

# Effect of Radiation on Combined Heat and Mass Transfer Flow of a Viscous Dissipative Fluid through a Porous Medium in A Non-Uniformly Heated Vertical Channel

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## ABSTRACT

We analyze the effect of radiation on mixed convective heat and mass transfer flow of a viscous fluid, incompressible fluid through a porous medium in a vertical channel bounded by flat walls. A non-uniform temperature is imposed on the walls and the concentration on these walls is taken to be constant. The viscous dissipation is taken in to account in the energy equation. Assuming the slope of the boundary temperature to be small. We solve the governing momentum, energy and diffusion equations by a perturbation technique. The velocity, the temperature, the concentration, the shear stress and the rate of heat transfer have been analyzed for different variations of the governing parameters. The dissipative effects on the flow, heat and mass transfer are clearly brought out.

**KEYWORDS** — Heat and Mass Transfer, Porous Medium, Viscous Fluid, Non-uniformly heated vertical channel.

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## 1. INTRODUCTION

Flows which arise due to the interaction of the gravitational force and density differences caused by the simultaneous diffusion of thermal energy and chemical species, have many applications in geophysics and engineering. Such thermal and mass diffusion plays a dominant role in a number of technological and engineering systems. The effect of viscous dissipation has been studied by Nakayama and Pop(10) for steady free convection boundary layer over a non-isothermal bodies of arbitrary shape embedded in porous media. They used integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. Prasad(12) has discussed the effect of dissipation on the mixed convective heat and mass transfer flow of a viscous

fluid through a porous medium in a vertical channel bounded by flat walls. Gehart(4), Gebhart and Mollen dorf(5) have shown that that viscous dissipation heat in the natural convective flow is important when the flow field is of extreme size or at extremely low temperature or in high gravitational field. Saffman (14) employing statistical method derived a general governing equation for the flow in a porous medium which takes into account the viscous stress. This analysis of heat transfer in a viscous heat generating fluid also important in engineering processes pertain to flow in which a fluid supports an exothermal chemical or nuclear reaction or problems concerned with dissociating fluids(8,9). Barletta(1) has pointed out that relevant effect of viscous dissipation on the temperature profiles and on the

Nusselt numbers may occur in the fully developed forced convection in tubes. The natural convection from horizontal cylinder embedded in a porous media has been studied by Fand and Brucker(3). They used integral method to show that the viscous dissipation results in lowering the level of the heat transfer rate from the body. Costa(2) has analyzed a natural convection in enclosures with viscous dissipation. They Rossidi schio(13) and Israel et al (7) have studied the effect of viscous dissipation on the convective flows past on infinite vertical plates and through vertical channels and Ducts. The combined effect of thermal and mass diffusion in channel flows has been studied in the recent times by (11,6).

## II. DERIVATION OF THE EQUATIONS

We analyze the steady motion of viscous, incompressible fluid through a porous medium in a vertical channel bounded by flat walls which are maintained at a non-uniform wall temperature in the presence of a constant heat source and the concentration on these walls are taken to be constant. The Boussinesq approximation is used so that the density variation will be considered only in the buoyancy force. The viscous, Darcy dissipations and the joule heating are taken into account in the energy equation. Also the kinematic viscosity  $\nu$ , the thermal conducting  $k$  are treated as constants. We choose a rectangular Cartesian system  $O(x, y)$  with  $x$ -axis in the vertical direction and  $y$ -axis normal to the walls. The walls of the channel are at  $y = \pm L$ . The equations governing the steady flow, heat and mass transfer are

Equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.1)$$

Equation of linear momentum:

$$\rho_e \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho g - \left( \frac{\mu}{k} \right) u$$

$$\rho_e \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \left( \frac{\mu}{k} \right) v \quad (2.3)$$

