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A Method for the Reduction Of Linear High Order MIMO Systems Using Interlacing Property and Factor Division Technique

G. Leela Prasad[1] | Ch. Vishnu Chakravarthi[2] | Dr.P.Mallikarjuna Rao[3]

[1]PG Student, Department of Electrical and Electronics Engineering, Sanketika Institute of Technology and Management, Visakhapatnam, Andhra Pradesh, India
[2]Assistant Professor, Department of Electrical and Electronics Engineering, Sanketika Institute of Technology and Management, Visakhapatnam, Andhra Pradesh, India
[3]Professor, Department of Electrical and Electronics Engineering, A U College of Engineering, Andhra University, Visakhapatnam, Andhra Pradesh, India

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ABSTRACT

This paper presents a new mixed method for the reduction of linear high order MIMO system. This method is based upon the interlacing property by which the denominator polynomial of the reduced order model is obtained and the numerator is obtained by using factor division method. In general, the stability of the high order system is retained in their models. Better approximation of the time response characteristics is attained by using this suggested method. The number of computations has been reduced when compared to several of the existing methods are in international literature. Another advantage of this method is that it is a direct method. The suggested procedure is digital computer oriented.

KEYWORDS: *Iterative in nature, Large Scale Systems, Order Reduction, Interlacing property & factor division Technique , MIMO Systems*

I. Introduction

Since recent years, Design, control and analysis of large scale systems is emerging as an essential area of research. Involvement of large number of variables in the high order system makes the design and analysis process computationally tedious. Majority of available analysis and design methods fail to give reasonable results when applied to large-scale advantageous systems. At this juncture the advantageous features of order reduction make the application of reduction procedures inevitable. The order reduction procedures are mainly classified in to either time domain category or frequency domain category. Basing on the simplicity and amicability the frequency domain dependent methods have become more prominent. Polynomial reduction methods are one of the important groups in the frequency domain category .The coefficients of the reduced order transfer functions are generated using various classical theories of mathematical approximations etc. like Routh approximation, Pade, Continued fraction approximation etc. The method "Routh approximations for reduction of LTI systems" Suggested by Hutton and fried land is one of the prominent methods in the Routh approximation group. "Model reduction by condensed continued fraction method" is given by T. N. Lucas. Shamash has suggested important methods using continued fraction expansion method. Unfortunately, very few methods are available for the reduction of Linear MIMO systems. Many of the methods available in the international literature can be easily extended for the reduced of linear MIMO (Multi input –Multi output systems).Earlier a method for the reduction MIMO methods was given by Bendekas et.at.,[5],which is mixed method of true moment matching and Pade approximates.Taiwoet.al.,[6]developed methods based upon pade approximated generalized time moments.Chen[7] has given a method based on matrix continued fraction technique .the Routh approximated method given by Huttom [1],was extended to MIMO systems by Sinha [8]

This paper suggests a new mixed method for the reduction of large scale linear systems. The denominator polynomial of the reduced order model is obtained by using pole clustering method and the numerator is obtained by matching the coefficients of higher order system with those of denominator of the reduced order model. In general, the stability of the higher order system is retained in their models addition to the matching of time response of system and its model this method is purely numerical and consist of non iterative procedure and can easily be implemented using a digital computer. Numerical examples are presented to show the effectiveness of proposed procedure.

II.PROBLEM FORMULATION

Consider an MIMO system, let it be

$$G(s) = \frac{1}{D_n(s)}\begin{bmatrix} g_{11(s)} & g_{12(s)} & g_{13(s)} & \cdots\cdots & g_{1j(s)} \\ g_{21(s)} & \cdots & \cdots\cdots & \cdots & g_{2j}(s) \\ . & . & \cdots & .. & . \\ . & \cdots & \cdots & . & . \\ g_{i1}(s) & \cdots & \cdots & \cdots & g_{ij}(s) \end{bmatrix}$$

Where

i =1,2, 3...........;j=1,2,3,.......

