

# Holts Exponential Smoothing Algorithm for Detection of R-Peak in ECG Signals

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## ABSTRACT

*Electrocardiogram (ECG) is one of the most common biological signals which play a significant role in the diagnosis of heart diseases. One of the most important parts of ECG signal processing is interpretation of QRS complex and obtaining its characteristics. R wave is one of the most important sections of this complex, which has an essential role in diagnosis of heart rhythm irregularities and also in determining heart rate variability (HRV). An adaptive threshold R- peak detection algorithm based on Holt's exponential smoothing model is proposed in this project. In this project, three stages are involved one is preprocessing, peak detection by using the R-peak threshold values that are selected using the proposed smoothing algorithm and finally parametric evaluation. For identifying of R-Peak MIT-BIH Arrhythmia database is used which is publicly available. Parametric values like precision, recall, F1-Score value are evaluated. The experimental results are performed using MATLAB software tool.*

**KEYWORDS:** *Electrocardiogram, Heart Rate Variability, Holts exponential smoothing, MIT-BIH Arrhythmia database*

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## INTRODUCTION

The heart is one of the most important organs in the human body, providing power to the body's circulatory system and enabling cells to maintain normal metabolism and function, and it is important to monitor the state of heart health [1]. One of the most effective tools available to detect the health of the heart is the electrocardiogram, which detects a series of electrophysiological changes produced by the heart through electrodes placed on the surface of the skin, and these ECG signals contain detailed information about the heart [2]. The morphological characteristics of the Rpeak are most evident in the ECG, which

represents the ventricles as they contract to their maximum. The interval between adjacent R-peaks is called the RR interval, which varies gently and dynamically, and the QRS complex wave, which usually lasts about 0.1 seconds, reflects the entire process of ventricular depolarization.

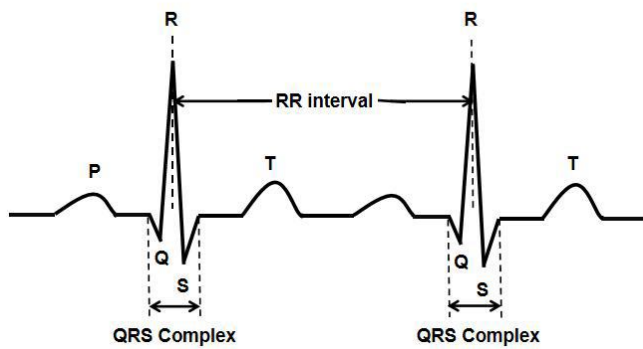


FIGURE 1. R-wave, QRS complex and RR interval in standard ECG signals

The wearable ECG bracelet is the representative product of portable ECG testing devices. Without affecting one's daily work life, users can check their heart rate, heart rate variability and other physiological status indicators through it anytime and anywhere, and then adjust their rest and diet patterns and seek medical treatment in time according to their own conditions. When using the wearable ECG bracelet to collect ECG signals, the user needs to wear the bracelet with the right hand and the electrode piece on the back against the skin of the wrist, while touching the electrode piece on the frontal bracelet with the left hand to form a closed loop circuit, and the ECG chip inside the bracelet can collect the bioelectric signals from the heart.

## RELATED WORK

It is challenging to create automated and broadly applicable algorithms because of the substantial morphological variance in ECG data. This challenge helps to explain why academics are always working on ECG signal processing. ECG detection algorithms are becoming as successful and dependable as skilled cardiologists, with the goal of increasing classification accuracy. Automatic R-peak recognition is a classic ECG signal processing challenge that has been thoroughly studied by researchers. Pan-Tompkins [6] presented the popular real-time R-peak detection technique. After a learning time, this traditional algorithm may process and provide the detection result for every sample. In contrast to more contemporary algorithms, the Pan-Tompkins method has a modest level of detection accuracy and a high level of complexity [7-8]. The majority of R-peak identification methods, however, have the drawback of potentially having a greater computing complexity and requiring unrealistic assumptions,