Equation of Energy:

$$\rho_e C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q + \mu \left( \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right) + \left( \frac{\mu}{\lambda k} + \sigma \mu_e^2 H_o^2 \right) (u^2 + v^2) - \frac{\partial (q_r)}{\partial y} \quad (2.4)$$

Equation Diffusion:

$$\left( u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_1 \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + k_{11} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (2.5)$$

Equation of State:

$$\rho - \rho_e = -\beta \rho_e (T - T_e) - \beta^* \rho_e (C - C_e) \quad (2.6)$$

where  $\rho_e$  is the density of the fluid in the equilibrium state,  $T_e, C_e$  are the temperature and Concentration in the equilibrium state,  $(u, v)$  are the velocity components along  $O(x, y)$  directions,  $p$  is the pressure,  $T, C$  are the temperature and Concentration in the flow region,  $\rho$  is the density of the fluid,  $\mu$  is the constant coefficient of viscosity,  $C_p$  is the specific heat at constant pressure,  $\lambda$  is the coefficient of thermal conductivity,  $k$  is the magnetic permeability of the porous medium,  $\beta$  is the coefficient of thermal expansion,  $\beta^*$  is the coefficient of expansion with mass fraction,  $D_1$  is the molecular diffusivity,  $Q$  is the strength of the constant internal heat source,  $q_r$  is the radiative heat flux and  $k_{11}$  is the cross diffusivity

Invoking Roseland approximation for radiation

$$q_r = -\frac{4\sigma^* \partial(T'^4)}{3\beta_R \partial y}$$

Expanding  $T'^4$  in Taylor's series about  $T_e$  neglecting higher order terms()  $T'^4 \cong 4T_e^3 T' - 3T_e^4$

where  $\sigma^*$  is the Stefan-Boltzmann constant  $\beta_R$  is the Extinction coefficient.

In the equilibrium state

$$0 = -\frac{\partial p_e}{\partial x} - \rho_e g \quad (2.7)$$

Where  $p = p_e + p_D, p_D$  being the hydrodynamic pressure.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined as

$$Q = \frac{1}{2L} \int_{-L}^L u \, dy \quad (2.8)$$

The boundary conditions for the velocity and temperature fields are

$$u = 0, v = 0 \quad \text{on } y = \pm L$$

$$T - T_e = \gamma(\delta x / L) \quad \text{on } y = \pm L$$

$$C = C_1 \quad \text{on } y = -L$$

$$C = C_2 \quad \text{on } y = +L \quad (2.9)$$

In view of the continuity equation we define the stream function  $\psi$  as

$$u = -\psi_y, v = \psi_x$$



the equation governing the flow in terms of  $\psi$  are

$$\left[ \frac{\partial \psi}{\partial x} \frac{\partial (\nabla^2 \psi)}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial (\nabla^2 \psi)}{\partial x} \right] = \nu \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right)^2 - \beta g \frac{\partial T}{\partial y} - \beta^* g \frac{\partial C}{\partial y} - \left( \frac{\nu}{k} \right) \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \quad (2.10)$$

$$\rho_e C_p \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \lambda \nabla^2 \theta + Q + \mu \left( \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} \right)^2 \right) + \left( \frac{\mu}{k} \right) \left( \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right) + \frac{4}{3N_1} \frac{\partial^2 \theta}{\partial y^2} \quad (2.11)$$

$$\left( \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = D_1 \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + k_{11} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (2.12)$$

Introducing the non-dimensional variables in (2.10)- (2.12) as

$$(x', y') = (x, y)/L, (u', v') = (u, v)/U, \theta = \frac{T - T_e}{\Delta T_e}, C^* = \frac{C - C_1}{C_2 - C_1} \quad (2.13)$$

$$p' = \frac{p_D}{\rho_e U^2}, \gamma' = \frac{\gamma}{\Delta T_e}$$

(Under the equilibrium state)

$$\Delta T_e = T_e(L) - T_e(-L) = \frac{QL^2}{\lambda}$$

The governing equations in the non-dimensional form (after dropping the dashes) are