$$D_n(s) = \sum_{l=0}^{n}[A_l]S^l$$

$$g_{ij}(s) = \sum_{l=0}^{n-1}[B_{ij}]_l S^l$$

$$\therefore G_{ij}(s) = \frac{\sum_{l=0}^{n-1}[B_{ij}]_l S^l}{\sum_{l=0}^{n}[A_l]S^l}$$

The reduced order model can be represented as,

$$R_{ij}(s) = \frac{\sum_{l=0}^{K-1}[b_{ij}]_l S^l}{\sum_{l=0}^{k}[a_l]S^l}$$

Where

i= 1, 2, 3...K; j =1, 2, 3...K

Let the transfer function of a higher order linear time invariant stable plant is given by

$$G_{11}(S)=\frac{N(s)}{D(s)}=\frac{B_0+B_1S+B_2S^2+\ldots\ldots\ldots\ldots+B_nS^n}{A_0+A_1S+A_2S^2+\ldots\ldots\ldots\ldots+A_{n-1}S^{n-1}} \quad \ldots\ldots\ldots\ldots(1)$$

The Denominator is polynomial of order n and numerator is polynomial of order (n-1).the aim is to obtain the kth order reduced order model. The respective procedural steps are as follows:

Step1: The Denominator polynomial is separated into even and odd parts.

For n is even:

$$D_n^{even}(S) = A_0 + A_2 s^2 + A_4 s^4 + \ldots\ldots + A_n s^n$$

$$\frac{D_n^{odd}(s)}{s} = A_1 + A_3 s^2 + A_5 s^4 + \ldots\ldots\ldots\ldots + A_{(n-1)} s^{(n-2)} \quad \ldots\ldots \quad (2).$$

For n is odd:

$$D(S) = A_0 + A_2 s^2 + A_4 s^4 + \ldots\ldots + A_{(n-1)} s^{(n-1)}$$

$$\frac{D^{odd}(S)}{S} = A_1 + A_3 s^2 + A_5 s^4 + \ldots\ldots\ldots\ldots + A_{(n)} s^{(n-1)} \quad \ldots\ldots\ldots \quad (3)$$

Let $(0 \pm \omega^d_{e,i})$ and $(0 \pm \omega^d_{o,i})$ denotes the roots of $D^{even}(S)$ and $\frac{D^{odd}(S)}{S}$ respectively. Then, it can be observed that

$$0 < \omega^e_{d,1} < \omega^o_{d,1} < \omega^e_{d,2} < \omega^o_{d,2} < \omega^e_{d,3} \ldots\ldots \quad (4)$$

Step2:

The Denominator polynomial of the kth order reduced order model is obtained as,

The even and odd polynomial for the reduced order polynomial can be written as,

For k is even:

$D_k^{even}(S)=(s^2+\omega^2_{e,1})(s^2+\omega^2_{e,2})$.........................$(s^2+\omega^2_{e,(k/2)})$ (5)

$\frac{D_k^{odd}(S)}{S}=(s^2+\omega^2_{o,1})(s^2+\omega^2_{o,2})$......................$(s^2+\omega^2_{o,(k/2)-1})$ .... (6)

For k is odd:

$D_k^{even}(S)=(s^2+\omega^2_{e,1})(s^2+\omega^2_{e,2})$.........................$(s^2+\omega^2_{e,(k-1)/2})$ .... (7)

$\frac{D_k^{odd}(S)}{S}=(s^2+\omega^2_{o,1})(s^2+\omega^2_{o,2})$.................$(s^2+\omega^2_{o,(k-1)/2})$ .... (8)

Obtain two constants namely I_1, I_2 by matching amplitude at $\omega=0$

$I_1=\frac{|D_n^{even}(S)|}{|D_k^{even}(S)|}$ at $\omega=0$

$I_2=\frac{|D_n^{odd}(S)|}{|D_k^{odd}(S)|}$ at $\omega=0$ ... (9)

Modified reduced denominators are

$D_m^{even}(S) = I_1 * D_k^{even}(S)$

$D_m^{odd}(S) = I_2 * D_k^{odd}(S)$ .... (10)

