

# Comparison between WESNR and Improved WESNR for Mixed Noise Removal

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## ABSTRACT

Mixed noise removal from natural images is a challenging task since the noise distribution usually does not have a parametric model and has a heavy tail. One typical kind of mixed noise is additive white Gaussian noise (AWGN) coupled with impulse noise (IN). Many mixed noise removal methods are detection based methods. They first detect the locations of IN pixels and then remove the mixed noise. However, such methods tend to generate many artifacts when the mixed noise is strong. In this paper, we propose a simple yet effective method, namely weighted encoding with sparse nonlocal regularization (WESNR), for mixed noise removal. WESNR method achieves leading mixed noise removal performance in terms of both low quantitative measures and low visual quality. This drawback can be overcome by considering the improved WESNR i.e Color-to-gray (C2G) image conversion is the process of transforming a color image into a grayscale one. Despite its wide usage in real-world applications, little work has been dedicated to compare the performance of C2G conversion algorithms. Subjective evaluation is reliable but is also inconvenient and time consuming. Here, we make one of the first attempts to develop an objective quality model that automatically predicts the perceived quality of C2G converted images. Inspired by the philosophy of the structural similarity index, we propose a C2G structural similarity (C2G-SSIM) index, which evaluates the luminance, contrast, and structure similarities between the reference color image and the C2G converted image. The three components are then combined depending on image type to yield an overall quality measure. Experimental results show that the proposed C2G-SSIM index has close agreement with subjective rankings and significantly outperforms existing objective quality metrics for C2G conversion.

**Index Terms :** Mixed noise removal, weighted encoding, nonlocal, sparse representation, Image quality assessment, color-to-gray conversion, perceptual image processing, structural similarity

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## I. INTRODUCTION

Bilateral filter (BF) is a well-known nonlinear filter, which preserves the information about the edges by estimating the denoised pixel as the neighboring pixels weighted average, but the weights are influenced by spatial and intensity similarity. An extension for the BF is non local means (NLM) filtering algorithm, in which the denoised pixel is estimated as the weighted average

of the all its standardized pixels of an original image and these weights are influenced by the similarity between them. BM3D approach has been proposed in by combining the similar non local patches into a 3D cube and applying transform based shrinkage.

Then after, by using these similar patches and grouping them into a matrix then applied principle component analysis (PCA) to denoise the AWGN image, which is known as LPG-PCA and it has been

proposed in. In recent years, an attractive attention has been made in image restoration and denoising algorithms by introducing dictionary learning and sparse representation schemes. The work proposed in initiates the dictionary learning from natural images to remove the AWGN and denoise the corrupted image using k- singular value decomposition (K-SVD). However, the mixture of both AWGN and IN increases the difficulties and makes much more complex to denoise the images. By proposing a mixed noise removal algorithm using median based signal dependent rank ordered mean (SDROM), but it produces better artifacts often. IN detection has been done by integrating the trilateral filter (TF) with absolute difference of rank order (ROAD) statistics into the BF.

COLOR-TO-GRAY (C2G) image conversion [1], also referred to as decolorization, has been widely used in real-world applications including black-and-white printing of color images, aesthetic digital black-and-white photography, and preprocessing in image processing and machine vision systems. Since color is fundamentally a multi-dimensional phenomenon described by the perceptual attributes of luminance, chroma and hue [2], C2G image conversion, which pursues a 1D representation of the color image, inevitably causes information loss. The goal of C2G conversion is to preserve as much visually meaningful information about the reference color images as possible, while simultaneously produce perceptually natural and pleasing grayscale images.

## II. EXISTING METHOD

### 2.1 WESNR Method

Denote an image by  $x \in R^p$ . We let  $x_m = R_m \in R^p$  as referenced in [13] be a patch size of stretched image vector, where  $R_m$  is the extracting patch  $x_m$  matrix operator at location  $m$ . As given in [37], we considered sparse representation theory to find the over-complete dictionary  $\Phi = [\Phi_1; \Phi_2; \dots; \Phi_p] \in R^{p,q}$  to sparsely code  $x_m$  where  $\phi_j \in R^p$  is the  $j^{th}$  atom of  $\Phi$ . The representation of  $x_i$  over the learning dictionary  $\Phi$  can be expressed as follows:

$$x = \Phi \alpha \quad (1)$$

Where  $\alpha$ =set of all coding vectors  $\alpha_i$

The main objective of de-noising an image is to estimate the desired image  $\hat{x}$  from  $y$  over the  $\Phi$ .