$$R \frac{\partial(\psi, \nabla^2 \psi)}{\partial(x, y)} = \nabla^4 \psi + \left( \frac{G}{R} (\theta_y + N C_y) - D^{-1} \nabla^2 \psi \right) \quad (2.14)$$

and the energy diffusion equations in the non-dimensional form are

$$P_1 R \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta + N_2 + \left( \frac{P_1 R^2 E_c}{G} \right) \left( \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \left( \frac{\partial^2 \psi}{\partial x^2} \right)^2 \right) + D^{-1} \left( \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right) \quad (2.15)$$

$$R S c \left( \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{S c S_o}{N} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \quad (2.16)$$

where

$$R = \frac{UL}{\nu} \quad (\text{Reynolds number}) \quad G = \frac{\beta g \Delta T_e L^3}{\nu^2} \quad (\text{Grashof number})$$

number)

$$P = \frac{\mu c_p}{k_1} \quad (\text{Prandtl number}), \quad D^{-1} = \frac{L^2}{k} \quad (\text{Darcy parameter}),$$

parameter),

$$E_c = \frac{\beta g L^3}{C_p} \quad (\text{Eckert number}) \quad N = \frac{\beta^* \Delta C}{\beta \Delta T}$$

(Buoyancy Number)

$$S_c = \frac{\nu}{D_1} \quad (\text{Schmidt number}) \quad S_o = \frac{k_{11} \beta^*}{\beta \nu} \quad (\text{soret parameter})$$

parameter)

$$N_1 = \frac{\beta_R \lambda}{4 \sigma_e T_e^3} \quad (\text{Radiation parameter})$$

$$N_2 = \frac{3N_1}{3N_1 + 4} P_1 = P N_2$$

The corresponding boundary conditions are

$$\psi(+1) - \psi(-1) = 1$$

$$\frac{\partial \psi}{\partial x} = 0, \quad \frac{\partial \psi}{\partial y} = 0 \quad \text{at } y = \pm 1 \quad (2.17a)$$

$$\theta(x, y) = f(\delta x) \quad \text{on } y = \pm 1 \quad (2.17b)$$

$$C = 0 \quad \text{on } y = -1$$

$$C = 1 \quad \text{on } y = 1 \quad (2.17c)$$

$$\frac{\partial \theta}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0 \quad \text{at } y = 0 \quad (2.18)$$

The value of  $\psi$  on the boundary assumes the constant volumetric flow in consistent with the hypothesis (2.8). Also the wall temperature varies in the axial direction in accordance with the prescribed arbitrary function  $\gamma(x)$ .

### III. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convection flow due to non-uniform slowly varying temperature imposed on the boundaries. We introduce the transformation

$$\bar{x} = \delta x$$

With this transformation the equations (2.14) - (2.16) reduce to

$$R \delta \frac{\partial(\psi, F^2 \psi)}{\partial(x, y)} = F^4 \psi + \frac{G}{R} (\theta_y + N C_y) - D^{-1} F^2 \psi - \quad (3.1)$$

and the energy & diffusion equations in the non-dimensional form are

$$P_1 R \delta \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) = F^2 \theta + N_2 + \left( \frac{P_1 R^2 E_c}{G} \right) \left( \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + \delta^2 \left( \frac{\partial^2 \psi}{\partial x^2} \right)^2 \right) + (D^{-1}) \left( \delta^2 \left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2 \right) \quad (3.2)$$

$$\delta R S c \left( \frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} \right) = F^2 C + \frac{S c S_o}{N} F^2 \theta \quad (3.3)$$

where

$$F^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

for small values of the slope  $\delta$ , the flow develops slowly with axial gradient of order  $\delta$  and hence we take

$$\frac{\partial}{\partial x} \approx O(1)$$

We follow the perturbation scheme and analyze through first order as a regular perturbation problem at finite values of  $R, G, P, Sc$  and  $D^{-1}$

