



# Design of a Linear and Non-linear controller for Induction Motor

P. Upendra Kumar<sup>1</sup> | J. Harish<sup>2</sup> | M. Ram Babu<sup>3</sup>

<sup>1</sup>Assistant Professor, Department of EEE, GMRIT, Rajam, Andhra Pradesh, India.

<sup>2</sup>Assistant Professor, Department of EEE, SVCE, Srikakulam, Andhra Pradesh, India.

<sup>3</sup>Assistant Professor, Department of EEE, GMRIT, Rajam, Andhra Pradesh, India.

## ABSTRACT

*This paper describes the controller design for the most widely used induction motors using advanced linear, non-linear and adaptive control techniques. Generally induction machine is a nonlinear one. Therefore, initially the considered nonlinear plant is linearized. Then PID control technique is applied to the considered induction motor along with LQR control law is also designed for the obtained linearized system. The non-linear plant dynamics are linearized by using dynamic feedback linearization techniques. By providing all these aspects, the speed control of induction motor is achieved in a wide range. To observe speed control, sinusoidal Ramp and Linear signals were taken as reference. Simulations are carried out and the performances of the designed controllers are compared. The design of these controllers is implemented in MATLAB/SIMULINK software.*

**KEYWORDS:** Non-linear and adaptive control, PID Control

Copyright © 2016 International Journal for Modern Trends in Science and Technology  
All rights reserved.

## I. INTRODUCTION

Induction motor is widely used in many applications because it is robust in nature; it has no brushes and contacts on rotor shaft. Operation of an Induction motor is based upon the application of Faraday's law and the Lorentz force on a conductor. High power/weight, lower cost/power ratio, and it is easy to manufacture, almost free maintenance, except for bearing and other external mechanical parts. Induction motors are simple, rugged, low cost and easy to maintain. The main disadvantage is that it has a high non-linearity and coupling structure. Basically in Induction motor, torque and flux cannot naturally decouple and can be controlled independently by the corresponding currents unlike DC motors. In this paper, a mathematical model for a sinusoidal wound, 2-phase, single pole-pair Induction motor with Squirrel cage rotor is specified. An Induction motor can either be a wound rotor or a Squirrel cage rotor. A Squirrel cage rotor is constructed using a number of short circuited bars arranged in a 'Hamster wheel' or Squirrel cage arrangement. The Squirrel cage rotor consists of copper bars,

slightly longer than rotor which is pushed into the slots. The ends are welded to copper end rings, so that all the bars are short circuited.

Conventional controllers like lag-lead, PID and may advanced control methods are mainly useful to control linear systems. In general, most of the plants are nonlinear in nature. Nonlinear control is one of the biggest challenges in modern control theory. In general, a nonlinear system has to be linearized so that initially an automatic controller can be effectively applied. This is typically achieved by adding a reverse non linear function to compensate nonlinear behavior resulting system input-output relationship becomes linear. In general, it is called feedback linearization. The objective of this paper is to design a controller based on LQR control, MRAC and then Feedback Linearization.

The model-reference adaptive system (MRAS) is an important adaptive controller. It may be regarded as an adaptive servo system in which the desired performance is expressed in terms of a reference model which gives a desired response to a command signal. This is a convenient way to give specifications to a servo problem.

The objective is to control the speed of Induction motor so as to damp out the transients as early as possible using Feedback Linearization by designing the linear and non-linear controllers. In this paper, the speed control of Induction Motor is achieved by using Linear & Non-Linear controllers. In specification of linear controller, we have developed LQR and PID controllers for the speed control of the Induction motor. Coming to a Non-Linear controller design; we adopted a Dynamic Feedback Linearization method which was applicable to speed control.