such as the requirement for global statistical knowledge of the complete input signal [9-11]. Two-parameter parabolic fitting has been effectively employed by researchers to identify ECG signal R points [12]. However, because these techniques rely on computationally demanding procedures like wavelet filtering, isoelectric line localisation, and high-order band-pass filters, they are not appropriate for wearable technology. Furthermore, a thorough comprehension of the entire ECG signal is required. The R-peak detection step locates the R-wave, and the neural network approach [16-18] and threshold method [13-15] are often employed techniques. The threshold method's benefits include its ease of calculation and the threshold's multilayer and adaptable capabilities. Adaptive signal thresholds and adaptive noise thresholds are used by the very traditional PT technique in the field of ECG QRS wave recognition to identify R-peaks [19], which performs exceptionally well on ECG samples with a high signal-to-noise ratio. Finding an appropriate threshold to separate the noise peak from the R-peak from the signal amplitude level is challenging, though, because the PT algorithm's noise threshold is poorly established and does not accurately represent the noise's size. Although the author in [20] greatly decreased the false and missed detection rates by localising the R peaks using dual thresholding of time and amplitude in conjunction with statistical false peak elimination, this method is still inapplicable to ECG signals with low signal-to-noise ratios because the threshold definition does not adequately account for the noise magnitude. This study proposes an adaptive threshold R-peak detection solution for the challenging R-peak detection issue, based on Holts exponential smoothing model.

## PROPOSED METHODOLOGY

The electrocardiogram (ECG) signal is one of the most important and well-known biological signals used for diagnosing people's health. Detection of QRS complex is one of the most important parts carried out in the ECG signal analysis. QRS detection, especially detection of R wave in heart signal, is easier than other portions of ECG signal due to its structural form and high amplitude. Electrocardiogram (ECG) is a powerful and non-invasive tool that can provide long-term cardiac information for the diagnosis of cardiac functions. Accurate and reliable classification of electrocardiogram (ECG) beats is most important in automatic ECG diagnosis applications.



### ECG Signals

The heartbeat is determined as Beats Per Minute (BPM). Fundamental segments are P, QRS, and T waves. P wave is utilized to speaks to the changes within the cells of the atria. The QRS is utilized to speaks to the changes in the cells of the ventricles and the T wave is to speak to the changes in the potentiality of the ventricles. U is said to be the successor of one of the fundamental segments termed as T wave. O is the source point.

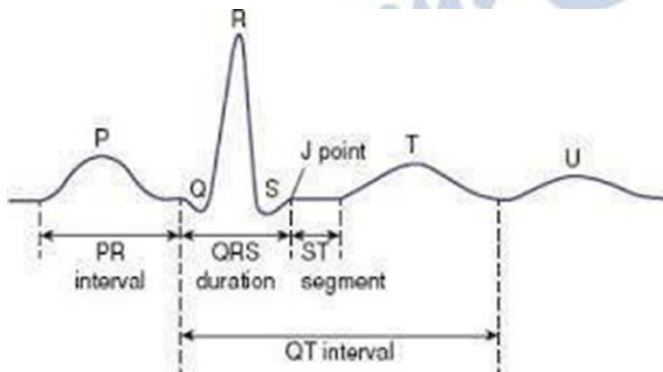


Figure: Representation of standard ECG signal

Table : Normal and Abnormal conditions of ECG Signal

Segment	Normal	Abnormal
QRS Complex	60-100ms	>100ms
R-R interval	0.6-1.2sec	>2sec
P wave	60-80msec	<50msec
PR interval	20-120msec	>120msec
QR interval	350-440msec	>540msec

### Segments in ECG signal P Segment:

Indicates the changes within the cells of the atria. The re-polarization is invisible because of its low amplitude, which is about 0.1-0.2mV. The duration is about 60-80ms.

### QRS Level:

Indicates the change within the cells of the left side of ventricle, right side ventricles are triggered one of the main siphoning in the heart. Immediately, Q-wave and R wave preceding the extra heartbeats that begin in one of your heart's two lower pumping chambers. S wave is the flow towards the down appears next after the pumping chambers.

### T Segment:

Indicates the left part and right part of ventricles within the cells. The amplitude of the T wave ranges is given as 0.1V to 0.3mV and its time duration is considered to be from 120ms to 160ms.

### PR segment:

It indicates the gap taken from atrial depolarization to ventricle depolarization. The duration is about 120-200ms.