Then the encoding model can be done using

$$\hat{x} = \arg \min_x \|y - x\|^2 + \lambda \cdot R(x) \quad (2)$$

By substitute the eq. (2) in above equation, we can obtain the encoding model

$$\hat{x} = \arg \min_x \|y - \Phi \alpha\|_2^2 + \lambda \cdot R(\alpha) \quad (3)$$

Where  $R(\alpha)$  denotes some regularization term that imposed on  $\alpha$  and  $\lambda$  is a parameter of regularization. The specific form of  $R(x)$  depends on the employed image priors.

### ALGORITHM: MIXED NOISE REMOVAL BY WESNR

- **Input:** Take Corrupted Image  $y$ ;
- Initialize  $e$  by Equation  $e(0)=y-x(0)$ ; and then Initialize  $W$  By Equation  $W_{ii} = \exp(-a \cdot e_{ii}^2)$ ; Initialize Nonlocal coding Vector  $\mu$  to zero and Generate Dictionary  $\Phi$ ;
- **Loop :** Iterate on  $k=1,2,3 \dots K$ ;
- 1. First Compute  $\alpha(k)$  by  $\alpha(k+1) = (\Phi^T W \Phi + (k+1))^{-1} (\Phi^T W y - \Phi^T W \phi \mu) + \mu$ ;
- 2. Compute  $x(k) = \Phi \alpha(k)$
- 3. Update the nonlocal coding vector  $\mu$ ;
- 4. Evaluate the residual  $e(k)=y-x(k)$ ;
- 5. Compute the weights  $W$  by  $e(k)$  using equation  $W_{ii} = \exp(-a \cdot e_{ii}^2)$ ;

**End**

## III. EXISTING SIMULATION RESULTS

Experimental results have been done in MATLAB 2013a version with 4GB RAM and i3 processor. To verify the performance of the proposed image denoising model using the WESNR algorithms with the existing denoising techniques such as AMF, NLM, we tested it with various images such as satellite, biometric, medical and more even natural images with different texture structures.

All the test images are intensity or gray-scale images with the pixels ranging from 0 to 255. We first discuss the parameter setting in our algorithm, and then compare the performance of proposed and its region based variants. Finally, experiments are conducted to validate its performance in comparison with the state-of-the-art denoising algorithms.

Several parameters are used in our proposed algorithm and they all can be fixed easily with our experience. First, the parameter  $\tau$  is the termination of iteration controlling. To balance the denoising results, we set it to 0.003. In eq. (12), the parameter that is used to control the weights decreasing rate w.r.t.  $e$ , this can be set it to 0.0008. Fig1 shows that the mixed noise removal from the Lena512.tif, it displayed all the denoised images obtained by using conventional AMF, NCSR, and proposed WESNR algorithms.



