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Pseudopotential analysis of dust acoustic solitons in complex plasmas with two-temperature nonthermal ions

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ABSTRACT

The present study explores the properties of arbitrary-amplitude nonlinear dust acoustic solitary waves (DASWs) in an unmagnetized dusty plasma system comprising two populations of nonthermal ions characterized by distinct temperatures. In order to analyze the nonlinear characteristics of DASWs, the Sagdeev-type pseudopotential approach, which draws upon a mechanical analogy to particle motion in a potential well, is employed. This method facilitates the examination of arbitrary-amplitude solitary structures by transforming the nonlinear wave problem into an energy-balance-like equation. The analysis indicates that only negative potential solitary structures can exist within the considered plasma configuration. The effects of key plasma parameters - namely, the degree of ion nonthermality, the Mach number, and the relative concentration of the higher-temperature ion species - on the characteristics of dust acoustic solitons are systematically analyzed and discussed. The study highlights how variations of these parameters influence the amplitude, width, and existence domains of solitary wave structures.

KEYWORDS: Dust acoustic waves; soliton; nonthermal distribution; Sagdeev pseudopotential.

1. INTRODUCTION

Dusty plasmas, comprising electrons, ions, and micron- to submicron-sized charged dust particles, have attracted considerable attention in recent decades due to their prevalence in various astrophysical and laboratory environments [1]-[5], such as planetary rings (e.g., Saturn's rings), cometary tails, interstellar clouds, fusion devices, plasma processing reactors and semiconductor manufacturing. The collective dynamics of dusty plasmas arise from the motion of their charged constituents, particularly the dust grains, whose interactions can give rise to a variety of wave phenomena among which the dust-ion acoustic wave (DIAW) and dust acoustic wave (DAW) are of significant interest in dusty plasma. Since the pioneering work of Rao et al. [6], which first predicted the existence of nonlinear DAWs in unmagnetized dusty plasmas, where the inertia is primarily contributed by charged dust particles and the restoring forces arise from the pressure of inertialess electrons and ions, numerous investigations have been conducted to explore the linear and nonlinear dynamics of low-frequency DAWs in both unmagnetized and magnetized plasma environments [7]-[11]. The nonlinear propagation of DAWs in dusty plasma environments can result in the emergence of localized solitary wave structures, or solitons, which may exhibit either compressive (positive amplitude) or rarefactive (negative amplitude) profiles. The polarity and characteristics of these solitons are strongly influenced by the interplay between the nonlinearity and dispersion inherent in the system, as well as by the relevant plasma parameters.

However, most early studies of dust acoustic solitary waves (DASWs) assumed that the ions in the plasma obey Maxwellian velocity distributions. In contrast, observations from satellites and space missions and laboratory experiments suggest that nonthermal distributions are prevalent in natural plasma environments. These nonthermal populations are characterized by enhanced high-energy tails and are more accurately represented by distributions such as those introduced by Cairns et al. [12, 13], which incorporate a nonthermal parameter to quantify the deviation from Maxwellian behavior. The presence of such nonthermal ions can significantly modify the collective response of the plasma, particularly affecting the amplitude, width, polarity, and even the stability of nonlinear structures. Ghosh et al. [14] investigated instability of DAW due to nonthermal ions in a charge varying dusty plasma. Lin and Duan [15] studied the characters of small but finite amplitude DASWs in a dusty plasma system with nonthermal ions through a Korteweg-de Vries (KdV) equation. Furthermore, in many realistic plasma environments, multiple ion species with distinct temperature distributions coexist. The coexistence of two-temperature ion populations introduces an additional layer of complexity, as the two species contribute differently to the plasma pressure and respond differently to potential perturbations. When these ion populations also follow nonthermal distributions, the resulting plasma system becomes highly nontrivial and demands a robust analytical treatment. Dorranian and Sabetkar [16] studied the properties of DASWs in a dusty plasma system with two kinds of nonthermal ions at different temperatures using reductive perturbation method. Emamuddin and Mamun [17] have carried out an investigation on DA shock waves in an unmagnetized dusty plasma with nonthermal ions by deriving nonlinear Burgers' equation.