A Proportional-Integral-Derivative (PID) controller which is designed to eliminate the need for continuous operate attention. This controller is used to automatically adjust some variable to hold the measurement (or process variable) at the set point. The variable being adjusted is called the manipulated variable which usually is equal to the output of the controller. The output of the PID controller will change in response to a change in measurement of response. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic functional is called the LQ problem. One of the main results in the theory is that the solution is provided by the linear-quadratic regulator (LQR). Linear quadratic regulator (LQR) problem is a regulator problem which is restricted to linear systems only (or linearized versions of non-linear systems) and chooses a function that is quadratic function of states and controls. In regulator problems all the states should be available for measurement and both states and inputs must converge to zero. LQR has many desirable properties among them are good stability margins and sensitivity properties.

## II. MATHEMATICAL MODEL OF PLANT

The mathematical modeling of a sinusoidal wound, 2-phase, and three pole pair induction motor is derived as follows.

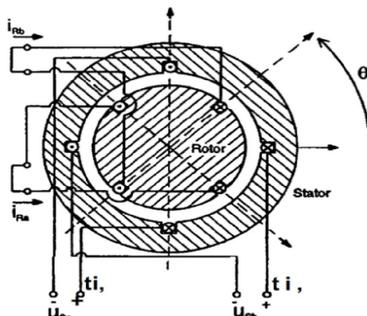


Fig. 1

Let  $i_{sa}$  and  $i_{sb}$  denote the currents in phases a and b of the stator,  $i_{Ra}$  and  $i_{Rb}$  denote the currents in phases a and b of the rotor .

Where,  $\theta$ = position of rotor;  $\omega$ =speed of the rotor;  $U_{sa}$ ,  $U_{sb}$  =applied voltages to phases a and b of the stator. The total flux linkage i.e. due to both stator and rotor currents in each of the stator phases a and b is given by

$$\lambda_{sa} = L_s i_{sa} + M(i_{Ra} \cos \theta + i_{Rb} \sin \theta) \quad ;$$

$$\lambda_{sb} = L_s i_{sb} + M(i_{Ra} \sin \theta + i_{Rb} \cos \theta)$$

Where,  $L_s$  is the self-inductance of each stator phase,  $R_s$  is the resistance of each stator phase,  $M$  is the coefficient of Mutual Inductance.

By Faradays and Ohms law, we have

$$U_{sa} = R_s i_{sa} + L_s \frac{di_{sa}}{dt} + M \frac{d}{dt}(i_{Ra} \cos \theta + i_{Rb} \sin \theta)$$

$$U_{sb} = R_s i_{sb} + L_s \frac{di_{sb}}{dt} + M \frac{d}{dt}(i_{Ra} \sin \theta + i_{Rb} \cos \theta)$$

Similarly the total flux in each of the rotor phases a and b is

$$\lambda_{Ra} = L_R i_{Ra} + M(i_{sa} \cos \theta + i_{sb} \sin \theta)$$

$$\lambda_{Rb} = L_R i_{Rb} + M(-i_{sa} \sin \theta + i_{sb} \cos \theta) \quad \text{respectively}$$

Where,  $L_R$  is the inductance of each rotor phase,  $R_R$  is the resistance of each rotor phase,

The Electrical equations are given by,

$$R_R i_{Ra} + L_R \frac{di_{Ra}}{dt} + M \frac{d}{dt}(i_{sa} \cos \theta + i_{sb} \sin \theta) = 0$$

$$R_R i_{Rb} + L_R \frac{di_{Rb}}{dt} + M \frac{d}{dt}(-i_{sa} \sin \theta + i_{sb} \cos \theta) = 0$$

Then the torque equation is given by

$$J \frac{d\omega}{dt} = M(i_{sb}(i_{Ra} \cos \theta + i_{Rb} \sin \theta)) - i_{sa}(i_{Ra} \sin \theta + i_{Rb} \cos \theta) - B\omega$$

Where  $J$ =moment of inertia of rotor;  $B$ =coefficient of viscous friction

To simplify the above equations, the dynamic equations are changed in terms of some equivalent rotor fluxes, and replace the value of  $\theta$  by  $n_p \theta$ .