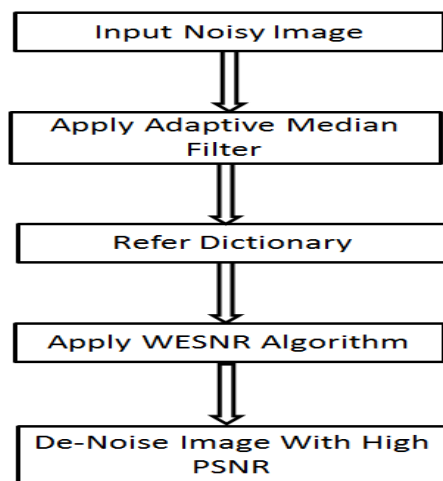


Fig 1: Block diagram of the algorithm

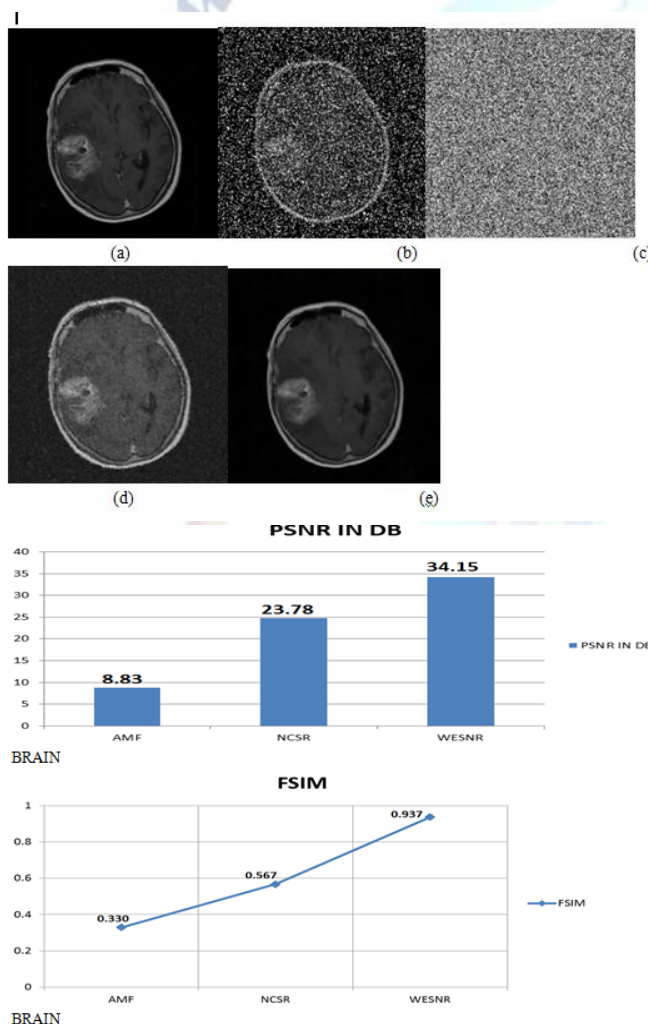


Fig 2 : (a) Original image, (b) AWGN (c) Mixed noise with sigma=5 and (d) AMF filtered image (e) WESNR method

We can observe that the visual quality of the proposed scheme is very good and much improved over the existing techniques

#### IV. EXISTING COLOR-TO-GRAY WORK

Most existing C2G conversion algorithms seek to preserve color distinctions of the input color image

in the corresponding grayscale image with some additional constraints, such as global consistency and grayscale preservation. As one of the first attempts, Bala and Eschbach introduced high frequency chrominance information into the luminance channel so as to preserve distinctions between adjacent colors [1]. This algorithm tends to produce artificial edges in the C2G image. Rasche et al. incorporated contrast preservation and luminance consistency into a linear programming problem, where the difference between two gray values is proportional to that between the corresponding color values [3]. Gooth et al. transformed the C2G problem into a quadratic optimization one by quantifying the preservation of color differences between two distinct points in the grayscale image [4]. By using predominant component analysis, Grundland and Dodgson computed prevailing chromatic contrasts along the predominant chromatic axis and used them to compensate the luminance channel [5]. A coloroid systembased [21] C2G conversion algorithm is proposed in [6], where color and luminance contrasts form a gradient field and enhancement are achieved by reducing the inconsistency of the field. Smith et al. developed a two-step approach that first globally assigns gray values incorporating the Helmholtz-Kohlrausch color appearance effect [22] and then locally enhances the grayscale values to reproduce the original contrast [8]. The algorithm performs the best based on Cadik's subjective experiment [16]. A mass-spring system is introduced in [9] to perform C2G conversion. Kim's method adopts a nonlinear global mapping to perform robust decolorization [7]. Song et al. incorporated spatial consistency, structure information and color channel perception priority into a probabilistic graphic model and optimized the model as an integral minimization problem [10]. Lu's method attempts to maximally preserve the original color contrast by minimizing a bimodal Gaussian function [11], [14]. However, contrast preservation or enhancement does not necessarily lead to perceptual quality improvement, but may produce some unnatural images due to luminance inconsistency [11]. Song et al. [12] and Zhou et al. [13] independently revisited a simple C2G conversion model that linearly combines RGB channels. The weights are determined based on predefined contrast preservation and saliency preservation measures. More recently, Eynard et al. assumed that if a color transformed image preserves the structural information of the original

image, the respective Laplacians are jointly diagonalizable or equivalently commutative. Using Laplacians commutativity as the criterion, they minimized it with respect to the parameters of a color transformation to achieve optimal structure preservation [15].

