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Graph Theory Matrix Approach in Selecting Optimal Combination of Operating Parameters

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ABSTRACT

In this paper Graph theory matrix approach is adopted as a decision making tool to find the optimal combination of operating parameters of a diesel engine. It is found that the load of 18A, 270 injection timing and 200 bar pressure forms the optimal combination of operating parameters of the engine under the specified conditions. The result of Graph Theory Matrix Approach is compared and analyzed with other multi-criteria decision-making methods including Simple Additive Weighting (SAW), Weighted Product Method (WPM) and Analytic Hierarchy Process (AHP). These methods provide selection of index for different alternatives considered. Finally, the optimal combination of operating parameters for the diesel engine is selected.

KEYWORDS: *Graph, GTMA, Matrix, Permanent function, Permanent index*

I. INTRODUCTION

Highlight A straight-line embedding of a graph G = (V, E) is an injective function π: V → R2 such that for any two distinct edges ab and cd the straight line segments π(a)π(b) and π(c)π(d) are internally disjoint. Many real world situations can conveniently be described by means of a graph consisting of a set of points together with lines joining certain pairs of these points. Graph theory matrix approach is applied to various fields of science and technology like civil engineering, software engineering, construction management, education, environmental engineering, operations research, decision science, mechanical engineering – manufacturing, energy, materials, maintenance, reliability, tribology, automotive, hydraulic, industrial engineering – quality, supply chain management, product design, costing etc. Gandhi et al [1] used Graph theory matrix approach in modeling and solving a decision making problem with multiple and interrelated attributes to show the reliability of a mechanical and hydraulic system. Venkatasamy and Agrawal [2] proposed Graph theory matrix approach for structural analysis of automobile components for selection of optimum structural design of an automobile system. Rao [3] employed Graph theory matrix approach for developing a performance evaluation system for technical education institutions, which is used for ranking the technical institutions. Mohan et al [4] developed a systematic approach for system and performance modeling of a coal based steam power plant and for optimum selection, benchmarking, maintenance strategy analysis, selection and performance comparison of alternative power plant design. Grover et al [5] considered human factors, behavioral factors, use of tools/techniques, non behavioral factors, functional areas and their interactions to develop a TQM index to quantify the degree of TQM concepts implementation in an industry utilizing Graph theory matrix approach. Prabhakaran et al [6] utilized Graph theory matrix approach for developing quality index for quality modeling and analyses of polymer composite products to evaluate and select composite product systems. Faisal et al [7] developed an information risk index by using Graph theory matrix approach to quantify information factors in supply chains. Upadhyay and Agrawal [8] proposed a systematic approach in order to model, evaluate and analyze intelligent mobile learning environments using Graph theory matrix approach. They considered intelligent tutoring systems, multi-agent intelligent system, mobile dimension system, environment and human aspect system in evaluating alternative mobile learning environments. Darvish et al [9] developed a contractor selection index to select most suitable contractor for a given construction project by considering work experience, technology and equipment, experience, financial stability, quality and reputation. Anand and Wani [10] proposed Graph theory matrix approach for product life cycle modeling and evaluation at the conceptual design stage to evaluate the life cycle design index which is useful in assessing the relative life cycle design value of product design alternatives.

The objective of this paper is to adopt Graph theory matrix approach to analyze and evaluate the optimal combination of operating parameters on a diesel engine which involve multiple attributes like Injection pressure, Injection timing and Load. In order to fulfill the above objective, the factors affecting the determined problem were identified through literature and management techniques. The relative importance between the factors was identified. Computer program was generated to simplify the calculation. Other multi attribute decision making methods like Simple additive weighting (SAW), Weighted products method (WPM) and Analytic hierarchy process(AHP) are as well used to compare and analyze the results obtained by Graph theory matrix approach.

