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Navigation of Mobile Inverted Pendulum via Wireless control using LQR Technique

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ABSTRACT

Mobile Inverted Pendulum (MIP) is a non-linear robotic system. Basically it is a Self-balancing robot working on the principle of Inverted pendulum, which is a two wheel vehicle, balances itself up in the vertical position with reference to the ground. It has four configuration variables (Cart position, Cart Velocity, Pendulum angle, Pendulum angular velocity) to be controlled using only two control inputs. Hence it is an Under-actuated system. This paper focuses on control of translational acceleration and deceleration of the MIP in a dynamically reasonable manner using LQR technique. The body angle and MIP displacement are controlled to maintain reference states where the MIP is statically unstable but dynamically stable which leads to a constant translational acceleration due to instability of the vehicle. In this proposal, the implementation of self balancing robot with LQR control strategy and the implementation of navigation control of the bot using a wireless module is done. The simulation results were compared between PID control and LQR control strategies.

KEYWORDS: *Mobile Inverted Pendulum, Navigation control, LQR controller, Wireless control*

I. Introduction

An inverted pendulum is a pendulum that has its centre of mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally and may be called a cart and pole. Most applications limit the pendulum to one degree of freedom (DOF) by affixing the pole to an axis of rotation. Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright; this can be done either by applying a torque at the pivot point, by moving the pivot point horizontally as part of a feedback system, changing the rate of rotation of a mass mounted on the pendulum on an axis parallel to the pivot axis and thereby generating a net torque on the pendulum, or by oscillating the pivot point vertically. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies.

The inverted pendulum is a single input multiple output (SIMO) system, where the input is given to the wheels as the single input where the position and the angle of the pendulum are taken as outputs. An ideal system has zero angle and position displacement. The controller should minimize both the displacement of the cart and the angle of the pendulum. The challenges in inverted pendulum are as follows.

- Highly Unstable – The inverted position is the point of unstable equilibrium as can be seen from the non-linear dynamic equations.

- Highly Non-linear – The dynamic equations of the MIP consists of non-linear terms.

- Non-minimum phase system – The system transfer function of MIP contains right hand plane zeros, which affect the stability margins including the robustness.

- Under actuated – The system has two degrees of freedom of motion but only one actuator i.e. the DC motor. Thus, this system is under-actuated. This makes the system cost effective but the control problem becomes challenging.

A. Existing systems

The system challenges can be solved using both linear and non linear controllers. The PI and PID controllers produce overshoots and oscillation. It reduces system complexity but is not suitable for a real time application. The main drawback of linear controllers are that they are non robust. The system moves from one place to another is driven manually or through joystick. Since the system is becomes unstable for a very small changes, thus it needs to be monitored and controlled for every instance of time the linear controller is not suitable. These systems produce noises which increases instability. System complexity and training time increases for neural schemes [21]

B. Proposed systems

In this paper, we propose Linear Quadratic Regulator (LQR) controlling technique for navigation control of the inverted pendulum. Firstly, this technique provides a robust control of the system. Secondly, this control provides better noise rejection and balances the robot. In addition to balancing the pendulum, the system employs a wireless module using which navigation or movement of the MIP is controlled. The navigation the MIP is commanded by the user. The LQR control reduces the system complexity and is very suitable for real time applications. Analysis has been made to study the performance of various controllers, simple solution for an inverted pendulum mounted on cart, using LQR controller for input tracking and disturbance rejection presents a balancing robot to vertically balance itself only.

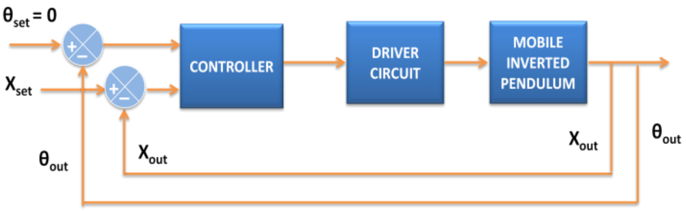

Fig 1. Block Diagram of Inverted Pendulum System

This paper is organized as follows. Section II explains the mathematical modelling of Mobile Inverted Pendulum while section III elaborates the controller design. The simulation results are presented in section IV followed by experimental setup explanation in section V and the conclusion in section VI.

II. Mathematical Analysis

A. System dynamics

The stabilization of inverted pendulum is a classical benchmark control problem. It is a simple system in terms of mechanical design only consisting of a D.C. Motor, a pendant type pendulum, a cart, and a driving mechanism. Figure 2. shows the basic schematic for the cart-inverted pendulum system.