#### 4.1 The SSIM Index

Suppose  $\mathbf{x}^1$  and  $\mathbf{y}^1$  are local image patches taken from the same location of two images being compared, the local SSIM index computes three components: the luminance similarity  $l(\mathbf{x}^1, \mathbf{y}^1)$ , contrast similarity  $c(\mathbf{x}^1, \mathbf{y}^1)$  and structure similarity  $s(\mathbf{x}^1, \mathbf{y}^1)$

Finally, the three measures are combined to yield the SSIM index

$$\text{SSIM}(\mathbf{x}', \mathbf{y}') = l(\mathbf{x}', \mathbf{y}')^\alpha \cdot c(\mathbf{x}', \mathbf{y}')^\beta \cdot s(\mathbf{x}', \mathbf{y}')^\gamma, \quad (4)$$

the simplified SSIM index that is widely used in practice is given by

$$\text{SSIM}(\mathbf{x}', \mathbf{y}') = \frac{(2\mu_{x'}\mu_{y'} + C_1)(2\sigma_{x'y'} + C_2)}{(\mu_{x'}^2 + \mu_{y'}^2 + C_1)(\sigma_{x'}^2 + \sigma_{y'}^2 + C_2)}. \quad (5)$$

It is widely recognized that SSIM is better correlated with the human visual system (HVS) than MSE [17], [18], [23] and has a number of desirable mathematical properties [24] for optimization purposes [23], [25]

### V. IMPROVED WESNR

The diagram of the proposed Improved WESNR C2G-SSIM index is shown in Fig. 3. First, we transform both the reference color image

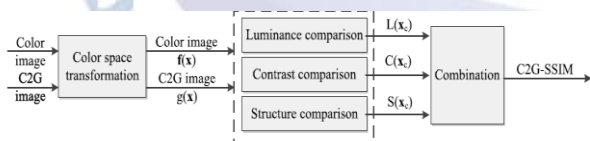


Fig 3. Three-stage structure of Improved WESNR

and the test C2G image into a color space, where the color representation is better matched to the HVS. Next, we measure luminance, contrast and structure distortions to capture perceived quality changes introduced by C2G conversion. Finally, we combine the above three measurements into an overall quality measure based on the type of image content.

Structure Similarity: The structure measure takes a similar form as in SSIM

$$S(\mathbf{x}_c) = \frac{\sigma_{fg}(\mathbf{x}_c) + C_3}{\sigma_f(\mathbf{x}_c)\sigma_g(\mathbf{x}_c) + C_3}, \quad (5)$$

where  $C_3$  is also a small positive constant introduced in both the denominator and numerator.

#### Overall Quality Measure

The luminance measure  $L(\mathbf{x}_c)$ , contrast measure  $C(\mathbf{x}_c)$  and structure measure  $S(\mathbf{x}_c)$  describe three different aspects of the perceptual quality of the C2G image.  $L(\mathbf{x}_c)$  quantifies the luminance consistency, whose importance in assessing the quality of C2G images varies according to the nature of image source, while  $C(\mathbf{x}_c)$  and  $S(\mathbf{x}_c)$  are more related to structural detail preservation of the C2G conversion. Specifically, for photographic images (PI) of natural scenes, human observers have strong prior knowledge about the luminance information.

Whether such information is maintained in C2G images is well reflected by the luminance measure  $L(\mathbf{x}_c)$ . On the other hand, for synthetic images (SI) generated via computer graphics, human observers have little prior knowledge about the luminance of the synthetic objects, and thus  $L(\mathbf{x}_c)$  is less relevant. To justify the above intuition, we carried out an informal test.