Introducing the asymptotic expansions

$$\psi(x, y) = \psi_0(x, y) + \delta \psi_1(x, y) + \delta^2 \psi_2(x, y) + \dots$$

$$\begin{aligned} \theta(x, y) &= \theta_0(x, y) + \delta\theta_1(x, y) + \delta^2\theta_2(x, y) + \dots \\ C(x, y) &= C_0(x, y) + \delta C_1(x, y) + \delta^2 C_2(x, y) + \dots \end{aligned} \quad (3.4)$$

On substituting (3.4) in (3.1) - (3.3) and separating the like powers of  $\delta$  the equations and respective conditions to the zeroth order are

$$\psi_{0,yyyy} - M_1^2 \psi_{0,yy} = -\frac{G}{R}(\theta_{0,y} + NC_{0,y}) \quad (3.5)$$

$$\theta_{0,yy} = -\frac{P_1 R^2 Ec}{G} \psi_{0,yy}^2 - \frac{P_1 M_1^2 Ec}{G} \psi_{0,y}^2 \quad (3.6)$$

$$C_{0,yy} = 0 \quad (3.7)$$

With

$$\psi_{0(+1)} - \psi_{0(-1)} = 1, \quad \psi_{0,y} = 0, \quad \psi_{0,x} = 0 \quad \text{at } y = \pm 1 \quad (3.7a)$$

$$\theta_0(\pm 1) = \gamma(x) \quad \text{at } y = \pm 1, \quad C_0(-1) = 0, \quad C_0(+1) = 1 \quad (3.7b)$$

and to the first order are

$$\psi_{1,yyyy} - M_1^2 \psi_{1,yy} = -\frac{G}{R}(\theta_{1,y} + NC_{1,y}) + R(\psi_{0,y} \psi_{0,xy} - \psi_{0,x} \psi_{0,yy}) \quad (3.8)$$

$$\theta_{1,yy} = P_1 R(\psi_{0,x} \theta_{0,y} - \psi_{0,y} \theta_{0,x}) - \frac{P_1 Ec}{G}(R^2 \psi_{1,yy}^2 + M_1^2 \psi_{1,y}^2) \quad (3.9)$$

$$C_{1,yy} = RSc(\psi_{0,y} C_{0,x} - \psi_{0,x} C_{0,y}) \quad (3.10)$$

$$\psi_{1(+1)} - \psi_{1(-1)} = 0$$

$$\psi_{1,y} = 0, \quad \psi_{1,x} = 0 \quad \text{at } y = \pm 1 \quad (3.10)$$

$$\theta_1(\pm 1) = 0 \quad \text{at } y = \pm 1 \quad (3.11)$$

$$C_0(-1) = 0, \quad C_0(+1) = 0 \quad (3.12)$$

Assuming  $Ec \ll 1$  to be small we take the asymptotic expansions as

$$\psi_0(x, y) = \psi_{00}(x, y) + Ec \psi_{01}(x, y) + \dots$$

$$\psi_1(x, y) = \psi_{10}(x, y) + Ec \psi_{11}(x, y) + \dots$$

$$\theta_0(x, y) = \theta_{00}(x, y) + Ec \theta_{01}(x, y) + \dots$$

$$\theta_1(x, y) = \theta_{10}(x, y) + Ec \theta_{11}(x, y) + \dots$$

$$C_0(x, y) = C_{00}(x, y) + Ec C_{01}(x, y) + \dots$$

$$C_1(x, y) = C_{10}(x, y) + Ec C_{11}(x, y) + \dots$$

$$(3.13)$$

Substituting the expansions (3.13) in equations (3.5)-(3.12) and separating the like powers of  $Ec$  we get the following equations