II. GRAPH THEORY MATRIX APPROACH

Graph theory matrix approach is a logical and systematical approach. The advanced theory of graphs and its applications are very well documented [11]. Graph theory matrix approach consists of the following components:

1. Digraph representation
2. Matrix representation
3. Permanent function

Digraph representation

Digraph model representation has proved to be useful in modeling and analyzing various kinds of systems in fields of science and technology. A digraph is a graph with directed edges. The performance attributes digraph used in the present study consists of BP, BSFC, BTE, NOx, HC, CO, CO2 and O2 as nodes and the interrelations between them are represented as edges. The performance attributes digraph is shown in Fig. 1 for quick visual appraisal. As the number of nodes and their relative importance increase, the digraph becomes complex. To overcome this difficulty, the digraph is represented in matrix form.

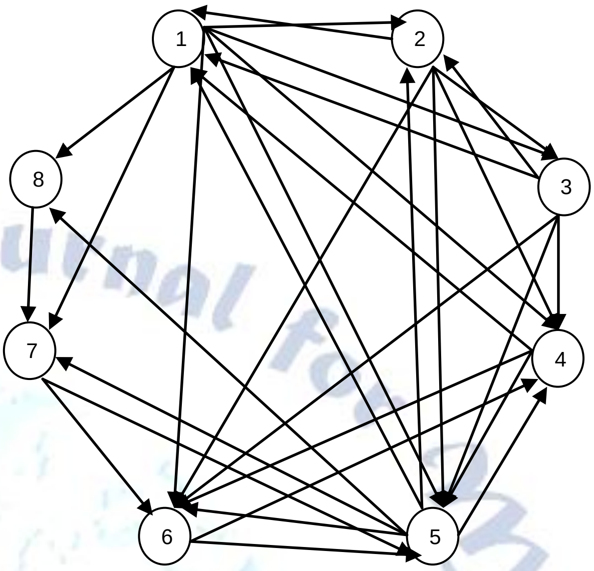

Figure 1: Performance attributes digraph

Matrix representation

The matrix representation of performance attributes digraph gives a one-to-one representation. The matrix approach is useful in analyzing the digraph expeditiously to derive the system function. A matrix called Performance attributes matrix (B) is defined, which is a MXM matrix and considers all of the attributes (Ri) and their relative importance (aij). The performance attributes matrix is shown in Eq. (1).

$$B=\begin{bmatrix} D_1 & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} & d_{17} & d_{18} \\ d_{21} & D_2 & d_{23} & d_{24} & d_{25} & d_{26} & d_{27} & d_{28} \\ d_{31} & d_{32} & D_3 & d_{34} & d_{35} & d_{36} & d_{37} & d_{38} \\ d_{41} & d_{42} & d_{43} & D_4 & d_{45} & d_{46} & d_{47} & d_{48} \\ d_{51} & d_{52} & d_{53} & d_{54} & D_5 & d_{56} & d_{57} & d_{58} \\ d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & D_6 & d_{67} & d_{68} \\ d_{71} & d_{72} & d_{73} & d_{74} & d_{75} & d_{76} & D_7 & d_{78} \\ d_{81} & d_{82} & d_{83} & d_{84} & d_{85} & d_{86} & d_{87} & D_8 \end{bmatrix} \quad (1)$$

Permanent Function

The permanent function is a standard matrix function and is used in combinatorial mathematics [15]. The application of permanent function leads to a better appreciation as no negative sign will appear in the expression and hence no information will be lost. The permanent function (Per) for Eq. (1). is written in Eq. 2

Performance Index

It is a measure of the ease which gives the optimal combination of operating parameters of an engine. It contains the measures of factors and their relative importance.