(a)



(b)

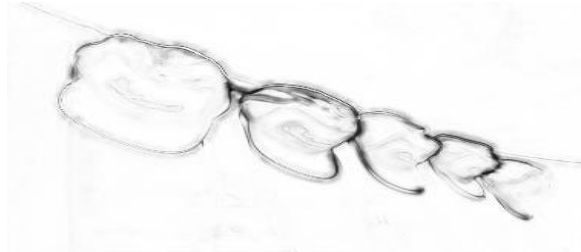


(c)

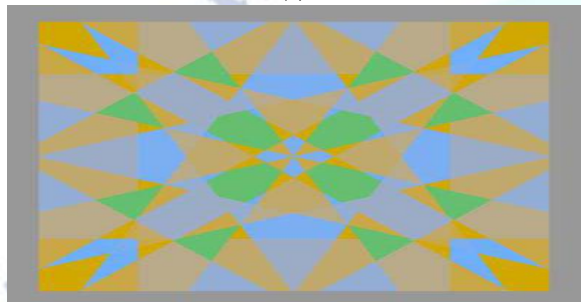




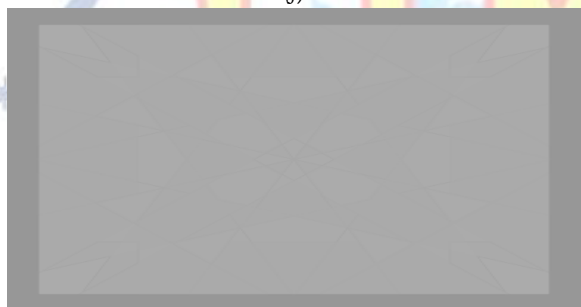
(d)



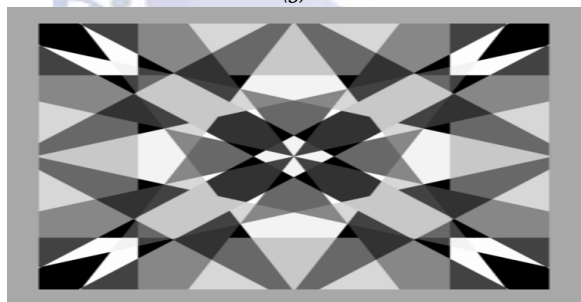
(e)



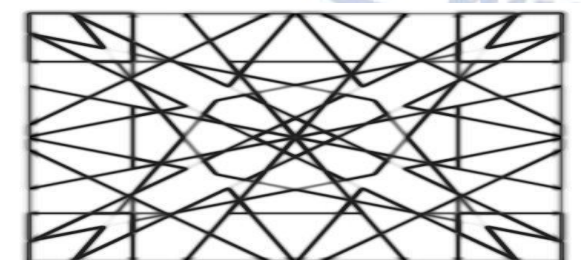
(f)



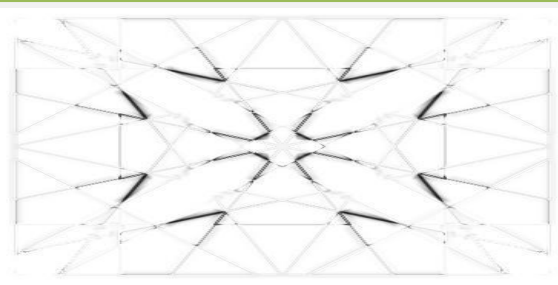
(g)



(h)



(i)



(j)

Fig. 4. C2G images and their I-WESNR-C2G-SSIM index maps. (a) and (f) are reference color images. (b), (c), (g) and (h) are C2G images created by the methods in [3] and [8], CIEY and [11], respectively. (d), (e), (i) and (j) are the corresponding I-WESNR-C2G-SSIM maps of (b), (c), (g) and (h), respectively. In all I-WESNR-C2G-SSIM index maps, brighter indicates better quality.

## VI. CONCLUSION AND FUTURE SCOPE

Here, previous denoising model for mixed noise using weighted encoding sparse non local regularization (WESNR). The mixed noise distribution i.e., Gaussian noise mixed up with random impulse noise is much more irregular over alone Gaussian noise also often has a heavy tail, and causes serious problems in image processing applications. To address this issue, we adopted a novel algorithm that removes the mixed noise more effectively and improves denoising system performance by increasing the PSNR and FSIM. But in FSIM over all image quality not upto the mark Proposed algorithm achieves promising results in enhancing the mixed noisy image while removing AWGN and IN. Most of the state-of-the-art denoising algorithms are based on the either local sparsity or nonlocal self-similarity priors of natural images. Unlike them, our proposed scheme used a kind of global prior, which is adaptively estimated from the given colour reference image.our proposed algorithm i.e Improved WESNR C2G-SSIM evaluates luminance, contrast and structure similarities between the reference color image and the C2G image. Image type dependent combination is then applied to yield an overall quality measure

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