Numerous mathematical techniques have been developed to analyze the propagation of solitary waves in plasma systems. Among these, the reductive perturbation method has been extensively employed for investigating nonlinear wave dynamics, particularly in the context of small-amplitude solitary waves [18]-[21]. This method relies on the assumption that wave amplitudes are sufficiently small, which inherently limits its applicability to weakly nonlinear regimes. Consequently, for studying arbitrary amplitude solitary waves, alternative analytical approaches are required. In this regard, Sagdeev [22] introduced a powerful method that remains effective for arbitrary amplitude solitary structures, known as the Sagdeev pseudopotential approach. This technique provides valuable qualitative and quantitative insights into nonlinear wave behavior. By transforming the fluid equations into a moving frame, and performing appropriate mathematical reductions, the system can be expressed as a single energy-like equation-often referred to as the energy balance equation. Pseudopotential approach is capable of capturing a wider spectrum of nonlinear behaviors and is especially suited for investigating solitary wave existence criteria, polarity, and stability. This approach has been adopted for studying nonlinear wave phenomena in various plasma environments by several authors [23]-[30]. However, to the best of our knowledge, no comprehensive study has yet been undertaken to explore the excitation of rbitrary amplitude DASWs a in a four-component dusty plasma system, where two distinct types of nonthermal ions, each with different temperatures, are present. In this paper, we investigate the characteristic features of electrostatic DAWs in an unmagnetized dusty plasma with two nonthermal ions at two different temperatures. The paper is structured in the following manner. The governing equations for the lowfrequency DA motion are described in Sec. 2. The energy balance equation for DAW is derived using the Sagdeev pseudopotential approach in Sec. 3. Section 4 is devoted to the parametric investigations and the concluding remarks and main findings of the study are summarized in Sec. 5.

2. MODEL DESCRIPTION

We focus on a four-component dusty plasma model consisting of negatively charged inertial dust grains, Boltzmann-distributed electrons and two nonthermal ions with different temperatures. The dynamics of such a plasma system are governed by the following set of normalized hydrodynamic equations:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) = 0, \qquad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = \frac{\partial \phi}{\partial x}, \qquad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_d + n_e - n_{il} - n_{ih} \qquad (3)$$

Here, n_d represents the dust particle number density normalized by the unperturbed dust particle number density n_{d0}. u_d denotes the velocity of the dust fluid normalized by the dust acoustic speed $C_d = \sqrt{Z_{d0}T_{eff}/m_d}$, Z_{d0} denoting the equilibrium dust-charge number, m_d represents the mass of the dust particles, T_{eff} is the effective temperature defined by $T_{eff} = Z_{d0}n_{d0}\left(\frac{n_{il0}}{T_{il}} + \frac{n_{ih0}}{T_{ih}} + \frac{n_{e0}}{T_e}\right)^{-1}$ with $n_{e0}(T_e)$, $n_{il0}(T_{il})$, $n_{ih0}(T_{ih})$ are the unperturbed number density (temperature) of electrons, lower temperature ions and higher temperature ions, respectively. The space and the time variables are normalized by using the Debye length $\lambda_D = (T_{eff}/4\pi Z_{d0}n_{d0}e^2)^{1/2}$ and dust plasma frequency $\omega_{pd} = (4\pi Z_{d0}n_{d0}e^2/m_d)^{1/2}$, respectively. $\emptyset$ denotes the electrostatic potential normalized by T_{eff}/e.

The normalized number density of electrons (n_e), lower temperature ions (n_{il}) and higher temperature ions (n_{ih}) are given as

$$n_e = \mu \exp(\beta_1 \sigma \emptyset), \qquad (4)$$

$$n_{il} = \mu\delta_1\left[1 + \gamma(\sigma\phi) + \gamma(\sigma\phi)^2\right]\exp(-\sigma\phi) \qquad (5)$$

$$n_{ih} = \mu\delta_2\left[1 + \gamma(\beta\sigma\phi) + \gamma(\beta\sigma\phi)^2\right]\exp(-\beta\sigma\phi), \qquad (6)$$