The rotor flux equations are defined as

$$\psi_{Ra} = \cos(n_p \theta) \lambda_{Ra} - \sin(n_p \theta) \lambda_{Rb}$$

$$= L_R \cos(n_p \theta) i_{Ra} - L_R \sin(n_p \theta) i_{Rb} + M i_{sa}$$

$$\psi_{Rb} = \sin(n_p \theta) \lambda_{Ra} + \cos(n_p \theta) \lambda_{Rb}$$

$$= L_R \sin(n_p \theta) i_{Ra} + L_R \cos(n_p \theta) i_{Rb} + M i_{sb}$$

In terms of state variables, the dynamic equations of the induction motor become

$$\frac{d\theta}{dx} = \omega$$

$$\frac{d\omega}{dt} = \mu(\psi_{Ra} i_{sb} - \psi_{Rb} i_{sa}) - \frac{B}{J} \omega$$

$$\frac{d}{dt}(\psi_{Ra}) = -\eta\psi_{Ra} - n_p\omega\psi_{Rb} + \eta Mi_{sa}$$

$$\frac{d}{dt}(\psi_{Rb}) = -\eta\psi_{Rb} + n_p\omega\psi_{Ra} + \eta M i_{sb}$$

$$\frac{d}{dt}(i_{sa}) = \eta\beta\psi_{Ra} + \beta n_p\omega\psi_{Rb} - \gamma i_{sa} + U_{sa} / (\sigma L_s)$$

$$\frac{d}{dt}(i_{sb}) = \eta\beta\psi_{Rb} - \beta n_p\omega\psi_{Ra} - \gamma i_{sb} + U_{sb} / (\sigma L_s)$$

Let  $(\psi_{Ra}, \psi_{Rb}, i_{sa}, i_{sb}, \omega) = (x_1, x_2, x_3, x_4, x_5)$

In State space form,

$$\dot{x}_1 = a_1x_1 + a_2x_2x_5 + a_3x_3 \quad (1)$$

$$\dot{x}_2 = a_1x_2 - a_2x_2x_5 + a_3x_4 \quad (2)$$

$$\dot{x}_3 = a_4x_1 + a_5x_2x_5 + a_6x_3 + U_{sa}a_9 \quad (3)$$

$$\dot{x}_4 = a_4x_2 - a_5x_1x_5 + a_6x_4 + U_{sb}a_9 \quad (4)$$

$$\dot{x}_5 = a_7(x_1x_4 - x_2x_3) + a_8x_5 \quad (6)$$

Where,

$a_1 = -\eta = -16.54$ ;  $a_2 = -n_p = -3$ ;  $a_3 = \eta \times M = 3.97$ ;  
 $a_4 = \eta \times B = 1525$ ;  $a_5 = -n_p \times B = 276.64$ ;  $a_6 = \gamma = -721.5$ ;  
 $a_7 = \mu = 314.69$ ;  $a_8 = -B/J = -0.3409$ ;  $a_9 = -1 / (\sigma \times L_s) = -99.90$   
 $1$ ;  $R_r = 4.3\Omega$ ;  $R_s = 6.37\Omega$ ;  $L_s = L_r = 0.26H$ ;  
 $L_m = 0.24H$ ;  $J = 0.0088kg \cdot m^2$ ;  $B = 0.003$ ;  $n_p = 3$ ;  
 $\sigma = (L_s \times L_r - M^2) / L_r = 0.03$ ;  $\eta = R_r / L_r = 16.5385$ ;  
 $\beta = M / (\sigma \times L_s \times L_r) = 92.215$ ;  $\mu = n_p \times M / (JL_r) = 314.6853$ ;  
 $\gamma = M^2 R_r / (\sigma \times L_s \times L_r^2) + 1 / (\sigma \times L_s) = 1002.3845$ .