$$\theta_{00,yy} = -1, \quad \theta_{00}(\pm 1) = f(\bar{x}) \quad (3.14)$$

$$C_{00,yy} = 0, \quad C_{00}(-1) = 0, \quad C_{00}(+1) = 1 \quad (3.15)$$

$$\psi_{00,yyyy} - M_1^2 \psi_{00,yy} = -\frac{G}{R}(\theta_{00,y} + NC_{00,y}), \quad (3.16)$$

$$\psi_{00(+1)} - \psi_{00(-1)} = 1, \quad \psi_{00,y} = 0, \quad \psi_{00,x} = 0 \quad \text{at } y = \pm 1$$

$$\theta_{01,yy} = -\frac{P_1 M_1^2}{G} \psi_{00,yy}^2 - \frac{P_1 R^2}{G} \psi_{00,y}^2, \quad \theta_{01}(\pm 1) = 0 \quad (3.17)$$

$$C_{01,yy} = 0, \quad C_{01}(\pm 1) = 0 \quad (3.18)$$

$$\psi_{01,yyyy} - M_1^2 \psi_{01,yy} = -\frac{G}{R}(\theta_{01,y} + NC_{01,y}), \quad (3.19)$$

$$\psi_{01(+1)} - \psi_{01(-1)} = 0,$$

$$\psi_{01,y} = 0, \quad \psi_{01,x} = 0 \quad \text{at } y = \pm 1$$

$$\theta_{10,yy} = RP_1(\psi_{00,y} \theta_{00,x} - \psi_{00,x} \theta_{00,y}), \quad \theta_{10}(\pm 1) = 0 \quad (3.20)$$

$$C_{10,yy} = RP_1(\psi_{00,y} C_{00,x} - \psi_{00,x} C_{00,y}), \quad C_{10}(\pm 1) = 0 \quad (3.21)$$

$$\psi_{10,yyyy} - M_1^2 \psi_{10,yy} = -\frac{G}{R}(\theta_{10,y} + NC_{10,y}) + R(\psi_{00,y} \psi_{00,xy} - \psi_{00,x} \psi_{00,yy}), \quad (3.22)$$

$$\psi_{10(+1)} - \psi_{10(-1)} = 0, \quad \psi_{10,y} = 0, \quad \psi_{10,x} = 0 \quad \text{at } y = \pm 1$$

$$\theta_{11,yy} = RP_1(\psi_{00,y} \theta_{1,x} - \psi_{1,x} \theta_{00,y}), \quad \theta_{11}(\pm 1) = 0 \quad (3.23)$$

$$C_{11,yy} = RSc(\psi_{00,y} C_{00,x} - \psi_{1,x} C_{00,y}), \quad C_{11}(\pm 1) = 0 \quad (3.24)$$

$$\psi_{11,yyyy} - M_1^2 \psi_{11,yy} = -\frac{G}{R}(\theta_{11,y} + NC_{11,y}) + R(\psi_{00,y} \psi_{11,xy} - \psi_{00,x} \psi_{01,yy} + \psi_{01,y} \psi_{00,xy} - \psi_{01,x} \psi_{00,yy}), \quad (3.25)$$

$$\psi_{11(+1)} - \psi_{11(-1)} = 0, \quad \psi_{11,y} = 0, \quad \psi_{11,x} = 0 \quad \text{at } y = \pm 1$$

#### IV. SOLUTION OF THE PROBLEM

Solving the equations (3.14) - (3.22) subject to the relevant boundary conditions we obtain