$$
\begin{aligned}
&Per(B) = \\
&\prod_{i=1}^{M} D_i + \sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \cdots\cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{ji})D_kD_lD_mD_nD_o....D_tD_m \\
&+\sum_{i=1}^{M-2}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M} \cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{ki}+d_{ik}d_{kj}d_{ji})D_lD_mD_nD_o....D_tD_M \\
&+[\sum_{i=1}^{M-3}\sum_{j=i+1}^{M}\sum_{k=i+1}^{M-1}\sum_{l=i+2}^{M} \cdots\cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{ji})(d_{kl}d_{lk})D_mD_nD_o.....D_tD_M + \\
&\sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=j+1}^{M} .... \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{kl}d_{li}+d_{il}d_{lk}d_{kj}d_{ji})D_mD_nD_o.....D_tD_m] \\
&+[\sum_{i=1}^{M-2}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-1}\sum_{m=l+1}^{M} \cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{ki}+d_{ik}d_{kj}d_{ji})(d_{lm}d_{ml})D_nD_o......D_tD_m \\
&+\sum_{i=1}^{M-4}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=i+1}^{M}\sum_{m=j+1}^{M} \cdots\cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{kl}d_{lm}d_{mi}+d_{im}d_{ml}d_{lk}d_{kj}d_{ji})D_nD_o......D_tD_m] \\
&+[\sum_{i=1}^{M-3}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=j+1}^{M}\sum_{m=1}^{M-1}\sum_{n=m+1}^{M} \cdots\cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{kl}d_{li}+d_{il}d_{lk}d_{kj}d_{ji})(d_{mn}d_{nm})D_o......D_tD_m \\
&+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=j+1}^{M}\sum_{l=1}^{M-2}\sum_{m=l+1}^{M-1}\sum_{n=m+1}^{M} \cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{ki}+d_{ik}d_{kj}d_{ji})(d_{lm}d_{mn}d_{nl}+d_{ln}d_{nm}d_{ml})D_o.......D_tD_m \qquad (2) \\
&+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M}\sum_{k=i+1}^{M-3}\sum_{l=i+2}^{M}\sum_{m=k+1}^{M-1}\sum_{n=k+2}^{M} \cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{ji})(d_{kl}d_{lk})(d_{mn}d_{nm})D_o.......D_tD_m \\
&+\sum_{i=1}^{M-5}\sum_{j=i+1}^{M-1}\sum_{k=i+1}^{M}\sum_{l=i+1}^{M}\sum_{m=i+1}^{M}\sum_{n=j+1}^{M} \cdots\cdots \sum_{M=t+1}^{M}(d_{ij}d_{jk}d_{kl}d_{lm}d_{mn}d_{ni}+d_{in}d_{nm}d_{ml}d_{lk}d_{kj}d_{ji})D_o.....D_tD_m] \\
&+\cdots\cdots\cdots
\end{aligned}
$$

As the permanent function contains only the positive terms, the higher values of Ri and aij will result in increased value of performance index. The values of Ri which are obtained from the experimental results are shown in Table 1.

The diagonal elements of the matrix B, Ri are quantitative and hence they are normalized in the range of 0 and 1. The normalized values of Ri are shown in Table 2.The off diagonal elements (relative importance) of the matrix B, aij are assigned between 0 and 10 on a qualitative scale as shown in Table 3. If a_{ij} represent the relative importance of the i^{th} attribute over the jth attribute, then the relative importance of the jth attribute over the ith attributeis evaluated using the Eq. (3).The relative importance between i,j and j,i is given as

aji = 1/aij (3)

The performance index for each experiment shown in Table 1 is evaluated using the Eq.(1) by substituting the values of Ri and aij. The performance index values for all experiments are arranged in descending order to rank them. The Exp. No. for which the value of performance index is highest is the optimal combination of operating parameters to obtain the best performance out of the engine.

From the Table 4, it is evident that the Exp. No. 25 has the highest value of index as calculated by all the MADMs and ranked no. 1 among all experiments. The same sort of approach and comparison was adopted by Paramasivam using AHP and ANP in selecting the milling machine [12].