Following is the list of parameters used in the derivation of Inverted Pendulum dynamics

M – Mass of cart in kg
m – Mass of Pendulum in kg
I – Moment of Inertia of pendulum in kg-m2
l – Length of Pendulum in m
b – Cart friction coefficient in Ns/m
g – Acceleration due to gravity in Nm/s^2
u – Input force applied in N
θ – Angular displacement
x – Horizontal displacement

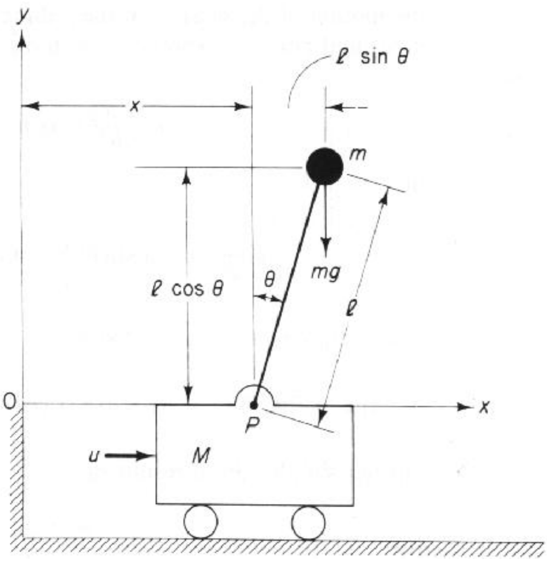

Fig 2. Schematic Diagram of Inverted Pendulum

Considering the free body diagram of cart where horizontal and vertical forces act on it we get,

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = u \qquad (2.1)$$

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Now considering the forces acting upon the pendulum we get,

$$(I+ml^2)\ddot{\theta} - mglsin\theta = - ml\ddot{x}\cos\theta \qquad (2.2)$$

For small θ, ie., cosθ = 1, sinθ = θ. Therefore equation (2.1) becomes

$$(M+m)\ddot{x} + b\dot{x} + ml\ddot{\theta} = u \qquad (2.3i)$$

$$(I+ml^2)\ddot{\theta} - mgl\theta = -ml\ddot{x} \qquad (2.3ii)$$

The angular acceleration and displacement acceleration is given by

$$\ddot{x} = \frac{u}{M+m} - \frac{b\dot{x}}{M+m} - \frac{ml\ddot{\theta}}{M+m} \qquad (2.4)$$

$$\ddot{\theta} = \frac{ml\ddot{x}}{I+ml^2} + \frac{mgl\theta}{I+ml^2} \qquad (2.5)$$

Sub equation (2.4) in equation (2.3ii)

$$\ddot{\theta} = \frac{mgl\,(M+m)\theta}{I(M+m)+ml^2(M+m)+m^2l^2} \qquad (2.6)$$

In order to obtain the state model we are assuming the states to be as the cart position x, cart linear velocity $\dot{x}$, pendulum angle θ, pendulum angular velocity $\ddot{\theta}$. The state space is of the form

$$\dot{x} = Ax + Bu$$

The above equation is the state space equation, where the system parameters can be represented as follows

$x_1 = x$

$x_2 = \dot{x} = \dot{x_1}$

$x_3 = \theta$

$x_4 = \dot{\theta} = \dot{x_3}$

$y_1 = x$

$y_2 = \theta$

By substituting we get,

$$\dot{x_4} = \frac{mgl\,(M+m)}{I(M+m)+Mml^2}x_3 + \frac{mlb}{I(M+m)Mml^2}x_2 - \frac{mlu}{I(M+m)+Mml^2} \qquad (2.7)$$

$$\dot{x_2} = \frac{-gm^2l^2}{I(M+m)+Mml^2}x_3 - \frac{b(I+ml^2)}{I(M+m)+Mml^2}x_2 + \frac{I+ml^2u}{I(M+m)+Mml^2} \qquad (2.8)$$

The state space equation for the system is

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

The output equation is given by

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

III. Controller Design

The main design objective is to minimize the cost equation which is an integral over time of the weighted inputs and outputs. There are two cost equations that are typically used for this design technique. The LQR is one of the most widely used static state feedback methods, primarily as the LQR based pole placement helps us to translate the performance constraints into various weights in the performance index. This flexibility is the sole reason for its popularity. The inverted pendulum system has many physical constraints both in the states and in the control input. Hence, the LQR design is attempted. The choice of the quadratic performance indices depends on physical constraints and desired closed loop performance of the control system. Any state feedback can be generalized for an LTI system as given below:

$$\dot{x} = Ax+Bu$$

$$y = Cx \qquad (3.1)$$

If all the n states are available for feedback and the states are completely controllable then there is a feedback gain matrix K, such that the state feedback control input is given by