$$\theta_{00} = 0.5(1 - y^2) + \gamma(\bar{x})$$

$$C_{00} = 0.5(y + 1)$$

$$\psi_{00} = a_4 Ch(M_1 y) + a_5 Sh(M_1 y) + a_6 y + a_7 + a_3 y^3 + d_2 y^2$$

$$\theta_{01} = 0.5 P M_1^2 (y^2 - 1)$$

$$C_{01} = a_6 (y^2 - 1)$$

$$\psi_{01} = a_{10} Sh(M_1 y) + a_{11} y + a_{12} + a_8 y^3$$

$$\theta_{10} = a_{24}y^2 + a_{25}y^3 + a_{26}y^4 + a_{27}y^5 + a_{28}y^6 +$$

$$(a_{20} + ya_{22})Ch(M_1y) + (a_{21} + ya_{23})Sh(M_1y)$$

$$C_{10} = a_{31}(y^2 - 1) + a_{32}(y^3 - y) + a_{33}(y^4 - 1) +$$

$$a_{34}(y^5 - y) + a_{35}(y^6 - 1) + (a_{36} + ya_{38})(Ch(M_1y) -$$

$$Ch(M_1)) + a_{37}(Sh(M_1y) - yShM_1) +$$

$$a_{39}(ySh(M_1y) - Sh(M_1))$$

$$\psi_{10} = b_8Ch(M_1y) + b_9Sh(M_1y) + b_{10}y + b_{11} + \phi(y)$$

$$\phi(y) = a_{70}y^2 + a_{71}y^3 + a_{72}y^4 + a_{73}y^5 + a_{74}y^6 +$$

$$+ a_{75}y^7 + (b_1y + b_3y^2 + b_5y^3)Ch(M_1y) +$$

$$(b_2y + b_4y^2 + b_6y^4)Sh(M_1y) + b_7y^4Sh(M_1y)$$

$$\theta_{11} = b_{15}y^2 + b_{16}y^3 + b_{17}Ch(2M_1y) + b_{18}Sh(2M_1y) + b_{19}y^4 + b_{20}y^6 +$$

$$b_{21}y^8 + b_{22}y^{10} + b_{23}y^{11} + b_{24}y^{12} + b_{25}y^8Sh(2M_1y) +$$

$$b_{26}y^7Ch(2M_1y) + b_{27}y^6Sh(2M_1y) + b_{28}y^5Ch(2M_1y) +$$

$$b_{29}y^4Sh(2M_1y) + b_{30}y^3Ch(2M_1y) + b_{31}y^2Sh(2M_1y) +$$

$$b_{32}yCh(2M_1y) + b_{33}Sh(2M_1y) + b_{34}y + b_{35}$$

$$\psi_{11} = b_{54} + b_{55}y + b_{56}Ch(M_1y) + b_{57}Sh(M_1y) + \phi_1(y)$$

$$\phi_1(y) = b_{34}y^{14} + b_{35}y^{13} + b_{36}y^{12} + b_{37}y^{11} + b_{38}y^{10} + b_{39}y^9 + b_{40}y^6 + b_{41}y^5 +$$

$$b_{42}y^4 + b_{43}y^3 + b_{44}Ch(2M_1y) + b_{45}Sh(2M_1y) + b_{46}y^8Ch(2M_1y) +$$

$$b_{47}y^7Sh(2M_1y) + b_{48}y^6Ch(2M_1y) + b_{49}y^5Sh(2M_1y) +$$

$$b_{50}y^4Ch(2M_1y) + b_{51}y^3Sh(2M_1y) + b_{52}y^2Ch(2M_1y) +$$

$$b_{53}ySh(2M_1y)$$

### SHEAR STRESS AND NUSSELT NUMBER

$$(Nu)_{y=+1} = \frac{(d_{10} + \delta(d_{11} + d_{12}))}{(d_8 - \gamma(x) + \delta d_9)}$$

$$(Nu)_{y=-1} = \frac{(-d_{10} + \delta(d_{12} - d_{11}))}{(d_8 - \gamma(x) + \delta d_9)}$$

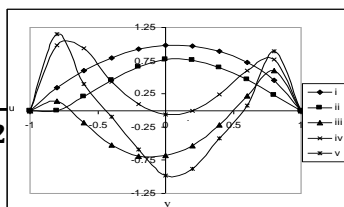


Fig.1 Variation of u with G

$D^{-1} = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $N = 1$ ,  $S_c = 1.30$ ,  $S_0 = 0.50$ ,  $E_c = 0.01$ ,  $x = \pi/4$