III. CONCLUSION

The proposed mathematical model, Graph theory matrix approach was used to find the optimal combination of operating parameters of the diesel engine. It was found that the load of 18A load, 270bTDC injection timing and 200 bar injection pressure forms the optimal combination of operating parameters of the diesel engine. The results of GTMA were compared with other MADM methods like AHP, SAW and WPM. It was found that the Exp.No 25 was ranked 1 by all the MADMs. Graph theory matrix approach considers all of the criteria simultaneously and gives the correct and complete evaluation of the attributes. In this approach any number of quantitative andqualitative attributes can be considered. A small variation in the attributes values leads to a considerable difference in the result using matrix permanent method. This helps in better appreciation of the criteria and their relative importance. Graph theory matrix approach provides not only the analysis of the alternatives, but also enables the visualization of interrelations among the attributes using graph representation.

Table 3. Relative importance of performance attributes

Class description	Relative importance of attributes	
	aij	aij= 1/aij
Two attributes are equally important	1	1.0
One attribute is strongly more important over the other	2	0.5
One attribute is very strongly more important over the other	3	0.33
One attribute is extremely important over the other	4	0.25
One attribute is exceptionally more important over the other	5	0.2

Table 1: Experimental results

Exp No.	Operating Parameters			Performance Characteristics			Emission Characteristics				
	Load (A)	IT (^{0}bTDC)	IP (bar)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NO_x (ppm)	HC (ppm)	CO (%)	CO_2 (%)	O_2 (%)
1	9	19	200	2.322	0.310	27	242	36	0.03	3.3	15.98
2	9	19	220	2.264	0.304	28	381	35	0.02	3.4	15.92
3	9	19	240	2.301	0.299	28	375	44	0.02	1.2	17.62
4	9	23	200	2.322	0.296	29	237	31	0.03	3.5	15.89
5	9	23	220	2.273	0.303	28	313	35	0.02	3.4	15.92
6	9	23	240	2.301	0.299	28	518	44	0.02	2.1	17.62
7	9	27	200	2.304	0.299	28	475	31	0.03	3.5	15.89
8	9	27	220	2.264	0.304	28	481	35	0.02	3.4	15.92
9	9	27	240	2.301	0.299	28	518	44	0.02	1.3	17.62
10	13	19	200	3.392	0.254	33	320	34	0.04	4.0	15.23
11	13	19	220	3.269	0.249	34	472	32	0.02	4.1	15.11
12	13	19	240	3.330	0.232	36	423	38	0.02	4.0	15.19
13	13	23	200	3.377	0.248	34	281	28	0.03	4.1	14.97
14	13	23	220	3.295	0.247	34	347	32	0.02	4.1	15.11
15	13	23	240	3.330	0.232	36	598	38	0.02	4.0	15.19
16	13	27	200	3.326	0.252	34	618	28	0.03	4.1	14.97
17	13	27	220	3.269	0.249	34	633	32	0.02	4.1	15.11
18	13	27	240	3.330	0.232	36	598	38	0.02	4.0	15.19
19	18	19	200	4.328	0.238	36	422	34	0.04	4.6	14.08
20	18	19	220	4.240	0.221	38	526	34	0.02	4.8	14.04
21	18	19	240	4.287	0.219	39	485	32	0.02	4.8	14.000
22	18	23	200	4.328	0.247	34	313	33	0.03	5.2	13.29
23	18	23	220	4.264	0.220	38	385	34	0.02	4.8	14.04
24	18	23	240	4.287	0.219	40	613	32	0.02	4.8	14.00
25	18	27	200	4.284	0.233	36	771	30	0.03	4.9	13.3
26	18	27	220	4.240	0.203	42	756	33	0.02	4.4	14.9
27	18	27	240	4.287	0.206	41	669	34	0.02	4.1	14.23