Now

$D_k(S) = D_m^{even}(S) + D_m^{odd}(S)$

Step3: The Numerator polynomial

$N_k(S) = \sum_{l=0}^{k-1}[a_{ij}]_l S^l$

of the reduced order model is obtained by using eqn.

$$b_{ij}=\frac{B_{ij}}{A_i}a_i \quad \left.\begin{matrix}\text{for } 0\le i\le k-1\\ \text{for } 0\le j\le k-1\end{matrix}\right\} \quad -(11)$$

Step4: Finally the k^th order reduced model is obtained as

$$R_k(s)=\frac{1}{D_k(s)}\begin{bmatrix} q_{11_{k-1}}(s) & q_{12_{k-1}}(S) & q_{13_{k-1}}(s) & \cdots & q_{1j_{k-1}}(s)\\ q_{21_{k-1}}(s) & \cdots & \cdots & \cdots & q_{2j_{k-1}}(s)\\ . & . & \cdots & .. & .\\ . & \cdots & \cdots & . & .\\ q_{i1_{k-1}}(s) & \cdots & \cdots & \cdots & q_{ij_{k-1}}(s)\end{bmatrix}$$

-(12)

Where

i= 1, 2, .k-1...; j =1, 2,.k-1

$$R_k(s)=\frac{N_k(s)}{D_k(s)}$$

Step5: similarly the remaining reduced order systems is obtained by using above steps .the reduced order model can be represented as,

$$R_{ij}(s)=\frac{\sum_{l=0}^{K-1}[b_{ij}]_l S^l}{\sum_{l=0}^{k}[a_l]S^l}$$

III. Numerical Example

EXAMPLE.1

Let the higher order MIMO system as

$$G(s)=\frac{1}{D(s)}[g_{11} \quad g_{12}]$$

$$G_{11}=\frac{2s^5+70s^4+763s^3+3611s^2+7700s+6000}{2s^6+41s^5+571s^4+3492s^3+10061s^2+13100s+6000}$$

$$G_{12}=\frac{3s^5+72s^4+762s^3+3610s^2+7701s+6000}{2s^6+41s^5+571s^4+3492s^3+10061s^2+13100s+6000}$$

The system will be reduced to k=4, The denominator polynomial of the 4th order reduced model is obtained by using the suggested procedure

$$D_4(s)=533.347499s^4+3330.706059s^3+10038.34662s^2+13100s+6000$$

The numerator polynomial is obtained using the suggested procedure as,

$$N_{41}(s)=622.0931104s^3+3588.346667s^2+7700s+6000$$

$$N_{42}(s)=621.08823s^3+3587.347333s^2+7701s+6000$$

The obtained reduced MIMO transfer function are

$$R_{41}=\frac{622.0931104s^3+3588.346667s^2+7700s+6000}{533.347499s^4+3330.706059s^3+10038.346662s^2+13100s+6000}$$

$$R_{42} = \frac{621.08823s^3 + 3587.347333s^2 + 7700s + 6000}{533.347499s^4 + 3330.706059s^3 + 10038.34662s^2 + 13100s + 6000}$$

The obtained reduced MIMO transfer function using least square method is

$$R_{21} = \frac{2.092549s + 4.507090}{s^2 + 6.148930s + 4.507090}$$

$$R_{22} = \frac{2.050627s + 7.694403}{s^2 + 8.975589s + 7.694403}$$

The step responses of original MIMO high order system and fourth reduced MIMO models obtained using the proposed method are compared in Fig 1.1 and 1.2

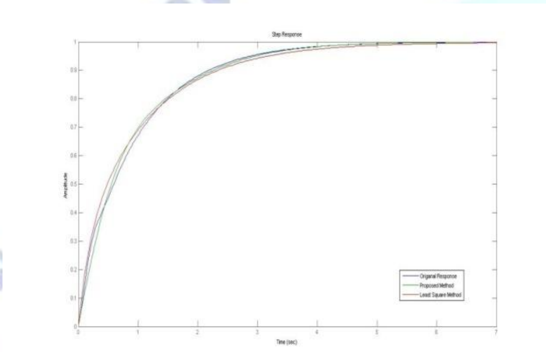

Fig.6.1.the step responses of original order and reduced order systems (1st output)