where $\mu = 1/(\delta_1 + \delta_2 - 1)$, $\gamma = \frac{4\alpha}{1+3\alpha}$, $\beta = \beta_1/\beta_2$ with $\beta_1 = T_{il}/T_e$ and $\beta_2 = T_{ih}/T_e$, $\delta_1 = n_{il0}/n_{e0}$, $\delta_2 = n_{ih0}/n_{e0}$, $\sigma = T_{eff}/T_{il} = [\mu(\beta_1 + \delta_1 + \delta_2\beta)]^{-1}$ and α represents the parameter that characterizes the number of nonthermal ions [12]. At equilibrium, the charge neutrality condition reads $n_{e0} + Z_{d0}n_{d0} = n_{il0} + n_{ih0}$. This may be written as $\delta_1 + \delta_2 - 1 \geq 0$. According to Verheest and Pillay [31], the applicability of the Cairns nonthermal distribution is restricted to values of the parameter $\alpha < 0.25$.

3. ARBITRARY AMPLITUDE SOLITARY WAVE THEORY

We predict the existence of traveling solitary waves with arbitrary amplitudes, which are waves that retain their shape and profile when observed in a reference frame moving with the excitation. This prediction is based on the assumption that all fluid variables in the governing evolution equations are functions of a single variable $\xi = x - Mt$, M represents the Mach number, i.e. the velocity of the solitary wave normalized by C_d. By applying this transformation into Eqs. (1)-(3), we derive a system of equations in the co-moving co-ordinate ξ. The resulting transformed system can be expressed as follows:

$$-M\frac{dn_d}{d\xi} + \frac{d}{d\xi}(n_d u_d) = 0, \qquad (7)$$

$$-M\frac{du_d}{d\xi} + u_d\frac{du_d}{d\xi} = \frac{d\phi}{d\xi}, \qquad (8)$$

$$\frac{d^2\phi}{d\xi^2} = n_d + n_e - n_{il} - n_{ih} \qquad (9)$$

Upon integrating Eqs. (7)-(9) and imposing the appropriate boundary conditions, namely $n_d \to 1$, $u_d \to 0$ and $\phi \to 0$ as $\xi \to \pm\infty$, one can obtain

$$u_d = M\left(1 - \frac{1}{n_d}\right) \qquad (10)$$

$$n_d = \frac{1}{\sqrt{1 + \frac{2\phi}{M^2}}}. \qquad (11)$$

Now substituting the density expressions (5) (4)-(6) and (11) in Poisson's equation (9) and using the boundary conditions $n_d \to 1$, $u_d \to 0$, $\phi \to 0$ and $\frac{d\phi}{d\xi} \to 0$ as $\xi \to \pm\infty$, we obtain a pseud-energy conservation equation in the form

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \psi(\phi) = 0, \qquad (12)$$

where the pseudopotential or Sagdeev potential [22] function $\psi(\phi)$ reads

$$\psi(\phi) = M^2\left(1-\sqrt{1+\frac{2\phi}{M^2}}\right) + \frac{\mu}{\beta_1\sigma}(1-\exp(\beta_1\sigma\phi)) + \mu\delta_1\left(\frac{1+3\gamma}{\sigma}\right) + \mu\delta_2\left(\frac{1+3\gamma}{\beta_1\sigma}\right.$$

$$-\mu\delta_1\exp(-\sigma\phi)\left(3\gamma\phi+\frac{1+3\gamma}{\sigma}+\gamma\sigma\phi^2\right) - \mu\delta_2\exp(-\beta\sigma\phi)\left(3\gamma\phi+\frac{1+3\gamma}{\beta\sigma}+\gamma\beta\sigma\phi^2\right)$$

Soliton solutions derived from Eq. (13) are obtained when the Sagdeev potential satisfies the following necessary conditions:

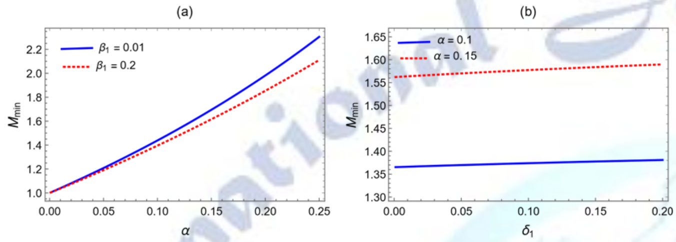

Figure 1: The variation of minimum Mach number M_{min} with α and δ_1. (a) Plot of M_{min} against nonthermal parameter α for different β_1, where $\delta_1 = 1$ and (b) Plot of M_{min} against δ_1 for different α, where $\beta_1 = 0.1$. The Other parameters are $\beta_2 = 0.3$ and $\delta_2 = 2$.

