International Journal for Modern Trends in Science and Technology
Volume: 03, Issue No: 06, June 2017
ISSN: 2455-3778
http://www.ijmtst.com

Noninner Automorphisms of Finite p-Groups Going Away the Center Element wise mounted

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To Cite this Article
D.Aruna and K.Srinivasa Rao, "Noninner Automorphisms of Finite p-Groups Going Away the Center Element wise mounted", *International Journal for Modern Trends in Science and Technology*, Vol. 03, Issue 06, June 2017, pp. 178-179.

ABSTRACT

A long conjecture asserts that each finite nonabelian p-group admits a noninner automorphism of order p. Let G be a finite nonabelian p-group. it's celebrated that if G is regular or of nilpotency category a pair of or the electric switch subgroup of G is cyclic, or G/Z(G) is powerful, then G has a noninner automorphism of order p effort either the middle Z(G) or the Frattini subgroup Φ(G) of G elementwise fastened. during this note, we tend to prove that the latter noninner automorphism will be chosen thus that it leaves Z(G) elementwise fastened.

I. INTRODUCTION

`One of the foremost wide famous, though nontrivial, properties of finite p-groups of order larger than p is that they invariably have a noninner automorphism α of p-power order. This reality was 1st proved by Gasch¨utz in 1966. Schmid extended Gasch¨utz's result by showing that if G may be a finite nonabelian p-group, then the automorphism α are often chosen to act trivially on the middle. A longstanding conjecture that had been raised even before Gasch¨utz's result's the subsequent

Conjecture 1. Each finite nonabelian p-group admits a noninner automorphism of order p.

Indeed, in 1964 Liebeck established that if p may be an odd prime and G is a finite p-group of sophistication two then G contains a noninner automorphism of order p acting trivially on the Frattini subgroup Φ(G). The corresponding result for 2-groups is fake normally, as Liebeck himself created AN example of a 2-group G of sophistication two with the property that each one automorphisms of order 2 exploit Φ(G) elementwise mounted ar inner. By a cohomological results of Schmid [9], it follows that finite regular nonabelian p-groups admit a noninner automorphism exploit the Frattini subgroup elementwise mounted. Deaconescu and Silberberg [4] established that if CG(Z(Φ(G))) 6= Φ(G), then the noninner automorphism are often chosen to act trivially on Φ(G). therefore the most results of [4] reduced the verification of Conjecture 1 to finite nonabelian p-groups G satisfying the condition CG(Z(Φ(G))) = Φ(G). In [1, 2, 3] it is proved that if G may be a finite nonabelian p-group of sophistication at the most three or G/Z(G) is powerful, then G has a noninner automorphism of order p exploit either Φ(G) or Ω1(Z(G)) elementwise mounted. Jamali and Viseh [6] established that each nonabelian finite 2-group with cyclic electrical switch subgroup contains a noninner automorphism of order 2 exploit either Φ(G) or Z(G) elementwise mounted. they need conjointly ascertained that the results of [1, 2] are often improved, that is, if G is of nilpotency category two or G/Z(G) is powerful, then G contains a noninner automorphism of order p exploit either the middle Z(G) or Frattini subgroup elementwise mounted. thus the subsequent result holds

Proposition 1.1. Let G be a finite nonabelian p-group satisfying one of the following conditions:
(1) G is regular;
(2) G is nilpotent of class 2;
(3) the commutator subgroup of G is cyclic;
(4) G/Z(G) is powerful.

Then G has a noninner automorphism of order p leaving either Z(G) or Φ(G) elementwise fixed. The main result of our paper is the following.

Theorem 1.2. Let G be a finite nonabelian p-group satisfying one of the following conditions:
(1) G is regular;
(2) G is nilpotent of class 2;
(3) the commutator subgroup of G is cyclic;
(4) G/Z(G) is powerful.
Then G has a noninner automorphism of order p leaving Z(G) elementwise fixed. 2. Proof of the main result

II. Proof of the main result

We need the following result which may be well-known. We prove it for the reader's convenience. Lemma 2.1. Let G be any finite p-group. Then G = AH for some subgroups A and H such that A ≤ Z(G) and Z(H) ≤ Φ(H). Proof. We prove Lemma by induction on |G|. If G is abelian then the assertion is clear, take A = G and H = 1. Now let G be a finite nonabelian p-group and assume that the assertion holds for all p-groups of order less than |G|. Moreover we may assume that Z(G) 6≤ Φ(G), otherwise one may take A = 1 and H = G to complete the proof. Thus there exist some element a ∈ Z(G) and a maximal subgroup M of G such that a 6∈ M. By induction hypothesis M = BH for some subgroups B and H of M such that B ≤ Z(M) and Z(H) ≤ Φ(H). Let A = ha, Bi. Therefore A ≤ Z(G) and G = AH. This completes the proof.

Remark 2.2 ([4, Remark 4.]). Let G be a central product of subgroups A and B; i.e., G = AB and [A, B] = 1. Suppose that α ∈ Aut(A) and β ∈ Aut(B) agree on A∩B. Then α and β admit a common extension γ ∈ Aut(G). In particular, if A has a noninner automorphism of order p which fixes Z(A) elementwise, then G has a noninner automorphism of order p leaving both Z(A) and B elementwise fixed. We are now ready to prove Theorem 1.2. Proof of Theorem 1.2. Let G be a finite nonabelian p-group. By Lemma 2.1, we have G = AH for some subgroups A and H of G such that A ≤ Z(G) and Z(H) ≤ Φ(H). If G is regular, or of nilpotency class 2, or with cyclic commutator subgroup, then so is H. Now, suppose that G/Z(G) is powerful. If p > 2, then H0Z(G)/Z(G) ≤ G0Z(G)/Z(G) ≤ GpZ(G)/Z(G). Thus H0 ≤ GpZ(G) = HpZ(G), since Gp = ApHp . Now if c ∈ H0 , then c = ba for some b ∈ Hp and a ∈ Z(G). But b −1 c = a ∈ Z(H). Therefore H0 ≤ HpZ(H) and this means that H/Z(H) is powerful. A similar argument shows that H/Z(H) is powerful for p = 2. Then, by Proposition 1.1, H has a noninner automorphism of order p fixing Z(H) elementwise.

Now it follows from Remark 2.2 that G has a noninner automorphism of order p leaving AZ(H) = Z(G) elementwise fixed. This completes the proof. We finish the paper with the following conjecture. Conjecture 2. Every finite nonabelian p-group admits a noninner automorphism of order p leaving the center element wise fixed.

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