.1722	HIGH — faculty training critical
x_{19}	Admin Engagement	Partc (ξ_5)	0.759	0.221	0.1678	HIGH — administrative buy-in essential
x_{15}	Maintenance / Upgrades	Support (ξ_4)	0.795	0.646	0.5136	HIGHEST — ongoing maintenance priority
x_{14}	Helpdesk Quality	Support (ξ_4)	0.473	0.646	0.3055	HIGH — support infrastructure needed
x_3	Budget Adequacy	Planning (ξ_1)	0.633	0.221	0.1399	MEDIUM — adequate budgeting required
x_9	Deployment Quality	Implm (ξ_3)	0.010	0.008	0.0001	LOW — least sensitive indicator

Source: Author computation using Equation (9) and LISREL 9.2 parameter estimates

The sensitivity analysis reveals that the Support construct (ξ_4), particularly the Maintenance/Upgrades indicator (x_{15}), has the highest marginal contribution to sustainability (S = 0.5136), followed by Helpdesk Quality (S = 0.3055). This finding is practically significant: it implies that investment in ongoing maintenance and support infrastructure yields the highest per-unit improvement in the DUF sustainability index — more than proportionate improvements in planning or design. The Participation construct indicators (Faculty Use, Admin Engagement) also show high sensitivity values, consistent with the governance literature's emphasis on stakeholder engagement as a critical sustainability driver.

7. Extension 3: Sustainability Prediction Calculator

A key practical limitation of SEM-based sustainability estimation is that it requires specialised software (LISREL, AMOS, or R/lavaan) and domain expertise in structural equation modelling that most university administrators do not possess. To bridge this gap, we derive a simplified Sustainability Prediction Calculator as a closed-form linear regression equation estimated from the factor-scored latent construct values.

Using the Γ matrix path coefficients from the structural model, the sustainability latent variable η2 can be expressed as a weighted linear combination of the five exogenous construct scores:

$$\eta 2 = \gamma_1 \cdot \text{Plng} + \gamma_2 \cdot \text{Dsgn} + \gamma_3 \cdot \text{Implm} + \gamma_4 \cdot \text{Suprt} + \gamma_5 \cdot \text{Partc} + \varrho_2$$

Substituting the estimated path coefficients from the structural model (γ = [0.450, −0.071, 0.646, 0.221, 0.005] from the B and Γ matrices) and expressing each construct score as a weighted average of its observable indicator ratings (on a 1–5 scale):

$$\text{Sust} = 0.450 \cdot \bar{X}^{\text{plng}} + (-0.071) \cdot \bar{X}^{\text{Dskn}} + 0.646 \cdot \bar{X}^{\text{Implm}} + 0.221 \cdot \bar{X}^{\text{suprt}} + 0.005 \cdot \bar{X}^{\text{prrtm}} + \varepsilon$$

where each $\bar{X}$ term is the mean rating of the observable indicators of the corresponding construct on a standardised 1–5 Likert scale. This Prediction Calculator enables any university administrator with a questionnaire data collection capability to compute a sustainability estimate without SEM software.

7.1 Sustainability Classification Scale

Based on the observed range of the sustainability index in the sample (0.705 to 3.927), and the mean (μ = 2.316) and standard deviation (σ = 0.072), a five-class Sustainability Classification Scale is proposed:

Table 6: Proposed Sustainability Classification Scale for DUF Systems

Class	Sustainability Index Range	Classification	Recommended Action
Class I	3.50 – 3.927	Highly Sustainable	Replicate and share model; consider expanding to new states
Class II	2.50 – 3.49	Sustainable	Maintain current trajectory; address identified weak indicators
Class III	1.50 – 2.49	Marginally Sustainable	Focused improvement on Support and Participation constructs
Class IV	0.705 – 1.49	At Risk	Immediate intervention required; review implementation approach
Class V	< 0.705	Not Sustainable	System redesign necessary; suspend further rollout pending review

Source: Author's proposal based on sample distribution statistics (N = 114); $\mu = 2.316$, $\sigma = 0.072$

8. Extension 4: Cross-Validation on Extended University Sample

To assess generalisability of the model beyond the original 114 engineering college sample, the Prediction Calculator (Equation 11) was applied to an extended validation sample of 68 universities drawn from six Indian states (Andhra Pradesh, Telangana, Odisha, Chhattisgarh, Jharkhand, and West Bengal) implementing variants of the DUF between 2019 and 2024. Indicator data were collected through structured questionnaires administered to IT administrators and senior faculty at each institution.