### RR Intervening:

Denotes the time between the successive ventricle with the cells. The duration should be less than 3 seconds

### ST segment:

It indicates the time during early ventricle re-polarization. ST segment starts from J, which lies in between QRS and ST. The duration should be less than 20ms

### Methodology

The design of proposed model is explained in this session. The steps of process flow of the work are shown in figure 4.2.

### A.Dataset Used

Datasets are used for the analyzing of ECG signal. We can obtain various kinds of ECG signals from different patients. Datasets are used for the study of detection of abnormalities and the ECG signal. Different normal and abnormal ECG signals taken from the standard MIT-BIH arrhythmia database. The dataset is available in physio net databank. The MIT-BIH database, which is given by the Massachusetts Institute of Technology and Boston's Beth Israel Hospital, includes 10 databases for different test purposes. One among them is the Arrhythmia Database that holds 48 half-hour recordings of two-channel ambulatory ECG signals. The database incorporates annotated ECG sampled at a rate of 360Hz with 11-bit resolution over a 10-mV range.

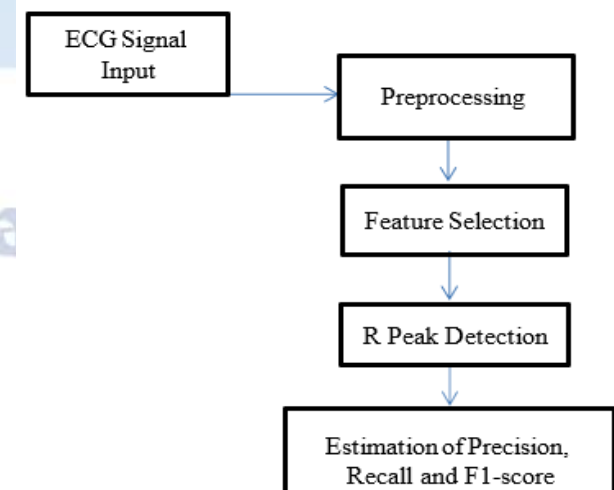


Figure : Block Diagram of Proposed Model

## B. Pre-Processing

The purpose of the preprocessing stage is to improve the signal-to-noise ratio and to highlight the R-peak morphological features. This stage uses frame splitting and bandpass filtering. The minimum heart rate value measured by standard ECG paper is 30 beats/min, i.e., the time of one heart beat is 2 seconds, so we set 3 seconds as a time period in order to ensure that there is at least one QRS complex wave in each frame. ECG signals are often contaminated with noise from various sources such as powerline interference, baseline wander, and muscle noise.

A Butterworth bandpass filter with 8th order cutoff frequency of 10Hz and 25Hz is used to filter out the noise.

## C. Feature Selection

After pre-processing, the wearable single-lead ECG signal will still have a large amount of noise interference, which very much affects the R-peak detection. In order to avoid the noise interference, we first observe the noise. The noise area is set as the area after the previous R-peak of 0.1 s and before the next R-peak of 0.1 s. The ECG signals in this area are considered as noise. After the observation of the noise, we can get the degree of noise interference at the signal amplitude level, so that accordingly we can set the R-peak detection threshold higher than the maximum noise value to avoid the impact of noise peaks. The noise area is illustrated in figure 4.3.

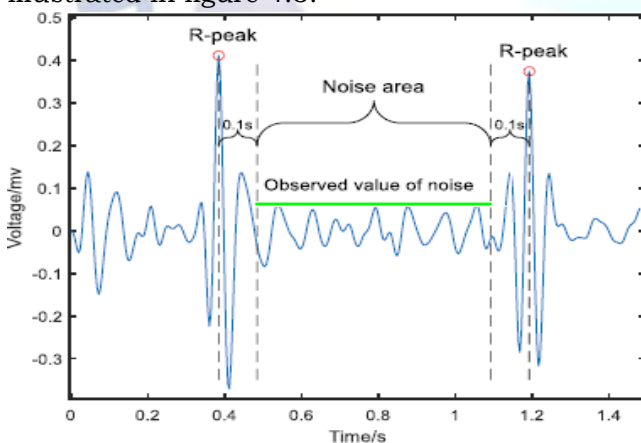


Figure 4.3 Noise area of ECG signal

### Thresholding of R-Peak:

In order to be able to distinguish between noise peaks and R-peaks at the signal amplitude level, the R-peak threshold should be located between the maximum noise peak amplitude and the R-peak amplitude. After each detection of an R-peak, the R-peak threshold suitable for detecting this R-peak can be inferred from the amplitude of this