(i) $\psi(\phi) = 0$, $\left(\frac{d\psi}{d\phi}\right) = 0$ and $\left(\frac{d^2\psi}{d\phi^2}\right) < 0$ at $\phi = 0$, and

(ii) $\psi(\phi) < 0$ for ϕ lying between 0 and ϕ_m, i.e., either for $0 < \phi < \phi_m$ (compressive) or $\phi_m < \phi < 0$ (rarefactive).

The first condition for the existence of solitary waves requires that the Mach number M satisfy the relation

$$M^2 > \frac{1}{\mu\sigma(1-\gamma)(\delta_1+\delta_2\beta)+\mu\beta_1\sigma}. \quad (14)$$

This condition guarantees that solitary wave solutions are physically admissible and spatially localized. From the above relation, it is evident that the lower bound for the Mach number is derived as

$$M_{min} = \sqrt{\frac{1}{\mu\sigma(1-\gamma)(\delta_1+\delta_2\beta)+\mu\beta_1\sigma}}. \quad (15)$$

This represents the minimum value required to sustain the formation of localized soliton structures.

4. PARAMETRIC ANALYSIS AND DISCUSSION

In this section, the numerical results derived from the study are analyzed and interpreted. A careful examination of Eq. (15) indicates that the formation of solitary waves requires the Mach number to exceed a certain minimum value. This minimum threshold is influenced by various plasma parameters, suggesting that the existence and properties of solitary waves are closely linked to the specific physical conditions of the plasma. Figure 1(a) illustrates the variation of the minimum Mach number (Mmin) with the nonthermal parameter α for different values of the ratio of lower temperature ions to electrons (β1), in the context of existence of solitary wave. The results indicate that Mmin increases as the ion nonthermality enhances. Figure 1(b) presents the dependence of Mmin on the lower temperature ion concentration (δ1) for various values of α. It is observed that Mmin increases with increasing δ1, highlighting the sensitivity of soliton formation to ion concentration and ion nonthermality. Figure 2 illustrates the variation of the pseudopotential ψ(φ) and the corresponding electrostatic potential φ for different values of α. As α increases, both the depth and the root of the pseudopotential decrease, indicating a weakening of the nonlinear potential well. This behavior is reflected in Fig. 2(b), which shows that the amplitude of the solitary wave diminishes, while its spatial width increases with higher values of α. Consequently, the solitary structure becomes broader and less pronounced as the ion nonthermality increases. Moreover, it is observed that the solitary wave solution no longer exists when α exceeds a critical value, specifically $\alpha = 0.21$. Physically, these energetic ions tend to respond more quickly to potential perturbations, distributing their energy over a wider spatial region and thereby weakening the nonlinear trapping mechanism required for the formation of stable solitary structures. As a result, the electrostatic potential associated with the solitary wave exhibits reduced amplitude and increased spatial widthindicating that the wave becomes broader and shallower. To investigate the influence of the Mach number M on solitary wave characteristics, Fig. 3 presents the variation of the pseudopotential ψ(φ) along with the corresponding electrostatic potential profiles for different values of M. The results reveal that as M increases, both the depth and the root of the pseudopotential decrease, leading to a reduction in the amplitude of the associated soliton structures. Additionally, with higher values of M, the solitary pulses become shorter in height and broader in width, indicating that the DAW becomes more flattened and spread out. It is also observed that solitary wave solutions cease to exist beyond a certain critical value of the Mach number, signifying a threshold for the propagation of such nonlinear structures. Figures 4 and 5 depict the characteristics of large-amplitude dust acoustic (DA) solitary waves for varying values of the ratio of higher temperature ions to electrons (β2) and the concentration of higher temperature ions (δ2). It is evident from both figures that an increase in either β2 or δ2 results in solitary pulses with greater amplitude and reduced width. This behavior suggests that higher values of β2 or δ2 inject more energy into the plasma system, thereby producing sharper and more pronounced solitary structures.