	I	II	III	IV	V
G	$1 \times 10^3$	$2 \times 10^3$	$3 \times 10^3$	$-2 \times 10^3$	$-3 \times 10^3$

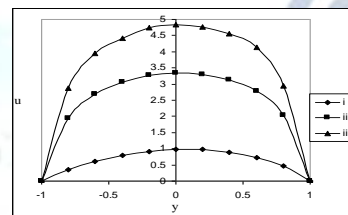


Fig.2 Variation of u with  $D^{-1}$

$G = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $N = 1$ ,  $S_c = 1.30$ ,  $S_0 = 0.50$ ,  $E_c = 0.01$ ,  $x = \pi/4$

$D^{-1}$	I	II	III
	$2 \times 10^3$	$3 \times 10^3$	$5 \times 10^3$

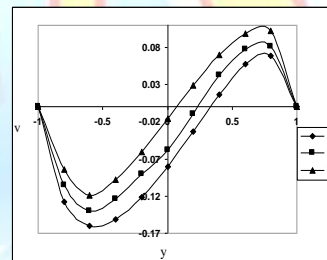


Fig.3 Variation of  $v \times 10^2$  with  $\alpha$

$D^{-1} = 2 \times 10^3$ ,  $G = 2 \times 10^3$ ,  $N = 1$ ,  $S_c = 1.30$ ,  $S_0 = 0.50$ ,  $E_c = 0.01$ ,  $x = \pi/4$

$\alpha$	I	II	III
	0.5	0.3	0.1

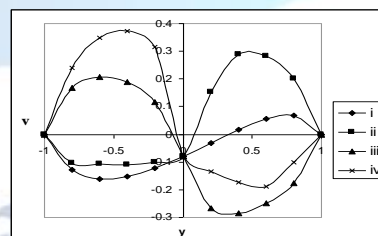


Fig.4 Variation of  $v \times 10^2$  with N

$D^{-1} = 2 \times 10^3$ ,  $G = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $S_c = 1.30$ ,  $S_0 = 0.50$ ,  $E_c = 0.01$ ,  $x = \pi/4$

N	I	II	III	IV
	1	2	-0.5	-0.8



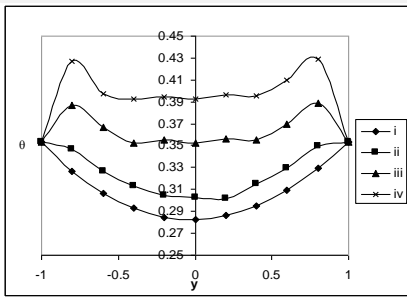


Fig.5 Variation Of  $\theta$  with  $S_c$   
 $D^{-1} = 2 \times 10^{-1}$ ,  $G = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $N = 1$ ,  $S_0 = 0.50$ ,  
 $E_c = 0.01$ ,  
 $x = \pi/4$

	I	II	III	IV
$S_c$	2.01	1.30	0.60	0.24

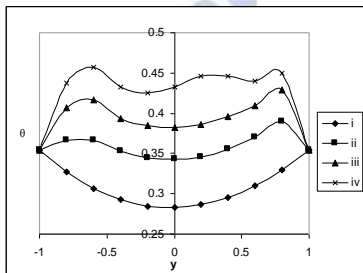


Fig.22 Variation of  $\theta$  with  $S_0$   
 $D^{-1} = 2 \times 10^{-1}$ ,  $G = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $N = 1$ ,  $S_c = 1.30$ ,  $E_c = 0.01$ ,  
 $x = \pi/4$