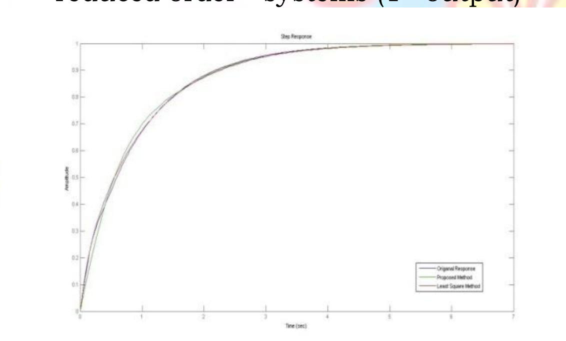

Fig.1.2.the step responses of original order and reduced order systems (1st output)

Table.1 Comparative analysis of step response of original system, methods used in literature and proposed models for example.1

Methods \ Specifications	Tr	Ts	Tp	Peak
G_{11}	0.0365	5.02	4.12	0.986
G_{12}	0.0509	5.02	5.02	0.994
R_{41}	0.0338	4.71	3.58	0.97
R_{42}	0.0431	4.95	4.98	0.994
R_{21}	0.036	5.02	4.1	0.977
R_{22}	0.0451	5.0	5.02	0.994

(Note: Tr-Rise Time; Ts-Settling Time; Tp-Peak Time)

IV.Conclusions

In this paper a new method has been proposed for the reduction of Linear high order MIMO system. This method is simple and efficient and gives a stable reduced order model for a stable high order MIMO system. The reliability & advantages of the proposed method have been illustrated by the examples. This new procedure is dependent on the interlacing property of systems and matching of coefficients of high order system and reduced order model. The denominator of the reduced order model is obtained by using the interlacing property and hence retention of the stability of original system in their reduced morel is guaranteed. The matching of coefficients gives better approximation of the time responses of original system and model. This method is digital computer oriented.

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VI.References

[1] Dr.P.MallikarjunaRao,"A Method for the reduction of MIMO system using interlacing Property and Coefficients Mtching", IJCA, Vol.1-No.9, 2010, (0975-8887).

[2] Hutton ,M.F and Friedland,B., "Routh Approximation for reducing order of linear time invariant systems", IEEE Trans. On Automatic Control, Vol.20, 1975, PP.329-337.

[3] Shieh,L.S and Goldman ,M.J, "Continued Fraction Expansion and Inversion of Cauer Third form", IEEE Trans. Circuits and Systems, May 1994, PP.341-345.

[4] Pal.J, "Stable Reduced order Pade Approximates using the Routh- Hurwitz Array", Electronic letters, Vol.15,No.8, April 1979, PP.225-226.

[5] Shamash, Y. "Linear System Reduction by Pade Approximation to allow retention of Dominant modes", International Journal of Control, Vol.21, 1975 PP.258-272.

[6] Bendekal DV.Papadopoulos D.P,"The Moment and Pade approximate method Applied to the order reduction of MIMO Linear system", Journal of Franklin. Vol .329.No.3, 1992.

[7] Taiwo O ;"the simplification of Multivariable padeaapproximation",International conference on control and Modeling Tehron , Iran,1990

[8] chen C.F.,"model reduction of multi variable Control systems by mixed continued Fraction", International journal of control, Vol.20, No.2, 1974.

[9] Sinha .N.K "reduction of multivariable system",Electrica letter .Vol.16,No.20,1980

[10] MAURICE F.HUTTON AND BERNARD FRIEDLAND,"Routh Approximations for Reducing order of Linear. Time- Invariant systems"IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL.AC-20, NO.3, JUNE1975.