Table 2. Normalized values

Exp No.	Operating Parameters			Normalized Performance and Emission Characteristics Values							
	Load (A)	IT (^{0}bTDC)	IP (bar)	BP (kW)	BSFC (kg/h kW)	BTE (%)	NO_x (ppm)	HC (ppm)	CO (%)	CO2 (%)	O2 (%)
1	9	19	200	0.54	1.00	0.64	0.31	0.82	0.75	0.36	0.83
2	9	19	220	0.52	0.98	0.67	0.49	0.80	0.50	0.35	0.83
3	9	19	240	0.53	0.96	0.67	0.49	1.00	0.50	1.00	0.75
4	9	23	200	0.54	0.95	0.69	0.31	0.70	0.75	0.34	0.84
5	9	23	220	0.53	0.98	0.67	0.41	0.80	0.50	0.35	0.83
6	9	23	240	0.53	0.96	0.67	0.67	1.00	0.50	0.57	0.75

7	9	27	200	0.53	0.96	0.67	0.62	0.70	0.75	0.34	0.84
8	9	27	220	0.52	0.98	0.67	0.62	0.80	0.50	0.35	0.83
9	9	27	240	0.53	0.96	0.67	0.67	1.00	0.50	0.92	0.75
10	13	19	200	0.78	0.82	0.79	0.42	0.77	1.00	0.30	0.87
11	13	19	220	0.76	0.80	0.81	0.61	0.73	0.50	0.29	0.88
12	13	19	240	0.77	0.75	0.86	0.55	0.86	0.50	0.30	0.87
13	13	23	200	0.78	0.80	0.81	0.36	0.64	0.75	0.29	0.89
14	13	23	220	0.76	0.80	0.81	0.45	0.73	0.50	0.29	0.88
15	13	23	240	0.77	0.75	0.86	0.78	0.86	0.50	0.30	0.87
16	13	27	200	0.77	0.81	0.81	0.80	0.64	0.75	0.29	0.89
17	13	27	220	0.76	0.80	0.81	0.82	0.73	0.50	0.29	0.88
18	13	27	240	0.77	0.75	0.86	0.78	0.86	0.50	0.30	0.87
19	18	19	200	1.00	0.77	0.86	0.55	0.77	1.00	0.26	0.94
20	18	19	220	0.98	0.71	0.90	0.68	0.77	0.50	0.25	0.95
21	18	19	240	0.99	0.71	0.93	0.63	0.73	0.50	0.25	0.95
22	18	23	200	1.00	0.80	0.81	0.41	0.75	0.75	0.23	1.00
23	18	23	220	0.99	0.71	0.90	0.50	0.77	0.50	0.25	0.95
24	18	23	240	0.99	0.71	0.95	0.80	0.73	0.50	0.25	0.95
25	18	27	200	0.99	0.75	0.86	1.00	0.68	0.75	0.24	1.00
26	18	27	220	0.98	0.65	1.00	0.98	0.75	0.50	0.27	0.89
27	18	27	240	0.99	0.66	0.98	0.87	0.77	0.50	0.29	0.93

Table 4: Performance index values

MADM Method											
GTMA			AHP			SAW			WPM		
Rank	Index	Exp. No	Rank	Index	Exp. No	Rank	Index	Exp. No	Rank	Index	Exp. No
1	58160	25	1	57190	25	1	8.535	25	1	0.057960	25
2	57390	19	2	56170	19	2	8.526	19	2	0.057280	19
3	56903	26	3	55138	9	3	8.518	26	3	0.056700	26
4	56523	9	4	55105	27	4	8.516	27	4	0.056500	27
5	56355	27	5	54717	26	5	8.508	24	5	0.056230	24
6	55843	3	6	54347	24	6	8.492	22	6	0.054930	21
7	55619	24	7	54338	3	7	8.491	21	7	0.054470	22
8	54866	16	8	53829	16	8	8.483	20	8	0.054240	20
9	54814	22	9	53575	10	9	8.478	23	9	0.053750	23
10	54803	10	10	53538	15	10	8.450	16	10	0.052220	16
11	54786	20	11	53464	20	11	8.447	10	11	0.051730	15
12	54511	15	12	53444	22	12	8.444	15	12	0.051730	18
13	54511	18	13	53276	18	13	8.444	18	13	0.051430	10
14	54433	21	14	53078	21	14	8.434	17	14	0.050910	17
15	54294	6	15	52979	6	15	8.424	12	15	0.050380	12
16	53845	17	16	52684	17	16	8.417	9	16	0.049780	11
17	53711	23	17	52286	23	17	8.416	11	17	0.049230	13
18	53085	12	18	51726	12	18	8.413	13	18	0.048640	14
19	52683	7	19	51602	7	19	8.408	3	19	0.045280	7
20	52569	11	20	51294	11	20	8.403	14	20	0.044990	6