### III. LINEAR CONTROLLER DESIGN

#### 3.1 Design of PID Controller

Controllers are designed in order to reduce the fatigue damage for the reduced order system like PID controller, LQR control and then Digital controller. A proportional-integral-derivative controller (PID controller) is a common feedback loop component in industrial control systems. The controller takes a measured value from a process or other apparatus and compares it with a reference set point value. The difference (or "error" signal) is then used to adjust some input to the process in order to bring the process measured value to its desired set point. The PID controller reduces the fatigue torque by reducing the maximum peak over shoot and it also reduces the settling time.

The PID algorithm can be implemented in several ways. The easiest form to introduce is the parallel or "non-interacting" form, where the P, I and D elements are given the same error input in parallel. Another representation of the PID controller is the

series, or "interacting" form. This form consists in essence of a PD and a PI controller in series.

Tuning a system means adjusting three multipliers  $K_p$ ,  $K_i$  and  $K_d$  adding in various amounts of these functions to get the system to behave in the desired way.

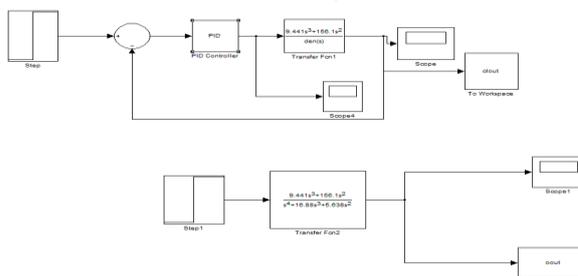
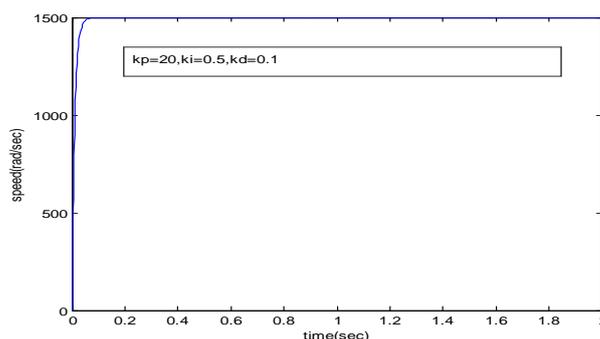
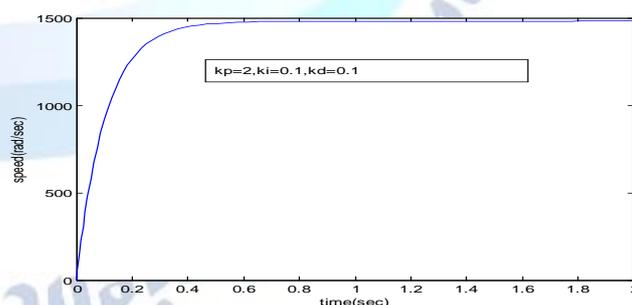
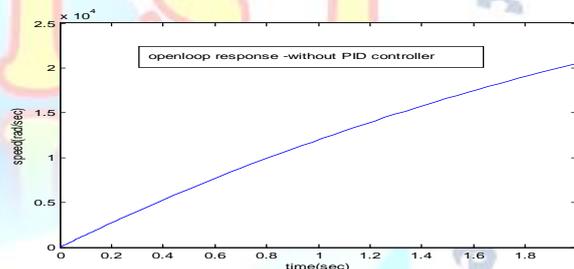


Fig. 3.1. Block diagram for the plant with and without PID controller.