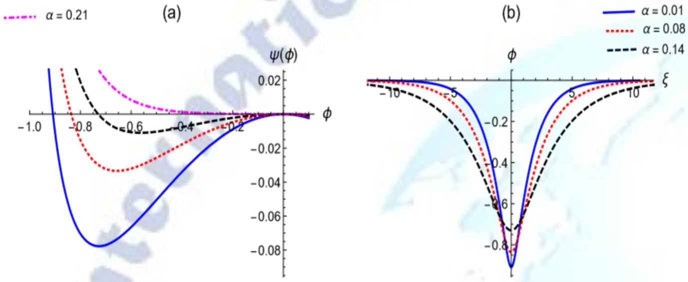

Figure 2: The profiles of (a) pseudopotential $\psi(\phi)$ with ϕ and (b) Electrostatic potential ϕ against ξ for different values of nonthermal parameter α, while the other parameters are $\beta_1 = 0.1$, $\beta_2 = 0.2$, $\delta_1 = 1$, $\delta_2 = 3$ and $M = 1.4$.

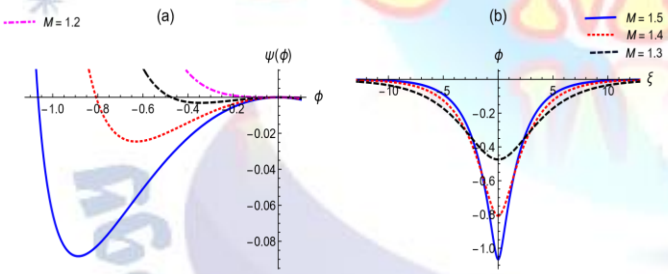

Figure 3: The profiles of (a) pseudopotential ψ(ϕ) with ϕ and (b) Electrostatic potential ϕ against ξ for different values of Mach number M, where $\alpha = 0.1$ and the other parameters are same as in Fig 2.

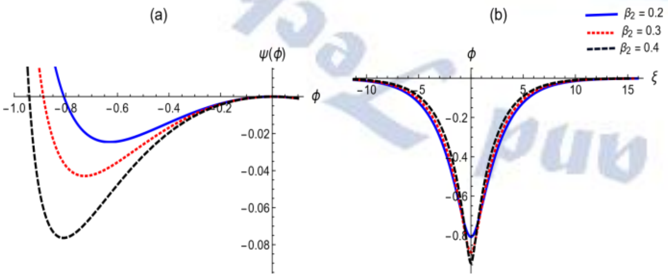

Figure 4: The profiles of (a) pseudopotential ψ(ϕ) with ϕ and (b) Electrostatic potential ϕ against ξ for different values of β2, where $\alpha = 0.1$ and the other parameters are same as in Fig 2.

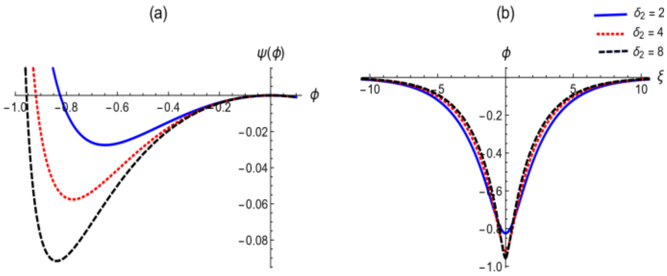

Figure 5: The profiles of (a) pseudopotential $\psi(\phi)$ with ϕ and (b) Electrostatic potential ϕ against ξ for different values of δ_2, where $\alpha = 0.1$ and the other parameters are same as in Fig 2.

5. CONCLUSION

We have investigated the existence and nonlinear behavior of arbitrary amplitude DASWs in a four-component dusty plasma system, which includes negatively charged dust grains, electrons following a Boltzmann distribution, and two ion species exhibiting nonthermal distributions at different temperatures. In order to explore the solitary wave characteristics, we have derived the energy balance equation based on our fluid dynamical model equations and constructed a Sagdeev-type pseudopotential for the plasma medium under consideration. We then numerically obtained the solitary potential structures by integrating the energy balance equation, allowing us to analyze the existence and nature of localized nonlinear wave solutions within the system. We have analyzed the parametric dependencies of the solitary potential structures by varying key plasma compositional parameters, such as the ion nonthermality and Mach number. The results reveal that the plasma system under consideration supports only rarefactive (negative potential) solitary wave structures. The present study provides valuable insights into the nonlinear structures of electrostatic disturbances in astrophysical and space plasma environments such as Saturn's rings, noctilucent clouds, interstellar molecular clouds, etc.-where the nonthermality of ions plays a significant role in plasma dynamics.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

REFERENCES

[1] M. Horanyi and D. Mendis, Astrophys J. 307, 800 (1986).