R-peak and the magnitude of the noise observed in the previous moment, and this R-peak threshold is defined as the observed value of the R-peak threshold with the following equation.

$$R_{thobserved} = N_{yobserved} + K(RPy - N_{yobserved})$$

where  $N_{yobserved}$  denotes the observed value of the noise,  $RPy$  denotes the R-peak amplitude,  $R_{thobserved}$  denotes the observed value of the R-peak threshold, and  $K$  denotes the interval coefficient between the R-peak amplitude and the observed value of the noise, which is taken as 0.5 in order to balance the influence of both. The observed value of the current R-peak threshold indicates the most suitable threshold size for the current R-peak detection.

### Holt's exponential smoothing (HES) algorithm

Due to the interference of dynamic strong noise, the R-peak detection algorithm based on fixed empirical thresholds cannot adapt to a variety of complex situations, so some thresholds and parameters need to be updated. However, the thresholds are updated mainly based on the R-peaks detected in the past and some observed quantities, so the threshold updating method is very important for the ability to x the observation errors and the response speed to the changes of the observed quantities. The Holts exponential smoothing model is chosen as the adaptive threshold update method in this project. Holt's exponential smoothing (HES) is a time series forecasting method that extends simple exponential smoothing to capture trends in the data. It involves two equations: one for the level and one for the trend. Holt's method is particularly useful for data with a linear trend.

• The keys components of HES are:

a. Level ( $lt$ )-

The smoothed value of the series at time  $t$ . The ( $lt$ ) is given as;

$$(lt) = (yt) + (1 - \alpha)(lt-1 + bt-1)$$

Here,  $yt$  is the actual value of time;  $\alpha$  is the smoothing parameter for the level  $0 < \alpha < 1$ ;

$lt-1$  is the level at time  $t-1$ ;  $bt-1$  is the trend at time  $t-1$ ;

b.Trend ( $bt$ )-

The estimated trend (slope) at time  $t$  and is given by,

$$bt = (lt - lt-1) + (1 - \beta)bt-1$$

Where  $\beta$  is the smoothing parameter for the level  $0 < \beta < 1$ ;  $t - lt-1$  is the difference between the current and previous level.

Compute the time interval between the consecutive R peaks for smoothing of threshold level.



#### D. R- Peak Detection

The purpose of the peak detection stage is to identify the R-peaks as accurately as possible. In this work, the Holts exponential smoothing model is used to update the R-peak threshold and predict the noise and RR interval. Based on the current amplitude variation of R peaks and noise and the slow change of RR interval, the design is carried out at the signal amplitude level and

RR interval time level to finally achieve robust detection of R peaks under dynamic strong noise interference.

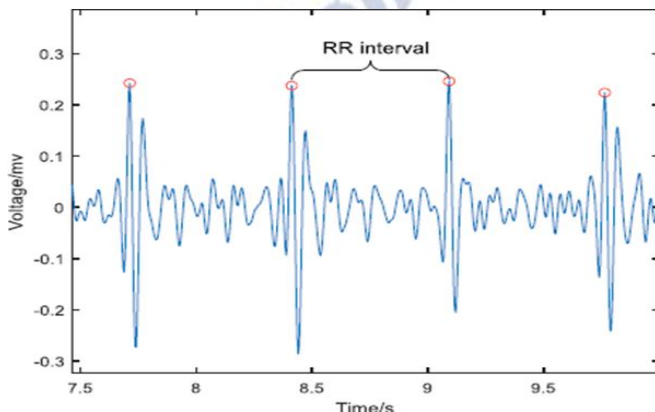


Figure 4.4 RR interval in ECG Signal

#### Peak Detection Based on R-peak Threshold

In this stage, the peak detection based on R-peak threshold is first performed for the ECG signal in this frame, and the RR interval threshold is set to 0.2 seconds. Then the detected local peaks are compared in size with the R-peak threshold in chronological order, and the local peak point that is larger than the R-peak threshold, we consider it as an R-peak. After each R- peak detection, this R-peak magnitude and noise magnitude are used to correct the R-peak threshold.