The above figure shows the arrangement for PID controller for the considered plant. As open loop simulation is performed on the plant, it is found that the system gives better response with including of PID controller. Different values for  $k_p$ ,  $k_i$  and  $k_d$  were assumed to get the desired response in optimum time. Finally we tuned  $k_p$ ,  $k_i$  and  $k_d$  to such values so as to obtain desired results. The simulation results as shown in below with different tuned parameters of  $k_p$ ,  $k_i$  and  $k_d$



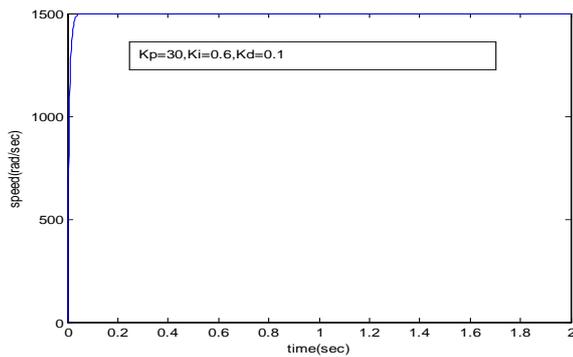


Fig. 3.2. Open loop performance of the plant with and without PID controller

### 3.2 Design of LQR controller

The theory of optimal control is concerned with operating a dynamic system at minimum cost. The case where the system dynamics are described by a set of linear differential equations and the cost is described by a quadratic functional is called the LQ problem. In laymans terms this means that the settings of a (regulating) controller governing either a machine or process (like an airplane or chemical reactor) are found by using an mathematical algorithm that minimizes a cost function with human (engineer) supplied weighting factors. The "cost" (function) is often defined as a sum of the deviations of key measurements from their desired values.

The LQR algorithm is at its core just an automated way of finding an appropriate state-feedback controller. Linear quadratic regulator (LQR) problem, which is restricted to linear systems only (or linearized versions of non-linear systems) and chooses a function interms of states and controls. In regulator problems all the states should be available for measurement and both states and inputs must converge to zero. LQR has many desirable properties among them are good stability margins and sensitivity properties. In effect this algorithm therefore finds those controller settings that minimize the undesired deviations, like deviations from desired altitude or process temperature. Often this means that controller synthesis will still be an iterative process where the engineer judges the produced "optimal" controllers through simulation and then adjusts the weighting factors to get a controller more in line with the specified design goals.

The following are the responses that are obtained while the linearized induction motor equations are simulated. It is found that without LQR controller the system becomes unstable. With

the help of controller the system reaches steady state.