$$u = - K(x-x_d) \qquad (3.2)$$

Let x_d be the vector of desired states. The closed loop dynamics using (3.2) in (3.1) becomes

$$\dot{x} = (A-BK)\,x + BK\,x_d \qquad (3.3)$$

Choice of K depends on the desired pole locations where one intends to place the poles such that the desired control performance is achieved. In the case of LQR the control is subjected to a Performance Index (PI) or Cost Functional (CF) given by

$$J = \frac{1}{2}[z(t_f) - y(t_f)]^T F(t_f)[z(t_f) - y(t_f)] + \frac{1}{2}\int_{t_0}^{t_f}\{[z-y]^T Q[z-y] + u^T Ru\}dt \quad (3.4)$$

Here z is the m dimensional reference vector and u is an r dimensional input vector. If all the states are available in the output for feedback then m becomes n. Since, the performance index equation (3.4) is in terms of quadratic terms of error and control it is called as quadratic cost functional. If our objective is to keep the system state to near zero then it is called as a state regulator system. Here the unwanted plant disturbances that need to be rejected e.g. Electrical Voltage Regulator System. If it is desire to keep the output or state near a desired state or output it is called a tracking system.

In equation (3.4), the matrix Q is known as the error weighted matrix, R is the control weighted matrix, F is known as the terminal cost weighted matrix.

A. *LQR Control Design*

The choice of Q and R is very important as the whole LQR state feedback solution depends on their choice. Usually they are chosen as identity values and are successively iterated to obtain the controller parameter. Bryson's Rule is also available for constrained system, the essence of the rule is just to scale all the variables such that the maximum value of each variable is one. R is chosen as a scalar as the system is a single input system.

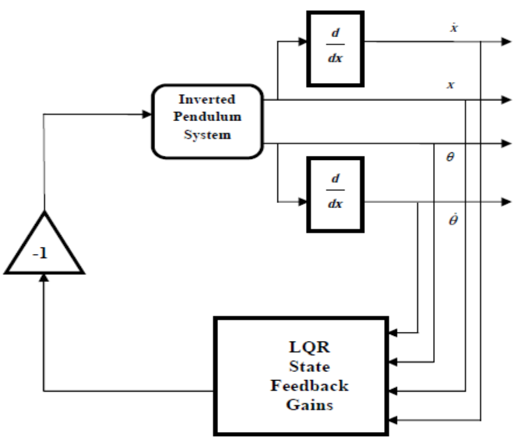


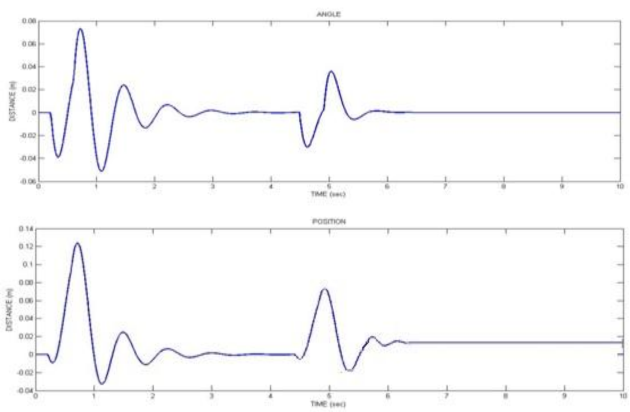
Fig 3. Schematic Diagram of Inverted Pendulum with LQR feedback

The excitation due to initial condition is reflected in the states can be treated as an undesirable deviation from equilibrium position. If the system described by Figure 3 is controllable then it is possible to drive the system into its equilibrium point. But it is very difficult to keep the control signal within bound as chances are such that the control signal would be very high which will lead to actuator saturation and would require high bandwidth designs in feedback that might excite the unmodelled dynamics. Hence, it required to have a trade-off between the need for regulation and the size of the control signal. It can be seen that the choice of control weighting matrix R comes handy in keeping the control signal magnitude small. It can be seen that larger the weight on R the smaller is the value of the control signal.

IV. Simulation Results

The proposed algorithm is simulated using the actual parameters of the system. The system parameters used in the simulation are Mass of cart (M) & pendulum (m) = 1.13, 0.03, l = 0.25, g = 9.81, I = 0.0041, b = 0.000039.

The simulation results for both PID control and LQR technique employed is presented in this section. Figure 4 & figure 5 shows variations of angle, position and disturbance given to the system when PID and LQR control is employed respectively. It can be observed that initial variations of the angle and position describe the balancing dynamics of the system. Furthermore, when disturbance of step input is given at time 4.5 sec, the system adjusts itself to the new operating point. The time taken by PID control is 3.7 sec with lot of oscillations. Rather the time taken by LQR control is 1.4 sec with very less oscillations.