#### Abnormal State Backtracking Judgement

The RR interval changes slowly, and this feature is used to fill the missed peaks and eliminate the wrong R peaks. First, the RR interval of the next moment is predicted based on the past RR interval, and then the detected RR interval value is compared with the predicted RR interval value to determine whether the current RR interval is abnormal, and analyze whether this abnormality is caused by the wrong R peak or the missed R peak, and finally fill the missed R peak and delete the wrong R peak correspondingly.

Step1: RR interval prediction based on Holt's exponential smoothing model.

Step2: RR interval abnormality judgment.

#### Abnormal R Peak Detection

When the difference between the current RR interval and the predicted RR interval value is greater than 0.4 times the predicted RR interval value, it is determined that the current detection R peak may be abnormal. When the current RR interval is larger, it indicates that the cause of RR interval abnormality may be caused by R-peak missed detection, and when the predicted RR interval value is larger, it indicates that the cause of RR interval abnormality may be caused by R-peak misdetection. Then the size of the next four RR intervals is tracked and recorded simultaneously, and the size of the previous four RR intervals are found out, with the purpose of using more before-and-after correlation information to accurately determine the R-peak abnormalities based on the smoothness of RR interval changes.

#### Filling of Undetected R-peaks

After determining that the cause of the RR interval anomaly is caused by the missed R- peak, the operation of filling the missed R peaks is started. The operation is to find the local peak point between the previous R-peak and the current R-peak that is larger than the predicted noise amplitude, and select the peak point with the largest amplitude as the missed R-peak, without considering other cases.

## RESULTS AND DISCUSSIONS

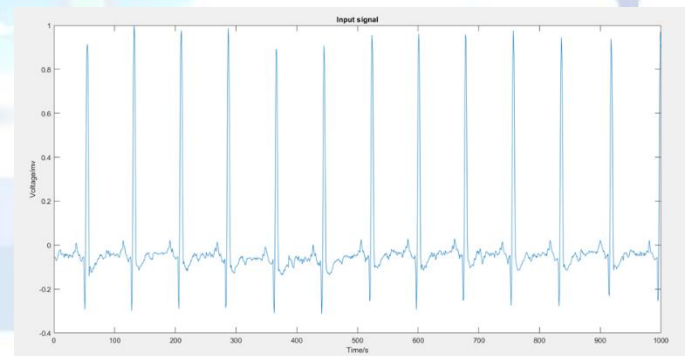


Fig 6.1 Original unfiltered input signal.

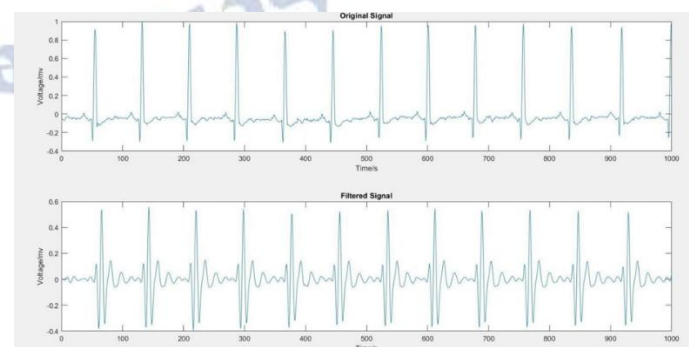


Fig 6.2 Original signal with filtered signal.

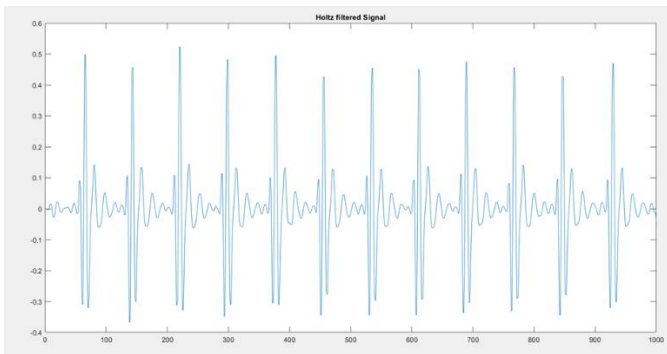


Fig 6.3 Holtz filtered signal.