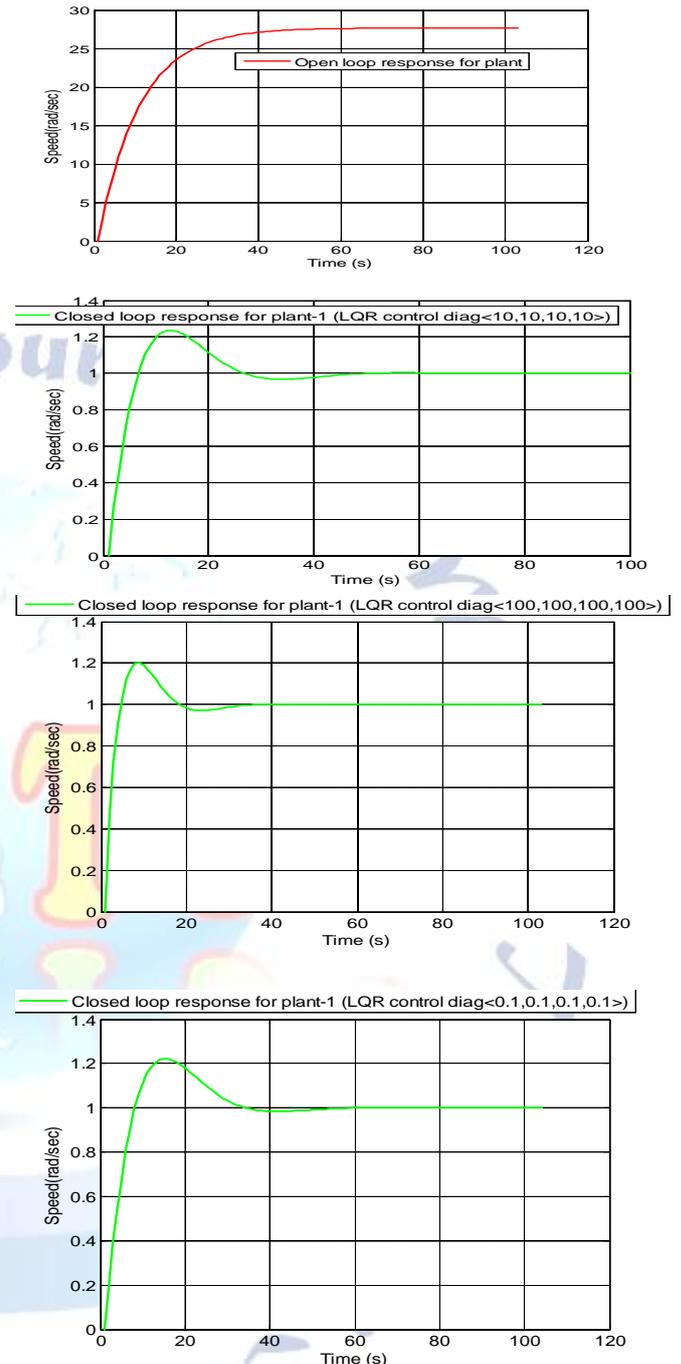


Fig. 3.3. Simulation results of induction motor with and without LQR

### IV. NON LINEAR CONTROLLER DESIGN

Physical systems are inherently non-linear. Thus, all control systems are nonlinear to a certain extent. Nonlinear control systems can be described by nonlinear differential equations. However, if the operating range of a control system is small and the involved nonlinearities are smooth, then the control system may be reasonably approximated by a linearized system, whose dynamics are described by a set of linear differential equations.

Basically in induction motor torque and flux cannot be naturally decouple and can be controlled independently by the corresponding currents unlike d.c. motors. To overcome these difficulties, present industry world adopt the technique named as field-oriented (vector control) technique for the design of Nonlinear controller.

**4.1 Field-Oriented Technique**

Field-Oriented Control algorithm generates voltage as a vector to control the stator current. By transforming the physical current into rotational vector using transformation techniques, the torque and flux Components become time invariant and allowing control with conventional techniques such as PI controllers, as like d.c. motor. This method consists in rewriting the dynamic equations of the induction motor in a reference frame that rotates with the rotor flux vector. In this new coordinate system, there is a linear relationship between a control variable and the speed by holding the magnitude of the rotor flux constant. A disadvantage of this method assumes that the magnitude of rotor flux is regulated to a constant value. Here the rotor speed is asymptotically decoupled from the rotor flux. For the speed control of the Induction motor, here a Dynamic Feedback Linearization technique is employed.

**4.2 Dynamic Feedback Linearization**

Consider the system as,

$$\dot{x} = f(x) + g(x) u$$

Where, x is the state vector; u is the control vector; f is a nonlinear state dependent function; g is nonlinear control function.

Dynamic Feedback Linearization is one of such techniques applicable to speed control, but not used for position control. Transformation of the nonlinear system into an equivalent linear system through substitution of variables and a suitable control input. The central idea of the approach is to algebraically transform a nonlinear system dynamics into a linear one, so that linear control techniques can be applied. These techniques can be viewed as ways of transforming original system models into equivalent models of a simpler form. These techniques can make the closed loop system to follow a prescribed transfer function. The Feedback Linearization is used in various applications such as, aircrafts, helicopters, industrial robots, biomedical devices.

**4.3 Field-Oriented and Input- Output Linearization Control**

By using Field-Oriented and Input-Output Linearization Control an equivalent model was

found using the following nonlinear state space transformation

$$d / dt(\theta) = w \dots(4.1)$$

$$d / dt(w) = \mu \psi_d i_{qr} - (B / J) w \dots(4.2)$$

$$d / dt(\psi_d) = -\eta \psi_d + \eta M i_{dr} \dots(4.3)$$

$$d / dt(\rho) = n_p w + \eta M i_{qr} / \psi_d \dots(4.4)$$

Where  $i_{dr}$  and  $i_{qr}$  are new inputs.

