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Solution of Non linear Heat Equation Using Differential Transform Method and Taylor Series Method

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ABSTRACT

This paper the application of differential transform method (DTM) and Taylor series method (TSM) to find the exact and approximate solutions of the heat equation with nonlinearity. Although the both of methods provide the solution in an infinite series, the DTM provides a fast convergent series of easily computable components and eliminates heavy computational work needed by TSM.

KEYWORDS: *Differential Transform Method, Nonlinear Heat Equation, Taylor Series Method.*

I. INTRODUCTION

The DTM is a semi analytical-numerical method that depends on Taylor series. It was introduced by zhou in 1986 for solving the linear and nonlinear initial value problem that appear in electrical circuits. It was found that, unlike other series solution methods, the DTM is easy to program in engineering problems, and provides solution terms without linearization and discretization. In [2] ,the advantage of the ADM has been expressed. The most recent application of the DTM and TSM lies in solving the nonlinear heat equation.

The DTM for the heat equation

Consider the following problem with solutions

$U_t(x,t)=u_{xx}+u^m$… (1)

$U(x,0)=f(x)$ … (2)

If the function u(x,t) is analytic and differential continuously with respect to time t and space x in the domain of interest , then let

$u_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]$… (3)

Where the t-dimensional spectrum function $u_k(x)$ is transformed of $u_k(x)$ is defined as

$u(x,t) = \sum_{k=0}^{\infty} u_k(x) t^k$ … (4)

From (1) and (2) the function u(x,t) can be described as

$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right] t^k$ … (5)

The differential transform method to obtain the solution of equation (1) and (2) by taking the differential transform on both sides we have,

$$(k+1)u_{k+1}(x) = \left[\frac{\partial^k}{\partial t^k} u_k(x) + \sum f_k(x)\right]\ldots (6)$$

$$U_0(x) = f(x)$$

Where $F_k(x)$ are,

$$F_0(x) = u_0^m(x)$$

$$F_1(x) = m u_0^{m-1}(x) u_1(x)$$

$$F_2(x) = \frac{1}{2} m(m-1) u_0^{m-2}(x) u_1^2(x) + m u_0^{m-1}(x) u_2(x)$$

$$F_3(x) = \frac{1}{6} m(m-1)$$
$$(m-2) u_0^{m-3}(x) u_1^3(x) + m(m-1) u_0^{m-2}(x) u_2(x) u_1(x) + m u_0^{m-1}(x) u_3(x)$$

From (4) we have,

$u(x,t) = u_0(x) + u_1(x)t + u_2(x)t^2 + u_3(x)t^3 + \cdots + u_n(x)t^n + \cdots$ (7)

$u_n(x,t) = \sum_{k=0}^{n} u_k(x)\, t^k \ldots$ (8)

Where n is order of approximation solution

Therefore, the exact solution of the equation (1) is given by,

$u(x,t) = \lim_{n\to\infty} u_n(x,t) \ldots$ (9)

The TSM for the heat equation

A Taylor series is a series expansion of a function based on the values of the values of the function and derivatives at one point one form for a taylor series expansion is,

$F(x)=f(x_0)+f'(x_0)(x-x_0)+f''(x_0)/2(x-x_0)^2+\ldots\ldots\ldots$ (10)

When $x_0=0$ it is also called Machaurin series for instance

$$\cos(x) = \cos(0) - \sin(0)\,(x) - \cos(0)\frac{x^2}{2!} + \cdots \quad (11)$$

A Taylor's series can also written in terms,

$f(x) = f(x_0) + f'(x_0) + f''(x_0) + \cdots$(12)

For a function of Taylor series derivatives of heat equation,

$f(x) = f(x_0) + \left(\frac{\partial f}{\partial x} x_0\right) + \frac{1}{2!}\left[\left(\frac{\partial^2 f}{\partial x^2}(x_0^2)\right) + 2\frac{\partial^2 f}{\partial x^2} x_0 +\right.$ *∂2f∂x2(x1)+13!∂2f∂x2(x0)3+3∂2f∂x2(x1)2+∂2f∂x2 x2+......* (13)

Applications

In this section, we solve nonlinear heat equation by DTM and TSM

Example

Consider the following nonlinear heat equation

$u_t(x,t) = u_{xx} - 2u^3$

$$u_0(x) = \frac{1+2x}{x^2+x+1}$$

Solution with DTM

$$F_0(x) = [\frac{1+2x}{x^2+x+1}]^3$$

$u_1(x) = \frac{-6(1+2x)}{(x^2+x+1)^2}$

$F_1(x) = \frac{-18(1+2x)^3}{(x^2+x+1)^2}$

$u_2(x) = \frac{36(1+2x)}{(x^2+x+1)^3}$

$$F_2(x) = \frac{216(1+2x)^3}{(x^2+x+1)^5}$$

$u_3(x) = \frac{-216(1+2x)}{(x^2+x+1)^4}$

From (7)

$$u(x,t) = \frac{1+2x}{(x^2+x+1)} - \frac{6(1+2x)}{(x^2+x+1)^2}t + \frac{36(1+2x)}{(x^2+x+1)^3}t^2 - \frac{216(1+2x)}{(x^2+x+1)^4}t^3 + \cdots$$

Solution with TSM

U(x,t)=f(x)

$U'(x_0)=\frac{-6(1+2x)}{(x^2+x+1)^2}$

$U''(x_0)=\frac{36(1+2x)}{(x^2+x+1)^3}$

$U'''(x_0)=\frac{-216(1+2x)}{(x^2+x+1)^4}$

From (13),

$$u(x,t) = \frac{1+2x}{(x^2+x+1)} - \frac{6(1+2x)}{(x^2+x+1)^2}t + \frac{36(1+2x)}{(x^2+x+1)^3}t^2 - \frac{216(1+2x)}{(x^2+x+1)^4}t^3 + \cdots$$

Example

Consider the equation,

$$\frac{x}{1-xy}$$

Solution with DTM

$u_0(x) = \left[\frac{x}{1-xy}\right]$ (13)

$$F_0(x) = [\frac{x}{1-xy}]^3$$

$u_1(x) = \frac{-6x}{(1-xy)^2}$

$$F_1(x) = \frac{-18x^3}{(1-xy)^2}$$

$u_2(x) = \frac{36x}{(1-xy)^3}$

$$F_2(x) = \frac{216x^3}{(1-xy)^5}$$ (14)

$u_3(x) = \frac{-216x}{(1-xy)^4}$

From (7)

$$u(x,t) = \frac{x}{1-xy} - \frac{6x}{(1-xy)^2} + \frac{36x}{(1-xy)^3} - \frac{216x}{(1-xy)^4} + \cdots$$

Solution with TSM (16)

$u(x_0) = \frac{x}{1-xy}$

$U'(x_0)=\frac{-6x}{(1-xy)^2}$ (17)

$U''(x_0)=\frac{36x}{(1-xy)^3}$

$U'''(x_0)=\frac{-216x}{(1-xy)^4}$

From (13), (18)

$$u(x,t) = \frac{x}{1-xy} - \frac{6x}{(1-xy)^2} + \frac{36x}{(1-xy)^3} - \frac{216x}{(1-xy)^4} + \cdots$$

II. CONCLUSION

This paper proposes the application of DTM and TSM for solving the nonlinear heat equations. The two series methods were applied separately to nonlinear heat equations.The study showed that the DTM is simple and easy to use and produces reliable results. The method also minimizes the computational difficulties of the TSM in that the components are determined elegantly by recurrence equations.

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