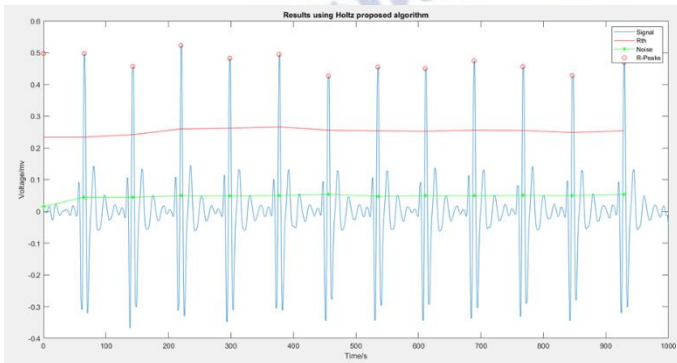


Fig 6.4 Results using Holtz Exponential Algorithm.

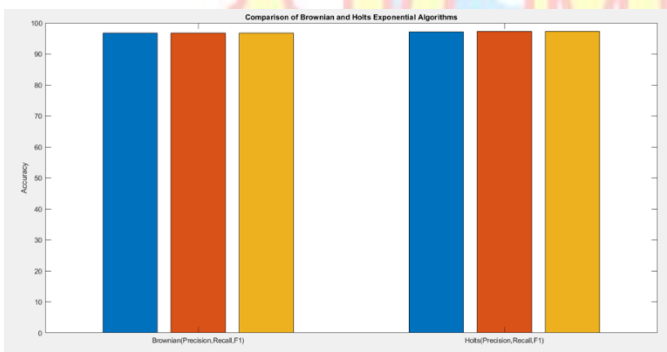


Fig 6.5 Comparison of Holtz Exponential Algorithm with another Exponential Algorithm.

## 6.2 Results and Discussion

The evaluation metrics used to assess the overall performance of the proposed method are precision, recall and F1 score, which are calculated as follows.

$$Precision = \frac{TP}{TP+FP}$$

$$Recall = \frac{TP}{TP+FN}$$

$$F1 - score = \frac{2 * Precision * Recall}{Precision + Recall}$$

where TP indicates the number of R-peaks that were correctly identified as R-peaks, FP indicates

the number of R-peaks that were incorrectly identified as R-peaks, and FN indicates the number of R-peaks that were not detected.

Method	Precision	Recall	F1 score
Holtz Exponential Smoothing	97.2	97.2	97.2
Brown Exponential Smoothing	96.7	96.8	96.7

Table 6.1: Comparison of parametric values.

## CONCLUSIONS

In this project an adaptive R-peak detection algorithm based on Holt's exponential smoothing model is proposed to address the problem of R-peak detection due to dynamic strong noise interference in wearable ECG bracelets. First, three features are selected based on the morphological characteristics and occurrence pattern of R peaks in wearable single-lead ECG signals, and the R peaks are detected at the signal amplitude level and time level to efficiently distinguish between noise peaks and R peaks. Then, the threshold parameters were updated according to the variation pattern of the selected features, using the Brown exponential smoothing model with its fast response speed to changes in the observed quantities and its ability to smooth out the observation errors. Finally, the accuracy of R-peak detection is further improved by backtracking the judgement method to eliminate the wrongly detected R-peaks and fill in the missed R-peaks.

## Future Scope

The R-peak detection technique developed in this project has the potential to be further enhanced and expanded in various ways. Implementing optimization algorithms, such as machine learning or deep learning, can improve the accuracy and robustness of R-peak detection. Additionally, this technique can be adapted for use with other medical signals, such as EEG, EMG, or blood pressure signals, and explored for use in other medical applications, including arrhythmia detection or cardiovascular disease diagnosis. A real-time implementation of the R-peak detection algorithm can be developed for clinical settings, and its integration with wearable devices or mobile health applications can be explored. Furthermore, clinical validation studies can be conducted to evaluate the performance of the R-peak detection algorithm in real-world clinical settings, and its



potential as a diagnostic tool for cardiovascular diseases can be investigated. Ultimately, the R-peak detection algorithm can be integrated with other diagnostic tools, such as echocardiography or cardiac MRI, to enhance its diagnostic capabilities.

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