[2] F. Verheest, Waves in Dusty Space Plasmas (Kluwer Academic, Dordrecht, 2000).

[3] P. K Shukla and A. A. Mamun, Introduction to dusty plasma physics (Bristol, Institute of Physics Publishing, 2002).

[4] G. E. Morfill and A. V. Ivlev, Rev. Mod. Phys. 81, 1353 (2009).

[5] V. Fortov and G. Morfill, Complex and Dusty Plasmas: From Laboratory to Space (CRC Press, Boca Raton, FL, 2009).

[6] N. N. Rao, P. K. Shukla, and M.Y. Yu, Planet. Space Sci. 38, 543 (1990).

[7] P. Bandyopadhyay, G. Prasad, A. Sen, P. K. Kaw, Phys. Rev. Lett. 101, 065006 (2008).

[8] P. K. Shukla and B. Eliasson, Rev. Mod. Phys. 81, 25 (2009).

[9] Y. Wang , C. Guo, X. Jiang, Z. Zhou, X. Ni, P. Qian, J. Shen, Phys. Plasmas 17, 113701 (2010).

[10] A. P. Misra and Y. Wang, Commun. Nonlinear Sci. Numer. Simulat. 22, 1360 (2015).

[11] H. A. Al-Yousefa, B. M. Alotaibia, R. E. Tolbab, W. M. Moslem, Results Phys. 21, 103792 (2021).

[12] R. A. Cairns, A. A. Mamun, R. Bingham, and P. K. Shukla, Phys. Scr. T63, 80 (1996).

[13] A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas 3, 2610 (1996).

[14] S. Ghosh, R. Bharuthram, M. Khan, and and M. R. Gupta, Phys. Plasmas 11, 3602 (2004).

[15] M.-mai Lin, W.-shan Duan, Chaos, Solitons and Fractals 33, 1189 (2007).

[16] D. Dorranian and A. Sabetkar, Phys. Plasmas 19, 013702 (2012).

[17] M. Emamuddin and A. A. Mamun, Phys. Plasmas 25, 013708 (2018).

[18] H. Washimi and T. Taniuti, Phys. Rev. Lett. 17, 996 (1966).

[19] T. Taniuti and C.-C. Wei, J. Phys. Soc. Jpn. 24, 941 (1968).

[20] A. M. El-Hanbaly, E. K. E-Shewy, A. I. Kassem, and H. F. Darweesh, Appl. Phys. Res. 8, 64 (2016).

[21] G. Khan, Z. Kumail, Y. Khan, I. Ullah, M. Adnan, Contrib. Plasma Phys. 60, e201900177 (2020).

[22] R. Z. Sagdeev, Rev. Plasma Phys. 4, 23 (1966).

[23] S. K. El-Labany, W. F. El-Taibany, N. A. El-Bedwehy, and M. M. El-Fayoumy, Eur. Phys. J. D 64, 375 (2011).

[24] H. Alinejad, Phys. Lett. A 375, 1005 (2011).

[25] M. Tribeche, R. Amour, and P. K. Shukla, Phys. Rev. E 85, 037401 (2012).

[26] M. Shahmansouri and M. Tribeche, Astrophys. Space Sci. 344, 99 (2013).

[27] S. A. El-Wakil, E. M. Abulwafa, and A. A. Elhanbaly, Phys. Plasmas 24, 073705 (2017).

[28] P. Bala, A. Kaur, and K. Kaur, Pramana- J. Phys. 95, 20 (2021).

[29] H. A. Al-Yousefa, B. M. Alotaibia, R. E. Tolbab, and W. M. Moslem, Results Phys. 21, 103792 (2021).

[30] M. Choudhury, Braz. J. Phys. 53, 110 (2023).

[31] F. Verheest and S. Pillay, Phys. Plasmas 15, 013703 (2008).