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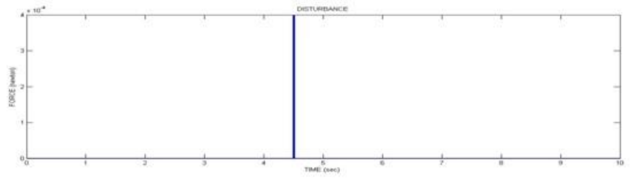

Fig 4. Simulation results for PID control (a) Angle (b) Position (c) Disturbance applied

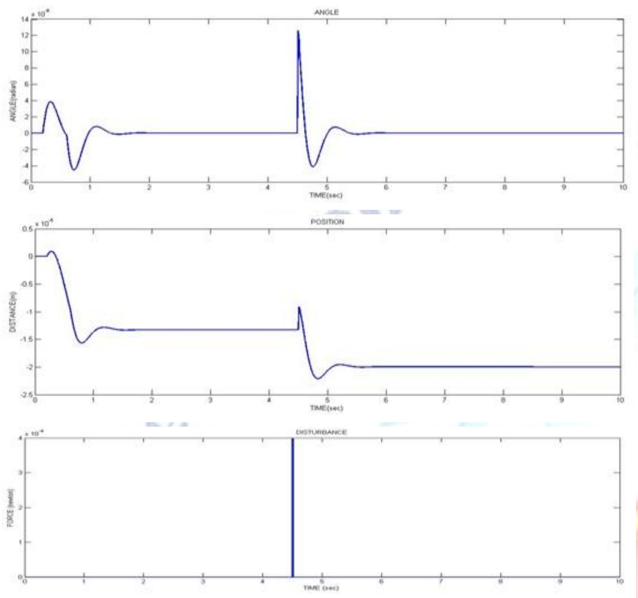

Fig 5. Simulation results for LQR control (a) Angle (b) Position (c) Disturbance applied

V. EXPERIMENTAL SETUP

The experimental setup for the mobile inverted pendulum is shown in the figure 6. The main elements of the system are DC motor, wireless module, battery, motor driver circuit, gyroscope and controller. There are two DC motors connected to each one of the robot wheels whose rating is of 7.5 V, 1 A, 300 rpm. Each of the DC motor is driven by a H-bridge circuit. The driving pulses to each H-bridge circuit are generated by controller using Pulse Width Modulation technique. Controller used in the system is ATMega 32. Two PWM channels are configured for driver circuit. The power supply to the robot is provided by Lithium polymer battery whose rating is of 12 V, 1.2 Ah.

The angular displacement (θ) is fed back to the controller based on which the balancing of the robot is performed. MPU – 6050 Accelerometer + Gyro is used to determine the tilt angle. It captures the x, y, and z channel at the same time. The sensor uses the SPI to interface with the ATMega. A remote control is used to navigate the robot. The wireless module used here is RFM75 operates at 2.5 GHz, 3.3 V. The command to the robot for navigation is sent by the transmitter. At the receiving end, the command signal is processed by controller. The controller decides the duty ratio of the PWM pulses sent to driver circuit. In this way, the robot balances itself and navigates per the command.

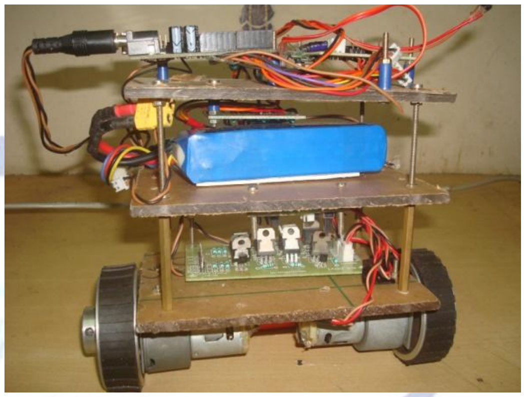

Fig 6. MIP model

VI. CONCLUSION

In this paper, self balancing MIP along with navigation control is presented. The LQR control adds the advantage of robustness and less recovery time during disturbances over linear controllers. PID controller is easier to implement but the main concern is the overshoots created by the 'I' (integral) factor in PID which makes the system prone to disturbance and is highly undesirable for this sort of non-linear system. Another major downside of PID is that its high gains yielded a control signal too high for the system to be practically implemented. LQR controller yielded results which were more practical alongside it was also stabilizing the system, it showed good results with the addition of disturbances.

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