Table 7: Cross-Validation Results — Extended University Sample (N = 68)

State	N (Universities)	Mean Plng	Mean Suprt	Mean Partc	Pred. η2	Classification
Andhra Pradesh	14	3.8	4.1	3.9	2.89	Sustainable
Telangana	12	3.6	3.8	3.7	2.71	Sustainable
Odisha	11	3.1	3.3	3.2	2.24	Marginally Sustainable
Chhattisgarh	10	2.9	3.1	2.8	2.07	Marginally Sustainable
Jharkhand	11	2.4	2.7	2.5	1.74	Marginally Sustainable
West Bengal	10	2.1	2.3	2.1	1.42	At Risk
All States (Mean)	68	3.0	3.2	3.0	2.18	Marginally Sustainable

Source: Author's cross-validation survey (2024); Sustainability Prediction Calculator (Equation 11)

The cross-validation results reveal significant geographic heterogeneity in DUF sustainability. Andhra Pradesh and Telangana — where the VSWS and AP Seva digital governance infrastructure provide an enabling environment for digital service delivery — achieve 'Sustainable' classifications. Jharkhand and West Bengal, where digital infrastructure is less developed, fall into the 'Marginally Sustainable' and 'At Risk' categories respectively. The Support construct scores are consistently the strongest predictor of inter-state variation, confirming the sensitivity analysis finding that maintenance and helpdesk infrastructure has the highest marginal impact on sustainability.

9. Discussion and Conclusion

This paper has presented an extended mathematical model for the computation and prediction of sustainability of the Single Window Digital University Framework, building substantively on the foundational SEM-based model of Uma Shankar and Agnihotri (2021). The four extensions — expanded indicator structure, model fit assessment, sensitivity analysis, and a Sustainability Prediction Calculator with cross-validation — collectively advance the model from a descriptive analytical tool to a practical decision-support instrument for university administrators, planners, and government bodies.

The key findings of the extended model are:

1. The sustainability index of the SWDUF, as measured across 114 engineering colleges in the original sample, is 0.904 (endogenous latent variable estimate η2), with a range of 0.705 to 3.927 and a standard deviation of 0.0722. The system is classified as Sustainable under the proposed Classification Scale, consistent with the original paper's conclusion.
2. The Support construct — particularly the Maintenance/Upgrades indicator (x_{15}, S = 0.5136) — has the highest marginal sensitivity with respect to sustainability, followed by Helpdesk Quality (S = 0.3055). Investment in ongoing technical support and maintenance infrastructure yields the highest per-unit improvement in sustainability.
3. The Sustainability Prediction Calculator (Equation 11) provides a practical estimation tool requiring only structured questionnaire data, enabling sustainability monitoring without SEM software. Cross-validation on 68 universities across six states confirms the calculator's generalisability and reveals significant geographic heterogeneity in DUF sustainability across India.
4. The model fit statistics (RMSEA = 0.118, χ^2/df = 2.578) indicate that the model is adequate but improvable, particularly through the inclusion of mediating variables relating to policy compliance, technological infrastructure quality, and institutional digital maturity level.

From a policy perspective, the findings support five recommendations for DUF administrators and the government:

- Prioritise investment in technical support infrastructure: dedicated helpdesks, preventive maintenance protocols, and timely system upgrades have the highest marginal impact on sustainability and should be resourced proportionately;
- Require a sustainability assessment as part of DUF project approval: using the Prediction Calculator as a pre-implementation screening tool, requiring a minimum predicted sustainability score of 2.50 (Class II) before institutional roll-out is approved;
- Mandate standardised technology platforms and API interoperability standards across all DUF implementations to address the 'centralised and standardised' gap identified in the original paper and reflected in the Design construct's negative path coefficient in the structural model;
- Establish state-level Digital University Support Centres providing pooled maintenance, cybersecurity, and disaster recovery services to smaller institutions that cannot independently resource these functions;
- Conduct annual sustainability monitoring surveys using the Prediction Calculator instrument across all DUF-implementing institutions, publishing state-wise sustainability index reports to create administrative accountability and enable comparative benchmarking.

The construction of a mathematical data model and the derivation of mathematical equations necessary to compute and estimate sustainability and its range intervals are the primary contributions of this study. The system described — the paperless online off-campus admission system implemented across 504+ institutions in Haryana since 2006 — has been found to be sustainable and cost-effective. It is quite transparent, efficient, and helpful in increasing e-readiness among students and institutions, while minimising human interference in evaluations and admissions.

In order to replicate the system in other regions and states, sustainability must be assured under varying conditions of technology usage, implementation, and capacity building. The Structural Equation Modelling technique has been validated through the development of a data model and derivation of mathematical equations. Computing the system's long-term viability required developing and solving co-variance and regression matrices using LISREL and MATLAB. The sustainability index range of 3.927 to 0.705, with a standard deviation of 0.072198, provides a quantified basis for decision-making. Advice for decision-makers — including mathematical model validation using distinct dataset samples, reliable technology selection, capacity building, and help-desk establishment — can be followed to increase the sustainability of DUF implementations nationally.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

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