21	52141	13	21	50835	13	21	8.391	6	21	0.044990	9
22	51867	8	22	50647	8	22	8.372	7	22	0.044220	8
23	51721	1	23	50587	4	23	8.361	8	23	0.04400	3
24	51597	14	24	50400	1	24	8.36	1	24	0.04343	4
25	51091	2	25	50234	14	25	8.350	2	25	0.04308	5
26	50917	4	26	49800	2	26	8.350	4	26	0.04300	1
27	50675	5	27	49336	5	27	8.346	5	27	0.04300	2

IV. REFERENCES

[1] O.P. Gandhi andV.P. Agrawal, FMEA – A digraph and matrix approach, Reliab engg syst saf, 35 (2002) 147-158.

[2] R. Venkatasamy and V.P. Agrawal, System and structural analysis of an automobile vehicle – a graph theoretic approach, J Veh Des, 16 (1997) 477-505.

[3] R.V. Rao, Graph theory and matrix approach for the performance evaluation of technical institutions, Indian J of Tech Educ, 23 (2) (2000) 27-33.

[4] M. Mohan, O.P. Gandhi and V.P. Agrawal, Systems modeling of a coal based steam power plant : a systems approach, In: Proceedings of the institution of mechanical engineers – Part A, Journal of power energy, 217 (2003) 259-277.

[5] S. Grover, V.P. Agrawal and I.A. Khan, A digraph approach to TQM evaluation of an industry, Int J Prod res, 42 (19) (2004) 4031-4053.

[6] R.T.D. Prabhakaran, B.J.C. Babu and V.P. Agrawal, Quality modeling and analysis of polymer composite products, Poly compos, 27 (2008) 329-340.

[7] M.N. Faisal, D.K. Banwet and R. Shankar, Information risks management in supply chains: an assessment and mitigation framework, J Entrp Inf Manag, 20 (6) (2007) 677-699.

[8] N. Upadhyay and V.P. Agrawal, Structural modeling and analysis of intelligent mobile learning environment a graph theoretic system approach, J Appl Quant Methods, 2 (2) (2007) 226-248.

[9] M. Darvish, M. Yasaei and A. Saeedi, Application of graph theory and matrix methods to contractor ranking, Intl J Proj Manag, 27 (2009) 610-619.

[10] A. Anand and M.F. Wani, Product life cycle modeling and evaluation at the conceptual design stage a digraph and matrix approach, J Mec Des,132 (9) (2010) DOI: 10.1115/1.4002241

[11] R.V. Rao, Decision making in the manufacturing environment: using graph theory and fuzzy multiple attribute decision making methods, Springer, London, (2007).

[12] V.Paramasivam. V.Senthil and N.Rajam Ramasamy, Decision making in equipment selection: an integrated approach with digraph and matrix approach, AHP and ANP, The international journal of manufacturing technology, 54 (2010) 1233-1244.

Nomenclature--

A : Ampere
BP : Brake power
BSFC : Brake specific fuel consumption
BTE : Brake thermal efficiency
CO : Carbon monoxide
CO2 : Carbon di oxide
GTMA: Graph theory matrix approach
HC : Hydro carbon
IP: Injection pressure
IT: Injection timing
NO : Nitric oxide
O2 : Oxygen
PI: Permanent index