The term Field-oriented refers to a new coordinate system whose angular position is  $\rho$  and the magnitude as  $\psi_d$ .

The state variables are

$$(\theta, \omega, \psi_d, \rho, u_1, u_2) = (z_1, z_2, z_3, z_4, i_{dr}, i_{qr})$$

$$Z_1' = z_2$$

$$Z_2' = c_1 * z_3 * u_2 + c_2 * z_2$$

$$Z_3' = c_3 * z_3 + c_4 * u_1$$

$$Z_4' = c_5 * z_2 + c_4 * u_2 / z_3$$

Where,

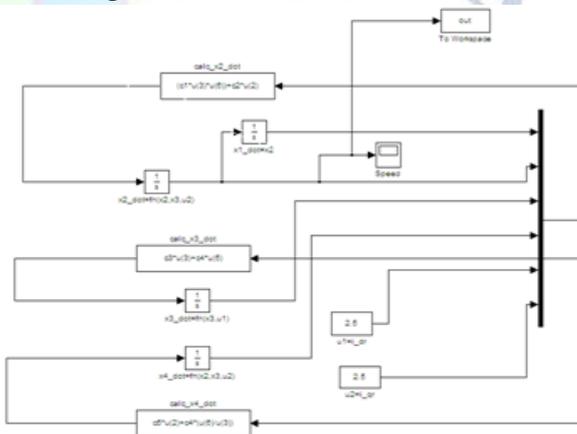
$$C_1 = \mu = 314.6853; C_2 = -B/J = -0.3409; C_3 = -\eta = 16.5385; C_4 = \eta * M = 3.97; C_5 = n_p = 3$$

Here in this Non-Linear controller design we have done open loop and closed loop simulations.

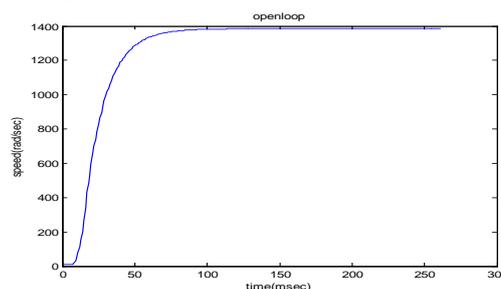
**V. RESULTS AND DISCUSSIONS**

**Open Loop Simulation**

The open loop block diagram of the nonlinear controller is given below.