	I	II	III	IV
$S_0$	0.5	1.00	-0.5	-1.00

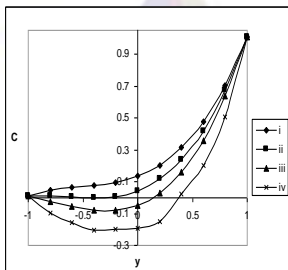


Fig.31 Variation of  $c$  with  $E_c$   
 $D^{-1} = 2 \times 10^{-1}$ ,  $G = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $N = 1$ ,  $S_c = 1.30$ ,  
 $S_0 = 0.50$ ,  
 $X = \pi/4$

	I	II	III	IV
$E_c$	0.01	0.03	0.05	0.07

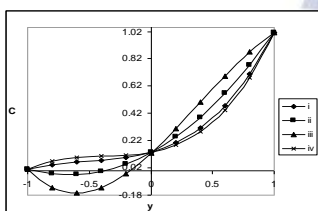


Fig.32 Variation of  $c$  with  $x$   
 $D^{-1} = 2 \times 10^{-1}$ ,  $G = 2 \times 10^3$ ,  $\alpha = 0.5$ ,  $S_c = 1.30$ ,  $S_0 = 0.50$ ,  
 $E_c = 0.01$ ,

$N = 1$

	I	II	III	IV
$x$	$\pi/4$	$\pi/2$	$\pi$	$2\pi$

## V. DISCUSSION OF THE NUMERICAL RESULTS

The equation of governing the flow, heat and mass transfer are solved by regular perturbation techniques with techniques with  $\delta$  the slope of the non-uniform boundary temperature as perturbation parameter. We take the Prandtl number  $P = 0.71$  and  $\delta = 0.01$ .

The actual axial velocity ( $u$ ) is shown in fig.1-2 the values of  $D^{-1}$ ,  $G$ ,  $S_c$ ,  $S_0$ ,  $\alpha$ ,  $N$ ,  $E_c$ , and  $x$ . Fig.1 represents the variation of  $u$  with Grashof number  $G$ . It is found that the actual axial velocity is in the vertically upward direction.  $u > 0$  is actual flow and  $u < 0$  is the reversal flow. It is noticed that the reversal flow occurs in region  $-0.5 < y < 0.5$  for  $G > 0$  while at  $G = 3 \times 10^3$  the transition occurs in the region  $(-0.4, 0.6)$ . Also  $|u|$  enhances with increase in  $G$ . The variation of  $u$  with  $D^{-1}$  shows that lesser the porous permeability of the porous medium larger  $|u|$  with maximum attained  $y = 0.6$ . An increase in the amplitude of the boundary temperature curve reduces  $u$  (fig2). The secondary velocity ( $v$ ) which is due to the non-uniform boundary temperature is The variation of  $v$  with amplitude  $\alpha$  of the non-uniform boundary temperature shows that  $|v|$  reduces with increase in  $\alpha$  in the region  $(-0.8, 0.5)$  and enhances in the remaining region (fig.3). The variation of  $v$  with buoyancy ratio  $N$  shows that when the molecular buoyancy force dominates over the thermal buoyancy force  $|v|$  enhances when the buoyancy forces act in the same direction and for the forces acting in opposite directions  $|v|$  enhances in the left half and reduces in the right half (fig.4). The non-dimensional temperature ( $\theta$ ) is shown in figs.5-6 for different parametric values. We follow the convention that the non-dimensional temperature is positive or negative according as the actual temperature is greater or lesser than the equilibrium temperature. we notice that lesser the molecular dissipative smaller the actual temperature in the flow region. An increase in the sort parameter  $S_0$  dissipates the actual temperature while it enhances with  $|S_0|$ . The non-dimensional concentration ( $C$ ) is shown in figs.7-8 for different parametric values. Fig.7 represents the variation of  $C$  with Eckert number  $E_c$  shows that higher the dissipative heat smaller the actual concentration in the flow region (fig.7). Moving

along the axial direction of the channel the actual concentration reduces with  $x < \pi$  and enhances with  $x > 2\pi$  in the left half while in the right half it enhance with  $x < \pi$  and enhances with  $x > 2\pi$  (fig 8).

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