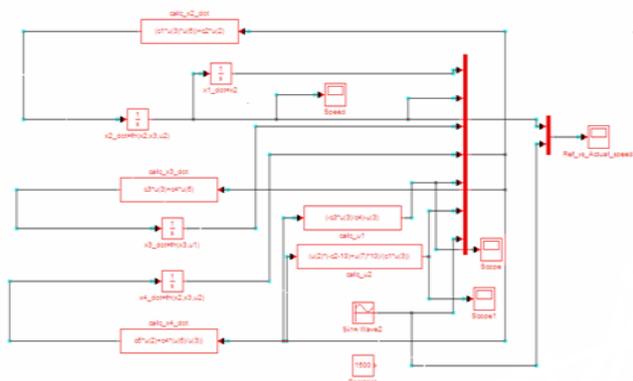


By simulating the above open loop block diagram, we get the response as



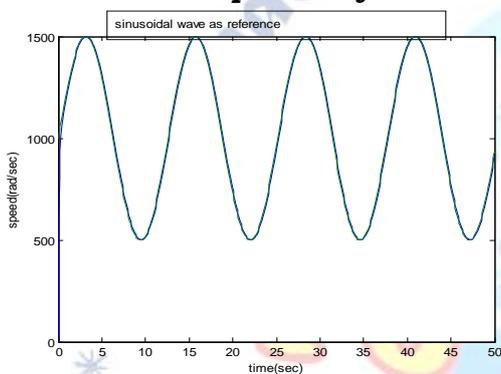
**Closed Loop Simulation**

The closed loop block diagram of non-linear controller is given below

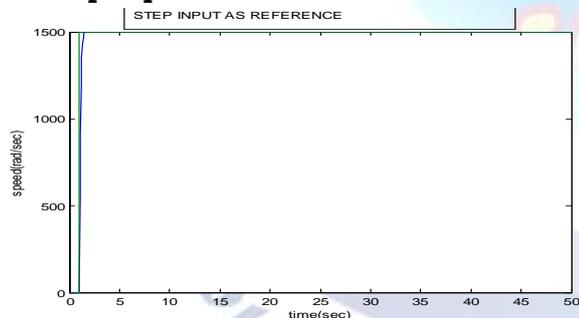


By simulating the above closed loop block diagram, we get the response as

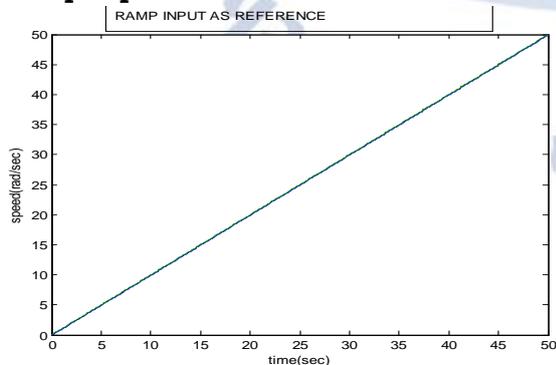
**For Sinusoidal Input as reference:**



**For Step Input:**



**For Ramp Input:**



**VI. CONCLUSION**

In this paper unstable non-linear induction motor response is stabilized by using PID controller and LQR controller. The considered non-linear

dynamic equations of the induction motor are linearized by using field oriented control using dynamic feedback linearization techniques. Then PID control techniques and LQR control techniques were implemented on obtained linear equations from non-linear controller. The response of the plant with the use of both the controllers were found satisfactorily stable than the response without controllers.

It has been assumed that the non-linearities are very small enough and hence neglected while designing the LQR controller, but in practical design these negligence of small nonlinearities may also cause severe disaster. As such a non-linear controller is designed by using dynamic feedback linearization. It is found that the speed output tracks the reference input waveform. Here it is assumed that the random input signals are sine, step and Ramp waveforms. Then the simulations are performed to find the error plot. The main aim is to reduce the error to a maximum extent using the mentioned control techniques. In these cases the error is minimized with respect to time with the help of the above mentioned controllers. Thus the present thesis recommends the use of controllers for the induction motor so as to obtain desired response.

**REFERENCES**

- [1] High-performance Induction Motor control Via Input-Output Linearization, Marc Bodson, John Chiasson and Robert Novotnak, Proceedings, IEEE control systems, 0272-1708/94/1994IEEE.
- [2] Input-Output Linearizing and Decoupling Control of an Induction Motor Drive; Dr K B Mohanty, vol 88, September 2007.
- [3] Pakki Bharani Chandra Kumar and M.Venkateswara Rao, Enhancement of excitation controllers using linear and non-linear control in Power Systems, proceedings of national conference on DREEM-06, pp 22-26, 01-02 Dec 06, GI-TAM, Vishakhapatnam, India.
- [4] High performance adaptive Field-Oriented Model Reference Control Of Induction Motor, Nouredine Golea, Amar Golea, Journal of Electrical Engineering, Vol. 57, NO.2, 2006, 78-86.
- [5] Applied Non linear Control, Jean -Jacques E. Slotine, Massachusetts Institute of Technology. Weiping Li Massachusetts Institute of Technology